ABSTRACT

Radiatively induced fermion resonance (RFR) is the resonance equivalent of the inverse Faraday effect (IFE), which is the magnetization of matter by circularly polarized radiation. The effect of gravitation on RFR is developed in this paper by considering the spin connection of Einstein Cartan Evans (ECE) field theory to be approximately dual to the tetrad. At spin connection resonance (SCR) the effect of gravitation is amplified, so the resulting gravitational shift in resonance frequency of RFR may become measurable. Similar considerations apply to all other forms of resonant spectroscopy.

Keywords: Radiatively induced fermion resonance (RFR), inverse Faraday effect (IFE), Einstein Cartan Evans (ECE) field theory, spin connection resonance (SCR), gravitationally induced shifts in atomic and molecular spectroscopy.
1. INTRODUCTION

The inverse Faraday effect (IFE) is well known to be the bulk magnetization of matter by circularly or elliptically polarized radiation. It is observable at all frequencies in all materials, and on the simplest level, one electron. The resonant equivalent of IFE is radiatively induced fermion resonance (RFR), which is fermion resonance induced by a circularly polarized electromagnetic field. Analogously, the resonant equivalent of bulk magnetization by a magnet (static magnetic field) is ESR or NMR. The resolution of RFR is much higher than ESR or NMR, and the RFR technique has a characteristic chemical shift pattern different from that of ESR or NMR. In this paper, Einstein Cartan Evans (ECE) field theory \(1-10\) is used to investigate the effect of gravitation on RFR and on atomic and molecular resonance spectra in general. In Section 2, the general theory is reviewed, particularly in respect to the ECE spin field. In Section 3, levels of approximation are described in the solution of the basic RFR equations, and in Section 4, spin connection resonance (SCR) is described as a possible means of amplifying gravitational induced spectral shifts so that they become observable in the laboratory.

2. GENERAL THEORY

Using indexless notation \(1-10\) for clarity of concepts, the effect of gravitation may be measured through the equations:

\[
\mathbf{F} = \mathbf{d} \wedge \mathbf{A} + \omega \wedge \mathbf{A} \quad - (1)
\]

and

\[
\mathbf{d} \wedge \mathbf{F} = \mu_0 \mathbf{j} \quad - (2)
\]

where
\[ j = \frac{A^{(s)}}{\mu_0} (R \wedge q_v - \omega \wedge T) \]  

and

\[ A = \hat{A}^{(s)} \hat{q}_v, \]

\[ F = \hat{A}^{(s)} \hat{T}. \]

Here, the various differential forms are expressed without their indices \{1-10\} so that the basic structure of the equations is revealed the most clearly. They are as follows: $F$ is the electromagnetic field, $A$ is the electromagnetic potential, $\hat{q}_v$ is the spin connection, $j$ is the homogeneous current, $R$ is the curvature, $\omega$ is the tetrad, and $T$ is the torsion. In these equations $cA$ is the primordial voltage, and $\mu_0$ is the vacuum permeability in the S.I. System of units. \{11\}.

The spin connection resonance (SCR) equation is, from Eqs. (1) and (2):

\[ d \wedge (d \wedge A + \omega \wedge A) = \mu_0 j = \hat{A}^{(s)} (R \wedge q_v - \omega \wedge T). \]  

The effect of gravitation on the electromagnetic field is governed by $j$, the homogeneous current of ECE theory. If there is no effect:

\[ j = 0 \]  

and

\[ d \wedge (d \wedge A + \omega \wedge A) = 0. \]

In this case, translational and rotational motions are independent. The translational motion governs the gravitational field and the rotational motion governs the electromagnetic field.
Einstein Hilbert (EH) field theory is governed by translational motion defined through the following Cartan geometry:

\[ R \wedge \sigma_V = 0, \quad - (q) \]
\[ T = 0, \quad - (r) \]
\[ D \wedge R = 0. \quad - (s) \]

Eq. (q) is the Ricci cyclic equation, and Eq. (r) is the second Bianchi identity. The well-known EH field equation is obtained from the second Bianchi identity (r) and Noether’s Theorem. As can be seen from Eq. (t) there is no Cartan torsion \( T \) in the EH theory. From Eqs. (q) and (t):

\[ j = 0 \quad - (12) \]

self-consistently, because EH is a theory of gravitation upon which there is no electromagnetic influence. Note that if \( \tilde{R} \) be the Hodge dual \( (1-10) \) of \( R \), then:

\[ \tilde{R} \wedge \sigma_V \neq 0. \quad - (13) \]

The rotational motion defines the electromagnetic field by:

\[ d \wedge F = 0 \quad - (14) \]

and its Hodge dual:

\[ d \wedge \tilde{F} = \mu_0 J \quad - (15) \]

where:

\[ J = \frac{e^{\nu}}{\mu_0} \left( \tilde{R} \wedge \sigma_V - \sigma \wedge \tilde{T} \right) \quad - (16) \]

is the inhomogeneous current of ECE theory. For rotational motion:
\[ R \wedge \sigma = \omega \wedge T, \quad -(17) \]
\[ \widetilde{R} \wedge \sigma = \omega \wedge \tilde{T}, \quad -(18) \]

and
\[ j_{\text{rotation}} = \widetilde{j}_{\text{rotation}} = 0 \quad -(19) \]

but from Eq. (13)
\[ j = A^{(o)} \left( \frac{R \wedge \sigma}{\mu} \right)_{\text{rotation}} \neq 0 \quad -(20) \]

Eqs. (14) and (15) give the ECE laws (1-10) of electrodynamics unaffected by gravitation. Eqs. (17) and (18) indicate that for pure rotational motion the rotational curvature R is the dual of the Cartan torsion T, and the tetrad is the dual of the spin connection (1-10).

From Eqs. (6) and (8) it is seen that the influence of gravitation on electromagnetism is to change \( \omega \) and through the presence of \( j \). This change also introduces the possibility of SCR through Eq. (6) (1-10) and its Hodge dual. If the effect of gravitation is very weak, (as in the laboratory), then, for rotational motion, \( \omega \) is dual to A in eq. (6) to an excellent approximation. We may thus consider the effect of gravitation to be a change in A produced by \( j \). The ECE spin field is defined by (1-10):
\[ B^{(s)} = -i j A \wedge A^* \quad -(21) \]

where \( A^* \) is the complex conjugate of A, and where:
\[ j = \frac{\nu}{A^{(o)}} \quad -(22) \]
where $\kappa$ is, in free space, a wave-number. Thus $B$ (switching to vector notation) is changed by gravitation, and the RFR resonance frequency is shifted by gravitation. At resonance from Eq. (6) it is seen that this RFR shift is greatly amplified, so may become measurable in the laboratory. Similar considerations apply for all types of atomic and molecular spectroscopy. We may also bring into consideration quantum electrodynamics (QED) through the ECE Lemma applied to $A$:

$$\Box A = RA - (2\gamma)$$

where:

$$R = -eT.$$ 

Here $\Box$ is the d’Alembertian, $R$ is the scalar curvature, $k$ is Einstein’s constant and $T$ is the index contracted canonical energy-momentum density. The latter is contained in general contributions from all fields and interaction terms.

3. LEVELS OF APPROXIMATION IN IFE AND RFR.

In general relativity both effects originate in the $\omega \wedge A$ term of:

$$F = dA + \omega \wedge A - (2\gamma)$$

and there are various levels of approximation that can be used to evaluate $\omega \wedge A$:

classical, semi-classical, special relativistic QED, and general relativistic ECE. For rigorously objective physics \{1-10\} general relativity must be applied to all equations and concepts without exception. This is the basic ECE philosophy needed to produce a generally covariant unified field theory. For electromagnetism free of gravitational influence the spin connection is dual to the potential, defining the ECE spin field. The IFE and RFR follow directly from

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the spin field, which in vector notation in the complex circular basis \{1-10\} is defined as follows:

\[
\underline{B}^{(3)} = -i \gamma B^{(1)} \times \underline{A}^{(2)} - (26)
\]

The IFE is magnetization due to the radiated spin field \(\underline{B}\), and RFR is resonance due to \(\underline{B}\). Analogously, bulk magnetization is due to a static magnetic field, and ESR and NMR are resonance phenomena due to a static magnetic field. In free space (no interaction with matter such as an electron), the spin field is defined \{1-10\} by:

\[
\underline{B}^{(3)} = B^{(0)} \underline{p} = \kappa A^{(0)} \underline{p} - (27)
\]

where:

\[
\gamma = \frac{\kappa}{\mu_0}.
\]

(28)

The conjugate product of non-linear optics \{1-10\} is defined by:

\[
A^{(1)} \times A^{(2)} = A \times A^{*} - (29)
\]

where \(A^{*}\) is the complex conjugate of \(A\). When \(\underline{B}\) interacts with matter, on the simplest level an electron, then:

\[
\gamma \rightarrow \gamma' - (30)
\]

where \(\gamma'\) is to be determined as follows from dynamics. The IFE is the magnetization:

\[
M^{(3)} = \frac{1}{\mu_0} B^{(3)} - i \gamma' A^{(1)} \times A^{(2)}
\]

\[
= \gamma' A^{(0)} \underline{p} - (31)
\]
The factor \( A \) can be related to the power density \( P \) (watts per square meter) of the electromagnetic field using the standard optical equation \( (12) \):

\[
A_\text{opt} = \frac{\mu_0}{c} \left( \frac{P}{\omega^2} \right).
\]

Therefore:

\[
\mathbf{m}_\text{opt} = \frac{\mathbf{g}'}{c} \left( \frac{P}{\omega^2} \right) \mathbf{p}.
\]

The factor \( \mathbf{g}' \) must also be calculated from general relativity in a fully self-consistent development. It must be calculated from the ECE wave equation in the presence of interaction between the electromagnetic field and one electron \( (1-10) \):

\[
\left( \gamma^\alpha \left( i \mathbf{\gamma}^\mu \mathbf{\nabla}_\mu - e \mathbf{A}_\alpha \right) - mc \right) q_\text{e} c = 0 \quad (34)
\]

where \( \gamma^\alpha \) is the Dirac matrix, \( \mathbf{\gamma}^\mu \) is the reduced Planck constant, \( e \) is the charge on the electron, \( m \) is the mass of the electron, \( c \) is the vacuum speed of light and \( q_\text{e} \) is the tetrad.

In the absence of interaction between the fermion and the gravitational field this equation reduces to the well known Dirac equation \( (1 - 10) \):

\[
\left( \gamma^\mu \left( i \mathbf{\gamma}^\nu \mathbf{\nabla}_\nu - e \mathbf{A}_\mu \right) - mc \right) \psi = 0 \quad (35)
\]

with the minimal prescription used to describe the interaction between the free fermion and the electromagnetic potential. This equation is well known to be the basis of QED and to successfully describe the Zeeman effect through the half integral spin of the fermion. It is the basis of ESX and NMR. In the non-relativistic quantum limit Eq. \( (35) \) reduces to the Schrödinger-Pauli equation:
\[ H \Phi = E \Phi \]  

(36)

in which the Hamiltonian is described using the Pauli matrices:

\[ H = \frac{1}{2m} \sigma \cdot \left( \frac{p + eA}{\hbar} \right) \sigma \cdot \left( \frac{p + eA^*}{\hbar} \right) + V. \]  

(37)

Here \( p \) is the classical momentum, \( A \) is the classical vector potential, \( \sigma \) denotes a Pauli matrix and \( V \) denotes the potential energy. For a static magnetic field \( (1-10) \):

\[ H \Phi = \frac{eB}{2m} \sigma \cdot B \Phi. \]  

(38)

and for an electromagnetic field:

\[ H \Phi = \frac{m^* e^2}{2m} \left( \frac{I}{\omega^2} \right) \sigma_2 \Phi. \]  

(39)

Electron spin resonance (ESR) is described by:

\[ \frac{\hbar}{\omega_{\text{res}}} = \frac{eB}{2m} \left( 1 - (-1) \right) B. \]  

(40)

with resonance angular frequency:

\[ \omega_{\text{res}} = \frac{e}{m} B. \]  

(41)

RFR is described by \( (1-10) \):

\[ \frac{\hbar}{\omega_{\text{res}}^2} = \frac{m^* e^2}{2m} \left( \frac{I}{\omega^2} \right) \left( 1 - (-1) \right) \]  

(42)

with resonance angular frequency:
\[ \omega_{\text{los}} = \left( \frac{m_{0}c^{2}}{\pm m} \right) \frac{1}{\omega^{2}}. \quad (4.3) \]

The IFE can be approximated by classical special relativistic limit of eq. (33), which is the special relativistic Hamilton Jacobi equation [1-10]:

\[ (\not{p} - e\not{A}) \left( \not{p} - e\not{A}' \right) = m^{2}c^{2} \quad (4.4) \]

The solution of Eq. (4.4) for N electrons in a sample volume V is:

\[ \frac{B}{\text{sample}} = \frac{N}{\sqrt{V}} \frac{\mu_{0}e^{3}c^{2}}{2m\omega^{2}} \left( \frac{\not{B}(0)}{(m^{2}c^{2} + e^{2}B(0))^{1/2}} \right) \frac{\not{B}}{\omega^{3}} \quad (4.5) \]

In the limit:

\[ m_{0} \gg eB(0) \quad (4.6) \]

we obtain:

\[ \frac{B}{\text{sample}} = \mu_{0} \frac{M_{\text{sample}}}{V} \rightarrow \frac{N}{\sqrt{V}} \left( \frac{\mu_{0}e^{3}c}{2m^{2}} \right) \frac{1}{\omega^{2}} \frac{B}{\omega^{3}} \quad (4.6) \]

Comparing equations (33) and (4.6):

\[ g' = \frac{N}{\sqrt{V}} \frac{e^{3}c^{2}}{2m^{2}} \quad (4.7) \]

in this approximation.
4. SPIN CONNECTION RESONANCE.

In the presence of gravitation the key resonance equation is:

\[ d \wedge (d \wedge A + \omega \wedge A) = \mu_0 j \]  \hspace{1cm} (4.8)

In the off-resonant condition the effect of gravitation in the laboratory is very small, so to an excellent approximation, and for rotational motion (1-10):

\[ \omega \wedge = - g \vec{A} \]  \hspace{1cm} (4.9)

Thus Eq. (4.8) becomes:

\[ d \wedge (d \wedge A - i g \vec{A} \wedge A^*) = \mu_0 j \]  \hspace{1cm} (5.0)

For circularly polarized radiation in free space (1-10);

\[ d \wedge \vec{A} = - i g \vec{A} \wedge A^* \]  \hspace{1cm} (5.1)

so:

\[ d \wedge (d \wedge \vec{A}) = \frac{\mu_0}{2} j \]  \hspace{1cm} (5.2)

Therefore are equations such as:

\[ d \wedge B^{(3)} = \frac{\mu_0}{2} j \]  \hspace{1cm} (5.3)

The effect of gravitation is to make \( \vec{B} \) space and time dependent, for example to make it precess in a cone as follows. Considering the space part of Eq. (5.3):

\[ \nabla \times \vec{B}^{(3)} = \frac{\mu_0}{2} j \]  \hspace{1cm} (5.4)
produces the precessional equations:
\[
\begin{align*}
\frac{d B_z^{(3)}}{d t} - \frac{d B_y^{(3)}}{d t} &= \frac{\mu_0}{2} j_x, \quad - (55) \\
\frac{d B_x^{(3)}}{d t} - \frac{d B_z^{(3)}}{d t} &= \frac{\mu_0}{2} j_y, \quad - (56) \\
\frac{d B_y^{(3)}}{d t} - \frac{d B_x^{(3)}}{d t} &= \frac{\mu_0}{2} j_z. \quad - (57)
\end{align*}
\]

Resonance equations can be derived from Eqs. (55) to (57) as follows. At resonance, the B field would be greatly amplified, meaning that the RFR line would be shifted enough by gravitation for the shift to become measurable in the laboratory. This might lead to a practical way of measuring the effect of gravitation on a spectrum in the laboratory. At present this is only possible by astronomy (red shifts for example).

Differentiating Eq. (55):
\[
\frac{d^2 B_z^{(3)}}{d y^2} - \frac{d}{d y} \left( \frac{d B_y^{(3)}}{d z} \right) = \frac{\mu_0}{2} \frac{d j_x}{d y}. \quad - (58)
\]

This becomes a resonance equation under the mathematical conditions:
\[
\frac{d}{d y} \left( \frac{d B_y^{(3)}}{d z} \right) = - \kappa_0^2 B_z^{(3)} \quad - (59)
\]
i.e.: \[ B_y^{(3)} = - \kappa_0 I_s \int B_z^{(3)} d z \, d y \quad - (60) \]
and \[ \frac{d j_x}{d y} = I_s \cos (\kappa_0 y) \quad - (61) \]
under which Eq. (58) becomes the undamped oscillator equation.

\[
\frac{d^2 b_z^{(5)}}{d\gamma^2} + \kappa_0^2 b_z^{(5)} = \mu z \int \frac{1}{2} \cos \left( \kappa Y \right) . \tag{62}
\]

At SCR from this equation, \( b_z \) is greatly amplified.
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