THE OBJECTIVE LAWS OF CLASSICAL ELECTRODYNAMICS: 
THE EFFECT OF GRAVITATION ON ELECTROMAGNETISM.

by

Myron W. Evans.

Alpha Institute for Advanced Study (AIAS),
emyrone@aol.com.

ABSTRACT

The four fundamental laws of classical electrodynamics are given in generally covariant form using the principles of differential geometry. In so doing it becomes possible to analyze in detail the effect of gravitation on electromagnetism. This development completes Einstein's generally covariant field theory of gravitation and shows that there is present in nature a source of electric power from the general four dimensional manifold. It is also shown that an electromagnetic field can influence gravitation, and there are major implications for power engineering and aerospace industries.

Keywords: generally covariant classical electrodynamics; Evans field theory; electric power from spacetime; the interaction of gravitation and electromagnetism.
1. INTRODUCTION

In order that physics be an objective subject it must be a theory of general relativity, in which ALL the equations of physics must be generally covariant. This is a well known and well accepted principle of natural philosophy first proposed by Einstein (1) who based his development on the philosophical ideas of Mach. Without this most fundamental principle there can be no objective knowledge (or science) of nature. However, the contemporary standard model does not conform correctly to this principle, because only gravitation is treated objectively. Classical electrodynamics in the standard model is a theory of special relativity, covariant only under the Lorentz transform (2-4) and unobjective under any other type of coordinate transformation. In other words electrodynamics in the standard model in general means different things to different observers. This is fundamentally unacceptable to natural philosophy and science, the objective observation of nature. In science, nature is objective to all observers, and if not we have no science (from the Latin word for “knowledge”). Furthermore the field theories of gravitation and electromagnetism in the standard model are conceptually different (4). Gravitation is essentially a special case of Riemann geometry within Einstein’s constant k, electromagnetism is a distinct, abstract, entity superimposed on the Minkowski (“flat”) spacetime. It is well known that the origins of contemporary classical electrodynamics go back to the eighteenth century, to an era when space and time were also considered as distinct philosophical entities, not yet unified into spacetime. The contemporary standard model is still based on this mixture of concepts and is the result of history rather than reason.

In order to unify electromagnetism and gravitation in a correctly objective manner, it has been shown recently (5-36) that physics must be developed in a four dimensional manifold defined by the well known principles of differential geometry, notably the two Maurer Cartan structure equations, the two Bianchi identities, and the tetrad postulate (3).
The Einstein field theory of gravitation is essentially a special case of differential geometry, and electromagnetism is described by the first Bianchi identity within a fundamental voltage. Gravitation and electromagnetism are unified naturally by the structure of differential geometry itself. This means that one type of field can influence the other, leading to the possibility of new technology as well as being a major philosophical advance. In the last analysis gravitation and electromagnetism are different manifestations of the same thing, geometry. This is hardly a new idea in physics, but the Evans theory \{5-36\} is the first correct unified field theory to be based on well accepted Einsteinian principles.

In this paper the four laws of classical electrodynamics are developed in correctly covariant form from the first Bianchi identity of differential geometry. In the standard model these four laws together constitute the Maxwell Heaviside theory of the electromagnetic sector. The standard model is fundamentally or qualitatively unable to analyze the important effects of gravitation on electrodynamics because the two fields are treated differently as described already. String theory makes matters worse by the introduction of adjustable mathematical parameters known optimistically as “dimensions”. These have no physical significance and this basic and irremedial flaw in string theory originated a few years after the Einstein theory of 1916 in the fundamentally incorrect introduction of an unphysical fifth dimension in an attempt to unify gravitation with electromagnetism. The Evans field theory \{5-35\} achieves this aim by correctly using only the four physical dimensions of relativity, the four dimensions of spacetime. String theory should therefore be abandoned in favor of the simpler and much more powerful Evans field theory, which is the direct and logical outcome of Einstein’s own work.

In Section 2 the correctly objective laws of classical electrodynamics are developed straightforwardly from the first Bianchi identity. The objective form of the Gauss law applied to magnetism and of the Faraday law of induction is obtained from the Bianchi identity itself,
and the objective form of the Coulomb law and Ampere Maxwell law is obtained from the appropriate Hodge duals used in the Bianchi identity. Therefore all four laws become a direct consequence of the first Bianchi identity of differential geometry. Within a scalar $A$ with the units of volt s/m the electromagnetic field is the torsion form and the electromagnetic potential is the tetrad form. In Section 3 a discussion is given of some of the major consequences of these objective laws of classical electrodynamics.

2. THE OBJECTIVE LAWS OF CLASSICAL ELECTRODYNAMICS.

The first Bianchi identity of differential geometry is well known to be [3]:

$$ D \wedge T^a = R^a \wedge \omega^b. \quad - (1) $$

Here $D \wedge$ denotes the covariant exterior derivative, $d \wedge$ is the exterior derivative, $T^a$ is the torsion form and $R^a \wedge \omega$ is the curvature form, also known as the Riemann form. The covariant exterior derivative is defined [3] as:

$$ D \wedge T^a = d \wedge T^a + \omega^a \wedge T^b. \quad - (2) $$

where $\omega^a \wedge \omega$ is the spin connection of differential geometry. As is customary in differential geometry [3] the indices of the base manifold are suppressed (not written out), because they are always the same on both sides of any equation of differential geometry. Therefore only the indices of the tangent bundle are given in Eq. (1). The first Bianchi identity is therefore:

$$ d \wedge T^a = - (\omega^b \wedge R^a \wedge \omega^c \wedge T^b) \quad - (3) $$
which implies the existence of the base manifold indices as follows:

\[ d \wedge T_{\mu \nu}^a = - \left( \omega^b_{\nu} \wedge R^{a \mu}_{\nu \lambda} + \omega^a_{\nu} \wedge T^b_{\mu \lambda} \right) \] 

The basic axiom of differential geometry is that in a given base manifold there is a tangent bundle to that base manifold at a given point \(3\). The tangent bundle was not used by Einstein in his field theory of gravitation, Einstein considered and needed only the restricted base manifold geometry defined by the Christoffel symbol and metric compatibility condition \(3\). These considerations of Einstein were sufficient to describe gravitation, but not to unify gravitation with electromagnetism. No one knew this better than Einstein himself, who spent thirty years (1925 - 1955) in attempting objective field unification in various ways.

The Bianchi identity \(3\) becomes the equations of electrodynamics using the following fundamental rules or laws:

\[ A^a_{\mu} = \mathcal{A}^{(s)}_{\mu} \]  
\[ F^a_{\mu \nu} = \mathcal{A}^{(s)}_{\mu} T^a_{\nu} \]  

defining the electromagnetic potential \(\mathcal{A}^a_{\mu}\), and the electromagnetic field \(F^a_{\mu \nu}\). These appellations are used only out of habit, because both \(A^a_{\mu}\) and \(F^a_{\mu \nu}\) have now become parts of the unified field, i.e. of electromagnetism influenced by gravitation (or vice versa). The homogeneous field equation of the Evans field theory (HE equation) is therefore:

\[ d \wedge F^a = - \mathcal{A}^{(s)} \left( \omega^b_{\nu} \wedge R^{a \mu}_{\nu \lambda} + \omega^a_{\nu} \wedge T^b_{\mu \lambda} \right) \] 

and the homogeneous current of the HE is:
\[ j^a = -A^{(\nu)}_{\mu} \left( \gamma^b \wedge R^c_{\nu} + \omega^c_{\nu} \wedge \gamma^b \right) \]  

(8)

When this current vanishes the HE becomes:

\[ d \wedge F^a = 0 \]  

(9)

and is for each index the homogeneous field equation of the Maxwell Heaviside theory:

\[ d \wedge F = 0 \]  

(10)

Equation (10) is a combination in differential form notation \( \{3\} \) of the Gauss law applied to magnetism:

\[ \nabla \cdot B = 0 \]  

(11)

and of the Faraday law of induction:

\[ \nabla \times E + \frac{\partial B}{\partial t} = 0 \]  

(12)

These two laws are well tested experimentally so the homogeneous current must be very small or zero within contemporary instrumental precision. These experimental considerations define the “free space condition”:

\[ R^c_{\nu} \wedge \gamma^b = \omega^c_{\nu} \wedge \gamma^b \]  

(13)

In general relativity however the homogeneous current may be different from zero, and so
general relativity means that the Gauss law and Faraday induction law are special cases of a more general theory. This is the objective theory given by the HE. Similarly it will be shown that the Coulomb and Ampere Maxwell laws are special cases of the inhomogeneous equation (IE) of the Evans field theory. The IE is deduced from the HE using the appropriate Hodge duals, those of $F^a$ and $R^a$. Therefore objective classical electrodynamics is deduced entirely from the Bianchi identity (1) using the rules (5) and (6).

Within contemporary instrumental identity the HE can therefore be written in differential form notation as Eq. (9).

In tensor notation Eq. (7) becomes:

$$\partial_\mu F^a_{\nu} + \partial_\nu F^a_{\mu} + \partial_\alpha F^a_{\mu \nu} = \epsilon_{\mu \alpha \nu \rho} \left( \partial_\rho F^a_{\nu} + \partial_\nu F^a_{\rho} + \partial_\rho F^a_{\mu} \right)$$ (14)

and this is the same equation as:

$$\partial_\mu \tilde{F}^{a\mu} = \epsilon_{\mu \alpha \nu \rho} \tilde{F}^{a\nu}$$ (15)

where:

$$\tilde{F}^{a\mu} = \frac{1}{2} \left| g \right|^{1/2} \epsilon^{\mu \rho \sigma} F_{\rho \sigma}^a$$ (16)

is the Hodge dual tensor defined (3) by:

$$\left| g \right| = \left| g_{\mu \nu} \right|$$ (17)

In Eq. (17) $\left| g \right|^{1/2}$ is the positive square root of the metric determinant (3).

However the Hodge dual of the current on the right hand side is defined by:

$$\tilde{\tilde{j}}^{a\sigma} := 3 \left( \frac{1}{6} \left| g \right|^{1/2} \epsilon^{\mu \rho \sigma} j_\rho \right)$$ (18)
and so \( \mathbf{j} \) cancels out on either side of Eq. (15). The simplest form of the homogeneous field equation is therefore:

\[
\mathbf{\nabla} \times \mathbf{A} = 0 \quad (19)
\]

and in vector notation this becomes:

\[
\mathbf{\nabla} \cdot \mathbf{B} = 0 \quad (20)
\]

and

\[
\frac{\partial \mathbf{B}}{\partial t} + \mathbf{\nabla} \times \mathbf{E} = 0 \quad (21)
\]

Experimentally it is found that the homogeneous current \( \mathbf{j} \) is very tiny.

Geometrically this implies that:

\[
\mathbf{R}^a \mathbf{\nabla} b = \omega^a_b \mathbf{\nabla} T^b \quad (22)
\]

Using the Maurer Cartan structure relations

\[
\mathbf{T}^a = D \mathbf{\nabla} \omega^a \quad (23)
\]

\[
\mathbf{R}^a_b = D \mathbf{\nabla} \omega^a \omega^b \quad (24)
\]

eq. (22) becomes:
\[
(0 \wedge \omega^a v^b) \wedge \gamma^c = \omega^a v^b \wedge (0 \wedge \gamma^c)
\]  \hspace{1cm} (25)

A possible solution of this equation is:
\[
\omega^a v^b = - \epsilon^a_{\ bc} \sigma^c \hspace{1cm} (26)
\]
in which the Levi Civita symbol is defined by:
\[
\epsilon^a_{\ bc} = g^{ad} \epsilon_{\ d\ bc} \hspace{1cm} (27)
\]

Here \( g^{ad} \) is the metric in the orthonormal tangent bundle spacetime. This defines the geometry of the Gauss Law and of the Faraday Law of induction in the Evans unified field theory.

In order to deduce the inhomogeneous field equations of the unified field theory it is first noted that the inhomogeneous field equations of the Maxwell-Heaviside field theory are encapsulated in the well known:
\[
d \wedge F = 0 \hspace{1cm} (28)
\]
\[
d \wedge \tilde{F} = \mu_0 J \hspace{1cm} (29)
\]

where \( \tilde{F} \) is the Hodge dual of \( F \) and where \( J \) is the charge current density three-form. The appropriate Hodge dual of Eq. (7) is:
\[
d \wedge \tilde{F}^a = - A^{(e)} (\sigma^a \wedge \tilde{R}^e + \omega^b \wedge \tilde{R}^b) \hspace{1cm} (30)
\]
and this is the inhomogeneous field equation of the Evans unified field theory (denoted UE).

When there is no field matter interaction the appropriate inhomogeneous field equation is:

$$\mathbf{\nabla} \wedge \overline{\mathbf{F}} = 0.$$  \hspace{1cm} (31)

Therefore for electromagnetic radiation in free space:

$$\left( \mathbf{\alpha}_V \wedge \overline{\mathbf{R}}^{a \ b} + \omega^{a \ b} \wedge \overline{\mathbf{F}}^{b} \right) = 0.$$  \hspace{1cm} (32)

For centrally directed gravitation (Einstein/Newton gravitation):

$$\mathbf{R}^{a \ b} \wedge \mathbf{\nabla}^{b} = 0$$  \hspace{1cm} (33)

$$\overline{\mathbf{R}}^{a \ b} \wedge \mathbf{\nabla}^{b} \neq 0.$$  \hspace{1cm} (34)

It is assumed that Eq. (32) continues to be true approximately in the presence of field matter interaction then the only term that contributes to the right hand side of the UE is that from the gravitational Eq. (35). (This approximation is analogous to the well known minimal prescription:

$$\mathbf{P} \rightarrow \mathbf{P} + e \mathbf{A}$$  \hspace{1cm} (36)

in which it is seen that the electromagnetic property ($\mathbf{A}$) is unchanged by the field matter interaction.) Therefore the UE becomes:

$$\mathbf{\nabla} \wedge \overline{\mathbf{F}}^{a} = - \mathbf{A}^{(\nu)} \left( \overline{\mathbf{R}}^{a \ b} \wedge \mathbf{\nabla}^{b} \right)_{\nu \mu \nu}.$$  \hspace{1cm} (37)
This is the inhomogeneous field equation linking electromagnetism to gravitation. Any type of electromagnetic field matter interaction is described by Eq. (37) provided Eq. (32) remains true for the electromagnetic field when the latter interacts with matter.

In tensor notation Eq. (37) is

\[ \partial_{\mu} F^{a\mu} + J^{a\mu} + \Lambda^{a\mu} \Phi \]

\[ = -\Lambda^{(0)} (R^{a}_{\mu
u} R_{\lambda\kappa} b^{\nu} b^{\kappa} + q_{\mu} R^{a}_{\nu\lambda} b^{\lambda} b^{\nu} + q_{\nu} R^{a}_{\mu\lambda} b^{\lambda} b^{\mu}) \]

which is the same equation as:

\[ \partial_{\mu} F^{a\mu} = -\Lambda^{(0)} R^{a}_{\mu\nu} b^{\nu} \]

where we have used:

\[ R^{a}_{\lambda\mu} = q_{\lambda} R^{a}_{\mu\nu} b^{\nu} . \]

Eq. (39) is the simplest tensor formulation of the IE.

In vector notation Eq. (39) becomes the Coulomb law of the Evans unified field theory and the Ampere Maxwell law of the Evans unified field theory. The Coulomb law is derived using

\[ \omega = 0, \mu = 1, 2, 3 \]

so give:

\[ \partial_{1} F^{a1} + \partial_{2} F^{a2} + \partial_{3} F^{a3} = -\Lambda^{(0)} R^{a}_{i0} \]

where summation over repeated indices i is implied. Now denote the fundamental voltage
\[ \phi^{(0)} = c A^{(0)} \quad - \quad (4.3) \]

to obtain:

\[ \nabla \cdot E^a = - \phi^{(0)} R^a_i i^0 \quad - \quad (4.4) \]

with charge density:

\[ \rho^a = - \varepsilon_0 \phi^{(0)} R^a_i \quad - \quad (4.5) \]

Eq. \((4.4)\) is the Coulomb law unified with the Newton inverse square law in the Evans unified field theory.

The units on both sides of Eq. \((4.4)\) are volt metre \(^{-2}\) and it is seen in Eq. \((4.5)\) that charge density originates in \(R^a_i\), the sum of three Riemann curvature elements. These elements are calculated from the Einstein field theory of gravitation in our approximation \((3.2)\). In the weak field limit it is well known that the Einstein field equation reduces to the Newton inverse square and so Eq. \((4.4)\) unifies the Newton and Coulomb laws in the weak field limit. Given the existence of \(\phi^{(0)}\) it is seen from Eq. \((4.4)\) and \((4.5)\) that an electric field can be generated from gravitation.

Similarly the Ampere Maxwell law in the Evans unified field theory is:

\[ \nabla \times B^a = \frac{1}{c^2} \frac{dE^a}{dt} + \mu_0 \frac{J^a}{c} \quad - \quad (4.6) \]

where

\[ J^a = J^a_x i + J^a_y j + J^a_z k \quad - \quad (4.7) \]
and where:

\[ J^{a}_x = -\tilde{A} \left( R^{a}_{\ 0}^{\ 10} + R^{a}_{\ 2}^{\ 12} + R^{a}_{\ 3}^{\ 13} \right) - (4.8) \]

\[ J^{a}_y = -\tilde{A} \left( R^{a}_{\ 0}^{\ 20} + R^{a}_{\ 1}^{\ 21} + R^{a}_{\ 3}^{\ 23} \right) - (4.9) \]

\[ J^{a}_z = -\tilde{A} \left( R^{a}_{\ 0}^{\ 30} + R^{a}_{\ 1}^{\ 31} + R^{a}_{\ 2}^{\ 32} \right) - (5.0) \]

From Eqs. (4.7) to (5.0) it is seen that current density also originates in sums over different Riemann tensor elements. This finding has the important consequence that electric current can be generated by spacetime curvature, the relevant Riemann tensor elements are again calculated from the Einstein theory of gravitation.

The unified Coulomb/Newton law (4.14) can be further simplified to:

\[ \nabla \cdot E = -\varphi \left( R^{a} \right) \quad -(5.1) \]

where

\[ R^{a} = R^{a}_{\ 1}^{\ 10} + R^{a}_{\ 2}^{\ 20} + R^{a}_{\ 3}^{\ 30} \quad -(5.2) \]

Here the units of \( \varphi \) are volts and the units of \( R^{a} \) are inverse square meters. The index a denotes a state of polarization and originates in the index of the tangent bundle spacetime. For example if \( E^{a} \) is in the Z-axis and if we use the complex circular basis a = (1), (2), (3) then:

\[ E^{a}_Z = E^{(3)} \quad -(5.3) \]

and we obtain:

\[ \frac{\partial E^{a}_Z}{\partial Z} = -\varphi \left( R^{a}_{\ 1}^{\ (3)} + R^{a}_{\ 2}^{\ (3)} + R^{a}_{\ 3}^{\ (3)} \right) \quad -(5.4) \]

\[ \frac{\partial E^{a}_Z}{\partial Z} = -\varphi \left( R^{(3)}_{\ 1}^{\ 10} + R^{(3)}_{\ 2}^{\ 20} + R^{(3)}_{\ 3}^{\ 30} \right) \]
Using the antisymmetry properties of the Riemann tensor gives the simple equation:

\[
\frac{\partial E_z}{\partial z} = - \phi_{(0)} R_z - (55)
\]

where

\[
R_z = R_{(1)}^{(3)} 10 + R_{(2)}^{(3)} 20 - (56)
\]

Eq. (55) is therefore the law that governs the interaction between two charges placed on two masses, and this law shows in the simplest way that an electric field is always generated by the scalar curvature \( R \) multiplied by the fundamental voltage \( \phi_{(0)} \). This product defines the charge density as:

\[
\rho = - \varepsilon_0 \phi_{(0)} R_z - (57)
\]

The minus sign in Eq. (55) indicates that charge density is a compression of spacetime.

The Einstein field equation indicates that:

\[
R_z = - \kappa_1 T_{zz} - (58)
\]

where \( \kappa_1 \) is a constant proportional to the Newton constant \( G \) and where \( T \) is an index contracted (i.e. scalar) canonical energy-momentum tensor. (The constant \( k_1 \) in Eq. (58) is not the same numerically as the Einstein constant \( k \) because \( R \) in Eq. (58) is not defined in the same way as the original \( R \) of the Einstein field theory.) Therefore Eq. (55) can be written as:

\[
\frac{\partial E_z}{\partial z} = \phi_{(0)} \kappa_1 T_{zz} - (59)
\]

where \( \kappa_1 \) is a fundamental voltage numerically different from \( \phi_{(6)} \). Eq. (59) shows that
the Coulomb law derives in the last analysis from spacetime energy momentum denoted T, and T can be transferred to an electric circuit. The curvature R is greatest near an electron and for a point electron R becomes infinite. There are however no infinites in nature so point electron are idealizations of the traditional theory of electrodynamics. The Evans field theory removes this infinity and also removes the need for renormalization and Feynman calculus in quantum electrodynamics.

If we repeat our consideration of the a = (3) index in the Ampere Maxwell law it is seen that current density is generated in the simplest way by a time varying electric field:

$$\mathbf{J}_z = - \varepsilon_0 \frac{\partial \mathbf{E}_z}{\partial t} = - \frac{\mathbf{A}^{(e)}}{\mu_0} \times \mathbf{R}_z - \mathbf{E}_0$$

because for a = (3):

$$\nabla \times \mathbf{B} = \mathbf{0}$$

It is seen that current density is generated by different curvature components from those that generate charge density in the Coulomb / Newton law (S5) of the Evans unified field theory. More generally the transverse a = (1) and (2) components are given by:

$$\nabla \times \mathbf{B}^{(1)} = \frac{1}{c^2} \frac{\partial \mathbf{E}^{(1)}}{\partial t} + \frac{\mathbf{J}^{(1)}}{\mu_0}$$

$$\nabla \times \mathbf{B}^{(2)} = \frac{1}{c^2} \frac{\partial \mathbf{E}^{(2)}}{\partial t} + \frac{\mathbf{J}^{(2)}}{\mu_0}$$

In free space there is no current density due to mass and so Eqs. (62) and (63) reduce to:

$$\nabla \times \mathbf{B}^{(1)} = \frac{1}{c^2} \frac{\partial \mathbf{E}^{(1)}}{\partial t}$$

$$\nabla \times \mathbf{B}^{(2)} = \frac{1}{c^2} \frac{\partial \mathbf{E}^{(2)}}{\partial t}$$
It has been shown that classical electromagnetism and classical gravitation can be unified with the well known methods of differential geometry, notably the Maurer Cartan structure relations and the Bianchi identities. The end result produces in this paper the four classical laws of electrodynamics unified with the classical laws of gravitation as given in the Einstein field theory. Charge density in the Coulomb law and current density in the Ampere Maxwell law have been shown to originate in sums over scalar components of the Riemann tensor of the Einstein gravitational theory. This result shows, for example, that the curl of a magnetic field and the time derivative of an electric field can produce gravitation through the current density of the Ampere Maxwell law. The gradient of an electric field produces gravitation through the charge density through the Coulomb law. Therefore an electromagnetic device can counter gravitation, and this is of clear importance to the aerospace industry. Conversely the curvature of spacetime as embodied in the Riemann tensor of gravitation can produce electromagnetic current density through the Ampere Maxwell law and charge density through the Coulomb law, and so can produce electric power. This is of clear interest to the electric power industry, because circuits can be designed in principle to produce electric power from gravitation in an original way. The vector formulation produced in this paper of the four laws of classical electrodynamics shows this clearly.

In deriving the simplest possible expressions of the unified field theory as given in this paper a type of minimal prescription has been used where it has been assumed that the electromagnetic field is unchanged in the interaction with matter. This produces a simple and clear result as described already. If this minimal prescription is not used the unified field equations become more complicated but still soluble numerically given the initial and boundary conditions.

In deriving the unified field equations in this paper no account has been taken of
polarization and magnetization. The development of these properties in the unified field theory will be the subject of future work. Essentially both polarization and magnetization become spacetime properties, and the electromagnetic field interacts with matter through molecular property tensors which are also spacetime properties and which in the unified field theory, incorporate the effects of gravitation. This is the subject of generally covariant nonlinear optics.

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