SOME NOTES ON THE I.E.

(INHOMOGENEOUS EVANS FIELD EQUATION)

The IE is deduced from the Bianchi identity of differential geometry:

\[ d \Lambda T^a = R_{a b} \Lambda v^b - \omega_{a b} \Lambda T^b \]  \quad \text{(1)}

in the general 4-D manifold or Evans spacetime. Here T^a is the torsion form, R_{a b} is curvature or Riemann form, \omega_{a b} is the spin connection and d \Lambda is the exterior derivative. The geometry is converted to the field theory using:

\[ A^a = A^{(0)} \Lambda v^a \]  \quad \text{(2)}

\[ F^a = A^{(0)} T^a \]  \quad \text{(3)}

Here A^{(0)} is the fundamental potential magnitude, with units of volt s/m, A^a is the potential form and F^a is the field form. From Eqn (1) to (3) we obtain the homogeneous Evans field equation (HE):

\[ d \Lambda F^a = R_{a b} \Lambda A^b - \omega_{a b} \Lambda F^b \]  \quad \text{(4)}

\[ = A^{(0)} (R_{a b} \Lambda v^b - \omega_{a b} \Lambda T^b) \]

with the geometrical constant:

\[ R_{a b} \Lambda v^b = \omega_{a b} \Lambda T^b \]  \quad \text{(5)}

Eqn (4) reduces to the homogeneous Maxwell-Heaviside field equation:
\[ d \nabla F^a = 0 \quad -(6) \]

for each index \( a \). The latter is the Weyl bundle index of differential geometry, and physically indicates states of polarisation. Thus, for each \( a \):

\[ d \nabla F = 0 \quad -(7) \]

Eqn (7) is the way in which the homogeneous MH eqn is usually written in differential geometry. It is seen that the homogeneous MH eqn. is a particular case of the homogeneous Einstein field eqn. (4). More generally, the homogeneous Einstein field eqn. (5) is not obeyed. These effects should be investigated experimentally. Using the structure equations:

\[
T^a = D \nabla q^a = d \nabla q^a + \omega^a_b q^b \\
R^a_b = D \nabla \omega^a_b = d \nabla \omega^a_b + \omega^a_c \omega^c_b \quad -(6)
\]

of differential geometry, eqn (5) becomes:

\[
(D \nabla \omega^a_b) \nabla q^b = \omega^a_b \nabla (D \nabla q^b) \quad -(7)
\]

one possible solution of eqn (7) is:

\[
\omega^a_b = \kappa \epsilon^a_b c q^c \quad -(8)
\]

\[
R^a_b = \kappa \epsilon^a_b c T^c \quad -(9)
\]

where \( \kappa \) is the wavenumber.
Eqs (8) and (9) mean that in free space, \( \omega^a b \) is an antisymmetric tensor dual to the vector \( \gamma^a c \), and \( R^a b \) is an antisymmetric tensor dual to \( T^a e \). These \( R^a b \) is an antisymmetric tensor dual to \( T^a e \). These

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In tensor notation, Eqn. (7) is:

\[
d_\mu F_{\mu \rho} + d \rho F_{\rho \mu} + d \sigma F_{\rho \mu} = 0 \quad - (10)
\]

or

\[
d_\mu F_{\mu \rho} = 0 \quad - (10a)
\]

where

\[
F_{\mu \rho} = \frac{1}{2} \epsilon_{\mu \rho \sigma} F_{\sigma \nu} \quad - (11)
\]

Eqn. (10a) is a consequence of two fundamental laws:

\[
\nabla \cdot B = 0 \quad - (12)
\]

\[
\nabla \times E + \frac{d B}{dt} = 0 \quad - (13)
\]

Eqn. (12) is the Gauss law applied to magnetism, and

Eqn. (13) is the Faraday law of induction.

These laws are therefore special cases of the

unified field theory, Eqn. (14).
Physically, these laws describe electromagnetism assuming that it is not influenced by gravitation. In the laboratory this is an excellent approximation, but in cosmology, a beam of light near a black hole will obey Eqn. (4), and small changes in the laws (12) and (13) are expected. This is a test of Einstein's general relativity and a prediction of the general field theory. Einsteins field theory of gravitation assumes that a 4-D manifold is described by a Ricci curvature. Einsteins unified field theory of all radiated and matter fields is described by Riemann spacetime. Einsteins field theory is governed by the assumption that fields are governed by the 4-D manifold in which torsion and curvature are defined by the structure relation of differential geometry. Eqns. (14) and (15) reduce to the Riemann spacetime used by Einstein when $T \alpha = 0 \quad (14)$ and $R^{a}{}_{b} N^{b} = 0. \quad (15)$

Eqn. (14) means that there is no electromagnetic
field present:
\[ F^a = 0. \quad - (16) \]

Eqn. (15) is the well known Bianchi identity used by Einstein:

\[ R_{\mu\nu\rho\sigma} + R_{\mu\rho\nu\sigma} + R_{\mu\rho\sigma\nu} = 0. \quad - (17) \]

Eqn. (17) is true if and only if:

\[ \Gamma^\kappa_{\mu\nu} = \Gamma^\kappa_{\nu\mu} \quad - (18) \]

and

\[ T^\kappa_{\mu\nu} = \Gamma^\kappa_{\mu\nu} - \Gamma^\kappa_{\nu\mu} = 0. \quad - (19) \]

Here \( R_{\mu\nu\rho\sigma} \) is the Riemann tensor, \( \Gamma^\kappa_{\mu\nu} \) is the Christoffel connection, \( T^\kappa_{\mu\nu} \) is the stress tensor. Eqn. (19) in tensor notation is equivalent to eqn. (14) ii.

This is because a differential form notation. Eqn. (1) is a generalization of eqn. (17) to 4-D Euclidean spacetime, a 4-manifold of differential geometry.

Before proceeding, let's discuss the following points. Firstly, we prove eqn. (10a) from eqn. (10)
Proof of Eq. (10a)

Consider the case:

\[ d_1 F_{23} + d_3 F_{12} + d_2 F_{31} = 0 \quad - (20) \]

Note that:

\[ e^{2301} d_1 F_{23} + e^{1203} d_3 F_{12} + e^{3102} d_2 F_{31} = 0 \quad - (21) \]

Because:

\[ e^{2301} = e^{1203} = e^{3102} = 1 \quad - (22) \]

Using the Leibniz theorem:

\[ d_1 (e^{2301} F_{23}) = (d_1 e^{2301}) F_{23} + e^{2301} d_1 F_{23} \]

\[ = e^{2301} d_1 F_{23} \quad - (23) \]

and

\[ (e^{2301}) d_1 F_{23} = \frac{1}{2} (d_1 (e^{2301} F_{23} + e^{3201} F_{32})) \]

\[ = d_1 F_{01} \quad - (24) \]

Therefore, eqn. (20) is the same as:

\[ d_1 F_{01} + d_2 F_{02} + d_3 F_{03} = 0 \quad - (25) \]

or:

\[ \nabla \cdot B = 0 \quad - (25a) \]

Q.E.D.
Similarly to HE, Eqn. (4), is the same as:

\[ \mu F_{\mu
u} = R_{\rho \mu \nu} \quad \text{Eqn. (4a)} \]

The geometrical contrivia for free space electromagnetism is therefore:

\[ V^\mu R_{\mu \nu} = \omega_{\rho \mu \nu} \quad \text{Eqn. (26)} \]

i.e. the dot products on the left and right must be the same. Eqn. (26) define free space electromagnetism free of any gravitational influence. More generally, Eqn. (26) does not hold, and electromagnetism and gravitation are mutually influential.

This case is very important for new technologies, and is the case where the dot products in Eqn. (26) are not the same, i.e.:

\[ \omega_{\rho \mu \nu} \neq \kappa E_{\rho \mu \nu} \quad \text{Eqn. (26a)} \]

\[ R_{\rho \mu \nu} \neq \kappa E_{\rho \mu \nu} \quad \text{Eqn. (26b)} \]

and:

\[ \nabla \cdot B \neq 0 \quad \text{Eqn. (26c)} \]

\[ \nabla \times E + \frac{\partial B}{\partial t} \neq 0 \quad \text{Eqn. (26c)} \]
Eqs. (265) and (260) mean, for example, that the polarization of a beam of light grazing an intensely gravitational object will be changed.

Secularly, we prove that if
\[
d \wedge F = 0 \quad - (7)
\]

then
\[
d \wedge \bar{F} \neq 0 \quad - (27)
\]

in general.

**Proof:** Eq. (27)

It must be proven that in general:
\[
d \mu F_{\mu \nu} + d \nu \bar{F}_{\mu \nu} + d \omega \bar{F}_{\mu \nu} \neq 0 \quad - (27a)
\]

If
\[
d \mu F_{\mu \nu} + d \nu \bar{F}_{\mu \nu} + d \omega \bar{F}_{\mu \nu} = 0 \quad - (7a)
\]

Consider the example:
\[
d \bar{F}_{\mu \nu} + d \frac{1}{2} \bar{F}_{\mu \nu} + d \frac{1}{2} \bar{F}_{\mu \nu}
\]

\[
= \frac{1}{2} \left( \bar{F}_{\mu \nu} + \bar{F}_{\mu \nu} + \bar{F}_{\mu \nu} + \cdots \right)
\]

\[
= \frac{1}{2} \left( \bar{F}_{\mu \nu} + \bar{F}_{\mu \nu} + \bar{F}_{\mu \nu} + \cdots \right)
\]

\[
= \bar{F}_{\mu \nu} \bar{F}_{\mu \nu} \bar{F}_{\mu \nu} \cdots
\]

\[
= \frac{\nabla \cdot \vec{E}}{\epsilon_0} = \rho / \epsilon_0 \quad - (28)
\]
Eqn. (28) is the Lorentz force law of electromagnetism. Eqn. (27a) is therefore the same as:

\[ \mu_0 F^\mu \sim \mu_0 J^\mu - (27b) \]

where \( J^\mu \) is the charge-current density four vector. Eqn. (27b) is the inhomogeneous Maxwell Heaviside field equation. In differential form notation it is:

\[ d\tilde{F} = J - (27c) \]

where \( J \) is the charge-current density four-form.

Q.E.D.

It is to be expected that eqn. (27c) is a special case of the more general inhomogeneous Einsteins field equation (IE):

\[
\begin{align}
    d\tilde{F}^a = & \tilde{R}^{ab} \wedge \Lambda^b - \omega^{ab} \wedge \tilde{F}^b \\
    = & \Lambda^{(o)}(\tilde{R}^{ab} \wedge q^b - \omega^{ab} \wedge \tilde{F}^b) \\
    = & -(29)
\end{align}
\]

where:

\[ \tilde{R}^{ab} \wedge q^b \neq \omega^{ab} \wedge \tilde{F}^b - (30) \]
Proof of Eqn. (29)

Eqn. (29) is derived from eqn. (4) by considering the duals of $F_{\alpha}$ and $R_{\alpha\beta}$ in Eano spacetime:

$$F_{\mu}^\alpha = \frac{1}{2} \sqrt{g} \left| 1 \right|^\frac{3}{2} \epsilon_{\mu\nu\rho\sigma} F_{\rho\sigma}^\alpha - (31)$$

$$R_{\mu}^\rho_{\alpha} = \frac{1}{2} \sqrt{g} \left| 1 \right|^\frac{3}{2} \epsilon_{\mu\nu\rho\sigma} R_{\nu\rho\sigma}^\alpha - (32)$$

where $\left| 1 \right|^\frac{3}{2}$ is the square root of the metric determinant and $\epsilon_{\mu\nu\rho\sigma}$ the 4-D Levi-Civita symbol.

In 4-D the Hodge dual, $F_{\alpha}$, of $F_{\mu}$ is another two-form. Similarly, the Hodge dual, $R_{\alpha\beta}$, of $R_{\mu}$ is also another two-form. Eqs. (31) and (32) are defined by the general definition of a Hodge dual in an n-dimensional manifold.

Using eqns. (31) and (32) the IE, eqn. (29) follows from eqn. (4) by correctly defining the Hodge duals on both sides of eqn. (4). This procedure generates the IE from the HE and the fundamental Branch identity (1). We therefore arrive at a generalization of Coulomb's law and Ampère-Maxwell law.
To check and illustrate this important result consider a particular term of eqn. (4) such as:

\[ d_1 F_{23} = R^a_{\,612} A^b_3 - \omega^a_{\,b1} F^b_{23}. \quad - (33) \]

Integrating:

\[ F^a_{23} = \int (R^a_{\,612} A^b_3 - \omega^a_{\,b1} F^b_{23}) \, dx^1. \quad - (34) \]

Therefore:

\[ F^a_{23} = \int (R^a_{\,612} A^b_3 - \omega^a_{\,b1} \tilde{F}^b_{23}) \, dx^1 \quad - (35) \]

and:

\[ d_1 F^a_{23} = R^a_{\,612} A^b_3 - \omega^a_{\,b1} \tilde{F}^b_{23}. \quad - (36) \]

Eqn. (35) is the only possible way of defining the Hodge dual so that indices match on both sides. The Hodge dual of the product \( R^a_{\,612} A^b_3 \) must be a one-form in 4-D and this cannot be the Hodge dual of \( A^b_3 \) equated to the two-form. The Hodge dual of \( A^b_3 \) must be a three-form, and this again does not give the right answer.

Therefore the correct Hodge dual structure of eqn. (4) is eqn. (29), A.E.O.
(2) The tensorial structure of the TE is:

\[ d_{\mu} \tilde{F}^a_{\nu\rho} + d_{\nu} \tilde{F}^a_{\rho\mu} + d_{\rho} \tilde{F}^a_{\mu\nu} \]

\[ = A(\tilde{R}^a_{\mu\nu\rho} - \tilde{R}^a_{\nu\rho\mu} + \tilde{R}^a_{\rho\mu\nu} - \tilde{R}^a_{\mu\nu\rho}) \]

which is the same as:

\[ d_{\mu} F^{a\mu, b} = A(\tilde{R}^a_{\mu\nu\rho} - \omega^b_{\mu\nu} T^\mu_{\nu, b}) \]

- (37a)

The Coulomb Law is E = \(d_{\mu} F^{a\mu, b} = \tilde{R}^a_{\mu\nu\rho} - \omega^b_{\mu\nu} T^\mu_{\nu, b} \)

The Coulomb Law is defined by:

\[ d_{1} \tilde{F}^a_{23} + d_{3} \tilde{F}^a_{12} + d_{2} \tilde{F}^a_{31} \]

\[ = d_{1} F^{a, 01} + d_{2} F^{a, 02} + d_{3} F^{a, 03} \]

\[ = \nabla \cdot E \]

\[ \nabla \cdot E^a = \phi(\tilde{R}^a_{\mu\nu\rho} + \tilde{R}^a_{\nu\rho\mu} + \tilde{R}^a_{\rho\mu\nu} - \tilde{R}^a_{\mu\nu\rho}) \]

\[ - \omega^b_{\mu\nu} T^\mu_{\nu, b} - \omega^b_{\mu} T_{\mu, b} - \omega^b_{b3} T^{03}_{\nu, b} \]

- (38)
In Eqn (38), $\phi'(0)$ is undefined.

Using the result:

\[ R_{a01} = R_{a'01} \wedge V_1 \]

Eqn (38) simplifies to:

\[ \nabla \cdot E = \phi'(0) \left( R_{a01} + R_{a02} + R_{a03} - \omega_{a'101,b} - \omega_{a'202,b} - \omega_{a'303,b} \right) \]

\[ = \rho / \varepsilon_0 \]

\[ \text{Eqn (40) is the Coulomb's Law in } \mathbb{R} \text{ even field theory.} \]

**Discussion**

Eqn (40) is the direct result of the Bianchi identity (1) of differential geometry, and reveals the origin of charge density $\rho$, in general relativity.

In the limit of Einstein's field theory of gravitation:

\[ E^a = 0 \text{ and } T^{\alpha\mu} = 0 \]

\[ \text{-(41)} \]
and so, in this limit:

$$R_1^{a_{11}} + R_2^{a_{03}} + R_3^{a_{03}} = 0. \quad -(42)$$

This is the limit where electromagnetism is absent, and eqn (42) is equivalent to eqn (17). In the limit of free space electromagnetism:

$$\nabla \cdot E^a = 0 \quad -(43)$$

$$E^a \neq 0 \quad -(44)$$

and so:

$$R_1^{a_{11}} + R_2^{a_{03}} + R_3^{a_{03}} = \omega_a b_1 T^{01, b} + \omega_a b_2 T^{02, b} + \omega_a b_3 T^{03, b}. \quad -(45)$$

The Coulomb law therefore indicates that the electromagnetic and gravitational fields are influencing each other, and this defines field-matter interaction. In this case neither eqn (42) nor eqn (45) is true.

These are important insights of the Einstein field theory which cannot be obtained from the Maxwell-Herzstein field theory, or Einstein theory.
These insights regarding gravitation can be used to influence electromagnetism and vice versa. The simplest example is the Coulomb law, one of the oldest laws of physics.

**Weak Field Limit**

The weak field limit may be defined by the absence of gravitation, and so is defined by eqn. (45).

From eqns (3) and (37a) the Coulomb law may also be expressed as:

\[
\nabla \cdot E^a = A^0 \left( d_1 T^{01, a} + d_2 T^{02, a} + d_3 T^{03, a} \right)
\]

\[
\nabla \cdot E^a = A^0 \nabla \cdot T^{0, a} = \rho / \epsilon_0 .
\]

Eqn. (46) shows that the Coulomb law originates in the concept of spacetime torsion. The latter is linked to curvature by the Bianchi identity (1), and the Coulomb law requires spacetime spinning and curving to be both present.
Using eqns. (40) and (45) it is seen that the Coulomb's law is a special case of the Bianchi identity (1):

\[
\begin{align*}
&d_1 T^{01,\alpha} + d_2 T^{02,\alpha} + d_3 T^{03,\alpha} \\
&\quad + \omega^{\alpha b_1} T^{01, b} + \omega^{\alpha b_2} T^{02, b} + \omega^{\alpha b_3} T^{03, b} \\
&= R^{\alpha}_{\ 01} + R^{\alpha}_{\ 02} + R^{\alpha}_{\ 03}. \\
\end{align*}
\]

This is the geometry of an empty spacetime that defines the Coulomb's law.

In the Einstein limit:

\[
R^{\alpha}_{\ 01} + R^{\alpha}_{\ 02} + R^{\alpha}_{\ 03} = 0 \quad - (48)
\]

\[
T^{01,\alpha} = T^{02,\alpha} = T^{03,\alpha} = 0. \quad - (49)
\]

It is seen clearly from eqn. (47) that the electromagnetic charge eqn. (48) of the Einstein theory, and so influence gravitation. This process is at the root of the Coulomb law, and yield — matter interaction in general.