PROOF OF THE FREE SPACE CONDITION:

\[ c^a b = c^b c^a \]

This fundamental condition is a solution of

\[ R^a_b \wedge q^b = c^a b \wedge T^b \]  \hspace{1cm} (1)

i.e.

\[ (D \wedge c^a b) \wedge q^b = c^a b \wedge (D \wedge q^b) \]  \hspace{1cm} (2)

or

\[ (D \wedge c^a b) \wedge q^b + (c^a b \wedge c^b) \wedge q^b = c^a b \wedge (D \wedge q^b) + c^a b \wedge (c^b c \wedge q^c) \]  \hspace{1cm} (3)

To Prove

\[ (D \wedge c^a b) \wedge q^b = c^a b \wedge (D \wedge q^b) \]  \hspace{1cm} (4)

Proof

For \( a = 1 \):

\[ (D \wedge c^1 2) \wedge q^2 + (D \wedge c^1 3) \wedge q^3 \]  \hspace{1cm} (5)

\[ = c^1 2 \wedge (D \wedge q^2) + c^1 3 \wedge (D \wedge q^3) \]

Eqn (5) is true if

\[ c^1 2 = \kappa c^1 23 q^3 = \kappa q^3 \]  \hspace{1cm} (6)

\[ c^1 3 = \kappa c^1 32 q^2 = -\kappa q^2 \]  \hspace{1cm} (7)
\[ (d \lor q^3) \lor q^2 = (d \lor q^3) \lor q^3 \]
\[ q^3 \lor (d \lor q^3) = q^3 \lor (d \lor q^3) \]
\[ \therefore (d \lor q^3) \lor q^2 = q^2 \lor (d \lor q^3) \]
\[ (d \lor q^3) \lor q^3 = q^3 \lor (d \lor q^3) \]
\[ \therefore (d \lor q^3) \lor q^3 = q^3 \lor (d \lor q^3) \]

Q.E.D.

To Prove

\[ (\omega^{a \lor c} \land \omega^{b}) \land q^b = \omega^b \land (\omega^{a} \land q^c) \]

Proof

For \( a = 1, b = 2, c = 3 \):

\[ (\omega^{1} \land \omega^{3}) \land q^2 = \omega^2 \land (\omega^{2} \land q^3) \]
\[ \omega^1 = \kappa q^3 \land \omega^2 = -\kappa q^3 \]
\[ \omega^3 = -\kappa q^1 \land \omega^2 = \kappa q^1 \]
\[ (q^2 \land q^1) \land q^3 = -q^3 \land (q^1 \land q^3) \]
\[ \therefore \omega^3 \land q^2 = -q^3 \land (-q^2) \]
\[ = q^3 \land q^2 \]

Q.E.D.
(3) For $o(3)$ electrodynamics we choose:
\[ a_{ab} = -\frac{1}{2} \varepsilon^{abc} q_{c} \]  
- (13)

In the structure relation:
\[ \mathbf{D} \mathbf{\nabla} q^a = \mathbf{D} \mathbf{\nabla} q^a + a_{ab} \mathbf{\nabla} q^b \]  
- (14)

Proof \[ \text{For } a = 1: \]
\[ \mathbf{D} \mathbf{\nabla} q^1 = \mathbf{D} \mathbf{\nabla} q^1 - \frac{1}{2} \left( \varepsilon^{1} 23 q^2 q^3 \right. \]
\[ \left. + \varepsilon^{1} 32 q^2 q^3 \right) \]  
- (15)

\[ \mathbf{D} \mathbf{\nabla} q^1 = \mathbf{D} \mathbf{\nabla} q^1 + \mathbf{\nabla} q^a \mathbf{\nabla} q^2 \mathbf{\nabla} q^3 \]  
- (15a)

In $o(3)$ circular complex basis this gives $o(3)$ electrodynamics.

This allows the retard of the free field to be identified as the potential, and also to be identified as the potential, and also the spin connection. $o(3)$ electrodynamics is a fundamental theory of general relativity.

Q.E.D.
Conversely, $\mathfrak{so}(3) \times \mathfrak{su}(2)$ is the fundamental theory. In which the spin connection and field are duals:

\[ \omega^i_2 = -\frac{1}{2} \mathbf{v}^i_3 \]  

at cyclicity

and:

\[
\begin{align*}
\omega^1 & = \omega^2_3 \\
\omega^2 & = \omega^3_1 \\
\omega^3 & = \omega^1_2
\end{align*}
\]

(17)

Eq (17) follows from the 4-D duality:

\[
\omega^0_1 = \frac{1}{2} \left( \epsilon^{012} \omega^2_3 + \epsilon^{013} \omega^2_2 \right)
\]

(18)

So it may be concluded that electromagnetism in free space is governed by the condition (1), the spin connection has the same symmetry as the field tensor, the retard for the same symmetry as the field components.

Electromagnetism is spinning spacetime.