

ECE theory of the Lamb shift in atomic hydrogen and helium

M.W. Evans^{1*} and H. Eckardt^{2}**

**Alpha Institute for Advanced Studies (AIAS) (www.aias.us); **Unified Physics Institute of Technology (UPITEC) (www.upitec.org)*

The ECE theory of the Lamb shift in atomic hydrogen and helium is developed using considerations of the centrifugal repulsion in atomic hydrogen and the Coulomb and exchange integrals in atomic helium. Radiative corrections are applied using the same method as for the anomalous g factor of the electron and as developed in previous work for atomic hydrogen without the centrifugal correction. The results are expressed systematically in terms of a parameter $r(\text{vac})$ that measures the effect of radiative correction on the radial and other orbitals. This method can be extended systematically in quantum chemistry packages, which can be used for example to numerically generate radial orbitals for use with this method. The latter may have important consequences in obtaining new sources of energy from space-time, (i.e. the source of radiative corrections).

Keywords: Einstein–Cartan–Evans (ECE) theory, Lamb shift in atomic hydrogen and helium, energy from space-time.

1 Introduction

Recently [1] the Einstein–Cartan–Evans unified field theory [2-12] has been applied to the determination of the Lamb shift in atomic hydrogen using a method that succeeded in reproducing the g factor of the electron to experimental uncertainty. The results of the calculation were expressed in terms of a radial parameter $r(\text{vac})$ which demonstrates the way in which the radiative correction affects each orbital. The Lamb shift in atomic H [13] was produced satisfactorily using this method, which has no free parameters. In this paper the method is extended to atomic hydrogen with consideration of centrifugal repulsion [14] as well as Coulombic attraction, and to atomic helium, which consists of two electrons and two protons. The results are again in satisfactory agreement with data, and can

¹e-mail: EMyrone@aol.com

²e-mail: horsteck@aol.com

be extended systematically using quantum chemistry packages such as density functional code. The overall aim is to find methods of amplifying the radiative correction by resonance, so that free electrons are released and used for power generation in circuits.

In Section 2 the method is used for the complete potential of atomic hydrogen, which is well known [14] to contain a centrifugal repulsion term as well as the Coulomb attraction between electron and proton. The result is expressed in terms of $r(\text{vac})$ and graphical data produced for the relevant orbitals. The original Lamb shift [15] was discovered between the 2s and 2p orbitals of hydrogen, and this method naturally removes the angular dependence, so that only radial orbitals need be considered [14].

In Section 3 the method is extended to atomic helium, and considerations given to the Coulomb and exchange integrals [14] in order to understand the basic features of multi electron atoms. If a method is found to resonate $r(\text{vac})$ to infinity, the atom will dissociate into free electrons which can be used for powering circuits. The source of this power is space-time, which is also the source of the radiative correction in ECE theory. Therefore the methods of this paper complement previous considerations [2-12] of ionization of atoms using spin connection resonance [16].

2 Atomic hydrogen with centrifugal effects

In this paper the non-relativistic quantum limit of the ECE wave equation [2-12] is used, the Schrodinger equation considered in a causal and objective interpretation of quantum mechanics rather than the Copenhagen interpretation. The Schrodinger equation of atomic hydrogen [14] is:

$$-\frac{\hbar^2}{2m}\nabla^2\Psi - \frac{e^2}{4\pi\epsilon_0 r}\Psi = E\Psi \quad (1)$$

where m is the reduced mass, \hbar is the reduced Planck constant, ϵ_0 is the S.I. vacuum permittivity, e is the charge on the proton, $-e$ is the charge on the electron, r is the radial distance between electron and proton, E is the total energy and Ψ is the wave-function. By expressing the laplacian in spherical polar coordinates, the equation of the spherical harmonics is introduced:

$$\Delta Y = -\ell(\ell+1)Y \quad (2)$$

where $Y(\theta, \Phi)$ are the spherical harmonics [14] and l is the angular momentum quantum number. The solution of Eq. (1) is:

$$\psi(r, \theta, \phi) = R(r)Y(\theta, \phi) \quad (3)$$

where $R(r)$ are the radial wave-functions. Table 1 gives the first five orbitals of hydrogen with normalization factored out for clarity. It is seen that the complete orbital is expressed in terms of the principal quantum number n , the angular momentum quantum number l and its components. However, from Eqs. (1) to (3) the Schrodinger equation can be written [14] as:

$$-\frac{\hbar^2}{2m} \frac{d^2 P}{dr^2} - V_{\text{eff}}^{(0)} P = EP \quad (4)$$

where the effective potential is:

$$V_{\text{eff}}^{(0)} = -\frac{e^2}{4\pi\epsilon_0 r} + \frac{\ell(\ell+1)\hbar^2}{2mr^2} \quad (5)$$

and where:

$$P = rR. \quad (6)$$

The second term in Eq. (5) is the centrifugal repulsion and the complete potential has a minimum when l is not zero (diagram (b)).

The radiative correction that leads to the Lamb shift [1] is now incorporated by changing Eq. (4) to:

$$-\frac{\hbar^2}{2m} \left(1 + \frac{\alpha}{4\pi}\right)^2 \frac{d^2 P}{dr^2} - V_{\text{eff}}^{(0)} P = EP \quad (7)$$

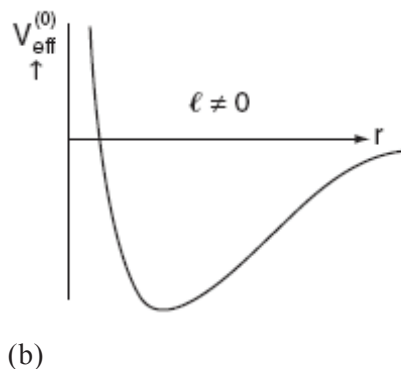
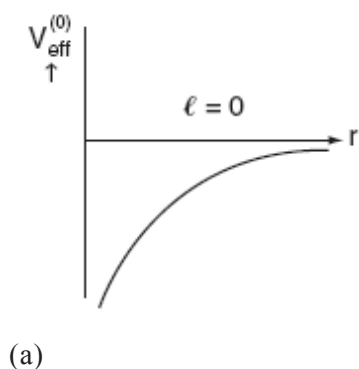


Table 1 First Five Orbitals of Atomic H

	n	ℓ	m_ℓ	$\psi_n(\ell m_\ell)(r, \theta, \phi)$
1s	1	0	0	$\exp\left(-\frac{r}{a}\right)$
2s	2	0	0	$\left(2 - \frac{r}{a}\right) \exp\left(-\frac{r}{2a}\right)$
$2p_z$	2	1	0	$\frac{r}{a} \cos \theta \exp\left(-\frac{r}{2a}\right)$
$2p_x$	2	1	1	$-\frac{r}{a} \sin \theta \exp(i\phi) \exp\left(-\frac{r}{2a}\right)$
$2p_y$	2	1	-1	$\frac{r}{a} \sin \theta \exp(-i\phi) \exp\left(-\frac{r}{2a}\right)$

where α is the fine structure constant. This equation is equivalent by hypothesis [1] to:

$$-\frac{\hbar^2}{2m} \frac{d^2 P}{dr^2} - V_{\text{eff}} P = EP \quad (8)$$

where:

$$V_{\text{eff}} = \frac{e^2}{4\pi\epsilon_0 (r + r(\text{vac}))} - \frac{\ell(\ell+1)\hbar^2}{2m(r + r(\text{vac}))^2}. \quad (9)$$

Here $r(\text{vac})$ is the extent to which the radiative correction perturbs the electron in each orbital. To first order in α :

$$-\frac{\hbar^2 \alpha}{4\pi m} \frac{d^2 P}{dr^2} = \left(V_{\text{eff}}^{(0)} - V_{\text{eff}} \right) P \quad (10)$$

where

$$P = P_0 \text{ (hydrogen)} \quad (11)$$

to a very good approximation, because the Lamb shift in hydrogen is very small compared with the total energy of the unperturbed orbitals. Therefore:

$$V_{\text{eff}}^{(0)} - V_{\text{eff}} = \frac{e^2}{4\pi\epsilon_0} \left(\frac{1}{r} - \frac{1}{r+r(\text{vac})} \right) - \frac{\ell(\ell+1)\hbar}{2m} \left(\frac{1}{r^2} - \frac{1}{(r+r(\text{vac}))^2} \right) \quad (12)$$

and Eq. (10) is solved for $r(\text{vac})$ for each orbital, the result being different for each orbital. Specifically there is a difference between the $2s$ and $2p$ orbitals, giving the Lamb shift. The experimental value of the latter is used to find $r(\text{vac})$. Computer algebra is used to solve Eq. (10) and the results are graphed in Section 4.

3 Atomic Helium

This atom consists of two electrons and two protons and its Schrodinger equation is:

$$H\Psi = E\Psi, \quad (13)$$

$$H = -\frac{\hbar^2}{2m} (\nabla_1^2 + \nabla_2^2) - \frac{2e^2}{4\pi\epsilon_0 r_1} - \frac{2e^2}{4\pi\epsilon_0 r_2} + \frac{e^2}{4\pi\epsilon_0 r_{12}} \quad (14)$$

where r_1 is the distance between electron 1 and the nucleus and similarly for r_2 . The Coulombic terms are doubled because there are two protons giving twice the attractive force. There is also an electron electron repulsion, where r_{12} is the distance between the two electrons. The inter-electron term can be expanded in terms of spherical harmonics:

$$\frac{1}{r_{12}} = \frac{1}{r_1} \sum_{\ell=0}^{\infty} \sum_{m_\ell=-\ell}^{\ell} \left(\frac{4\pi}{2\ell+1} \right) \left(\frac{r_2}{r_1} \right)^\ell Y_{\ell m_\ell}(\theta_1, \phi_1) Y_{\ell m_\ell}(\theta_2, \phi_2) \quad (15)$$

if $r_1 > r_2$, and inter-change r_1 and r_2 when $r_2 > r_1$. This gives rise [14] to the Coulomb integral:

$$J = \frac{e^2}{4\pi\epsilon_0} \int |\psi_1(r_1)|^2 \frac{1}{r_{12}} |\psi_2(r_2)|^2 d\tau_1 d\tau_2. \quad (16)$$

For example, if we consider the interaction energy between two electrons in a hydrogen like $1s$ orbital [14], then:

$$\psi(r_1) = \left(\frac{Z^3}{\pi a_0^3} \right)^{\frac{1}{2}} \exp\left(-Z \frac{r_1}{a_0}\right) \quad (17)$$

$$\psi(r_2) = \left(\frac{Z^3}{\pi a_0^3} \right)^{\frac{1}{2}} \exp\left(-Z \frac{r_2}{a_0}\right) \quad (18)$$

and it is found that:

$$\left. \begin{aligned} \frac{1}{r_{12}} &= \frac{1}{r_1} \langle r_1 \rangle r_2, \\ \frac{1}{r_{12}} &= \frac{1}{r_2} \langle r_2 \rangle r_1. \end{aligned} \right\} \quad (19)$$

These results can be used to calculate J analytically:

$$J = \frac{5}{8} \frac{e^2}{4\pi\epsilon_0} \frac{Z}{a_0}. \quad (20)$$

It is found that [14]:

$$J = \frac{5}{16} (2E_{1s}) \quad (21)$$

so the electron electron repulsion is a substantial fraction of the unperturbed total energy of the 1s orbital in helium. The aim of this section is to incorporate radiative corrections into the helium atom, which also contains the well known exchange integral leading to Fermi heap and hole theory [14]. The Schrodinger equation of atomic helium is:

$$H\psi(r_1, r_2) = E\psi(r_1, r_2) \quad (22)$$

where:

$$\psi(r_1, r_2) = \Psi_{n_1 \ell_1 m_{\ell_1} 1} \Psi_{n_2 \ell_2 m_{\ell_2}}. \quad (23)$$

It is assumed that there is an unperturbed hamiltonian [14] which is the sum of two hydrogen like hamiltonians:

$$H^{(0)} = H_1 + H_2 \quad (24)$$

where:

$$H_i = -\frac{\hbar^2}{2m} \nabla_i^2 - \frac{2e^2}{4\pi\epsilon_0 r_i}. \quad (25)$$

The factor two in the numerator of the second term comes from the fact that there are two electrons and two protons, so each electron is attracted by two protons. The assumption (24) means that the total wave-function must be the product [14]:

$$\Psi(r_1, r_2) = \Psi(r_1)\Psi(r_2). \quad (26)$$

The total unperturbed energy levels are [14]

$$E = -4hcR_\infty \left(\frac{1}{n_1^2} + \frac{1}{n_2^2} \right) \quad (27)$$

where R_∞ is the Rydberg constant.

To incorporate radiative corrections the first step is to develop Eq. (25) and write it as:

$$H_i = -\frac{\hbar^2}{2m} \frac{d^2}{dr_i^2} - V_{\text{eff},i}^{(0)} \quad (28)$$

where

$$V_{\text{eff},i}^{(0)} = -\frac{e^2}{2\pi\epsilon_0 r_i} + \frac{\ell(\ell+1)\hbar^2}{mr_i^2} \quad (29)$$

So there are two hydrogen like equations:

$$-\frac{\hbar^2}{2m} \frac{d^2 P}{dr_1^2} - V_{\text{eff},1}^{(0)} P = E_1 P \quad (30)$$

$$-\frac{\hbar^2}{2m} \frac{d^2 P}{dr_2^2} - V_{\text{eff},2}^{(0)} P = E_2 P \quad (31)$$

where:

$$V_{\text{eff},1}^{(0)} = -\frac{e^2}{2\pi\epsilon_0 r_1} + \frac{\ell(\ell+1)\hbar^2}{mr_1^2} \quad (32)$$

$$V_{\text{eff},2}^{(0)} = -\frac{e^2}{2\pi\epsilon_0 r_2} + \frac{\ell(\ell+1)\hbar^2}{mr_2^2}. \quad (33)$$

Radiative corrections are incorporated into Eqs. (30) to (31) as in Eqs. (8) to (11). For each r_1 and r_2 there will be a corresponding $r_1(\text{vac})$ and $r_2(\text{vac})$ and the total wave-function is the product as in Eq. (26). These radiative corrections may then be incorporated systematically into the Coulomb and exchange integrals using the expansion (15) of r_{12} in terms of r_1 and r_2 . With radiative corrections:

$$r_1 \rightarrow r_1 + r_1(\text{vac}), \quad (34)$$

$$r_2 \rightarrow r_2 + r_2(\text{vac}). \quad (35)$$

By hypothesis, Eqs. (30) and (31) become:

$$-\frac{\hbar^2}{2m} \frac{d^2 P}{dr_1^2} - V_{\text{eff},1} P = E_1 P \quad (36)$$

$$-\frac{\hbar^2}{2m} \frac{d^2 P}{dr_2^2} - V_{\text{eff},2} P = E_2 P \quad (37)$$

with:

$$\frac{d^2}{dr_1^2} \rightarrow \left(1 + \frac{\alpha}{4\pi}\right)^2 \frac{d^2}{dr_1^2} \quad (38)$$

$$\frac{d^2}{dr_2^2} \rightarrow \left(1 + \frac{\alpha}{4\pi}\right)^2 \frac{d^2}{dr_2^2} \quad (39)$$

where:

$$V_{\text{eff},1} = \frac{e^2}{2\pi\epsilon_0 (r_1 + r_1(\text{vac}))} - \frac{\ell(\ell+1)\hbar^2}{(r_1 + r_1(\text{vac}))^2} \quad (40)$$

$$V_{\text{eff},2} = \frac{e^2}{2\pi\epsilon_0(r_2 + r_2(\text{vac}))} - \frac{\ell(\ell+1)\hbar^2}{(r_2 + r_2(\text{vac}))^2}. \quad (41)$$

To first order in α :

$$-\frac{\hbar^2\alpha}{4\pi m} \frac{d^2P}{dr_1^2} = (V_{\text{eff},1}^{(0)} - V_{\text{eff},1}^{(0)})P \quad (42)$$

$$-\frac{\hbar^2\alpha}{4\pi m} \frac{d^2P}{dr_2^2} = (V_{\text{eff},2}^{(0)} - V_{\text{eff},2}^{(0)})P. \quad (43)$$

For two $1s$ electrons:

$$V = \frac{e^2}{4\pi\epsilon_0} \left(\frac{Z^3}{\pi a_0^3} \right) (2\pi)^2 Z^2 \int_0^\infty r_2 e^{-\frac{2Zr_2}{a_0}} \left(\int_0^{r_2} \frac{r_1^2}{r_2} e^{-\frac{2Zr_1}{a_0}} dr_1 + \int_{r_2}^\infty \frac{r_1^2}{r_1} e^{-\frac{2Zr_1}{a_0}} dr_1 \right) dr_2 \quad (44)$$

and this may be evaluated with computer algebra with radiative corrections. Finally a search may be made numerically for conditions under which $r(\text{vac})$ becomes very large. Under such conditions the atom may ionize, giving free electrons. This idea complements previous work [2–12] using spin connection resonance [17]. The overall method can be systematically extended by using quantum chemistry software to generate radial wave-functions with which radiative corrections can be considered.

4 Graphical Results for Hydrogen and Helium

In this section we present some graphical results for the spacetime interaction of hydrogen and helium. The corresponding calculations in the preceding paper are repeated with inclusion of the centrifugal term in the potential. As stated in section 2, this accounts for all angular momentum effects in the limit of averaged angular dependencies. The orbital averaging used in the preceding paper is no longer required. The method of calculating the equivalent radius function $r(\text{vac})$ for the shift of each orbital is the same as before: insert Eq. (12) into Eq. (10). With the wave functions of Table 1 computer algebra delivers the analytical solution for $r(\text{vac})$. For the $1s$ and $2s$ orbitals we obtain the same result as in the preceding paper since $l = 0$. For the $2p$ state the result is (in atomic units

for simplicity):

$$r(\text{vac})_{1,2} = \frac{1}{r^2 + (16\pi - 8)r - 16\pi + 8} \times \left(\pm 2\sqrt{2\pi r} \sqrt{(8\pi - 2)r^2 + (16 - 32\pi)r + 32\pi - 16} - r^3 + (8 - 8\pi)r^2 + (16\pi - 8)r \right) \quad (45)$$

with

$$\alpha_0 = \hbar = c = 4\pi\epsilon_0 = 1 \quad (46)$$

The dependence of $r(\text{vac})$ from the radius coordinate r was already shown in the preceding paper, Figs. 1c and 2c, for the $1s$ and $2s$ orbitals. The two solutions for $2p$ are graphed in Fig. 1 of this paper. The first solution shows a pole and is therefore considered to be non-physical. The second looks qualitatively similar to that obtained by the earlier averaging method (Fig. 3c of the preceding paper?????).

The Lamb shift itself is characterized by the difference of $r(\text{vac})$ values for $2s$ and $2p$. This is shown in Fig. 2. for both solutions of $2p$. The difference values for the second solution remain positive, indicating again the physical relevance of this solution.

In the simplest approach to He, hydrogen-like wavefunctions are being used (see section 3). In this approach the effective potentials decouple for both

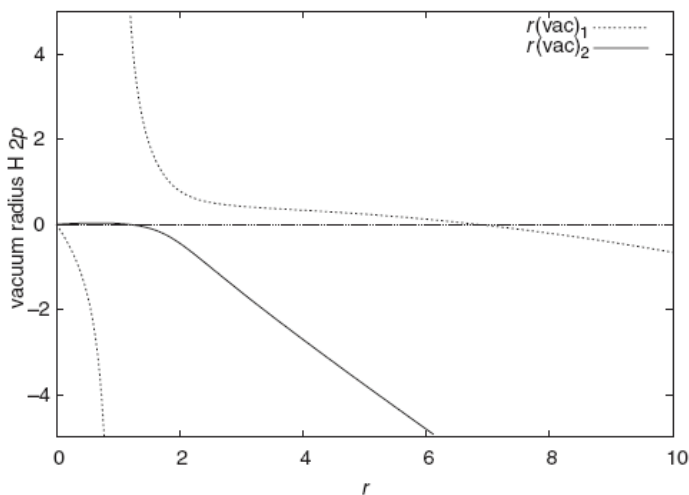


Fig. 1. Vacuum interaction radius $r(\text{vac})$ for H $2p$ orbital.

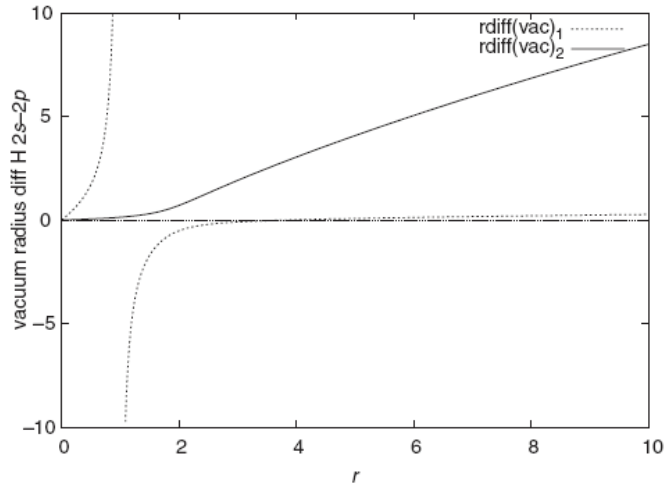


Fig. 2. Effective Lamb shift radius $r(\text{vac})(2s) - r(\text{vac})(2p)$.

electrons and $r(\text{vac})$ is the same in both cases. For the 1s state the Hydrogen-like wavefunction

$$\psi(1s)(r) = \sqrt{\frac{Z^3}{\pi a_0^3}} e^{-\frac{Zr}{a_0}} \quad (47)$$

is used which contains a parameter Z for the core charge. Solving Eqs. (42/43) for $r(\text{vac})$ then gives

$$r(\text{vac}) = \frac{r^2 Z - Z a_0 r}{r Z + 4 \frac{\pi m c}{\hbar} a_0^2 - Z a_0} \quad (48)$$

The curves of this function for both H and He are plotted in Fig. 3. It can be seen that the He curve is compressed in radial direction due to the higher core charge of He

Highly interesting is the effect of $r(\text{vac})$ on the exchange integral (16). Its definition then is to be modified to

$$J = \frac{e^2}{4\pi\epsilon_0} \int \frac{|\psi_1(r_1)|^2 |\psi_2(r_2)|^2}{|r_1 + r(\text{vac})(r_1) - r_2 - r(\text{vac})(r_2)|} d\tau_1 d\tau_2 \quad (49)$$

For two 1s electrons this gives in analogy to Eq. (44):

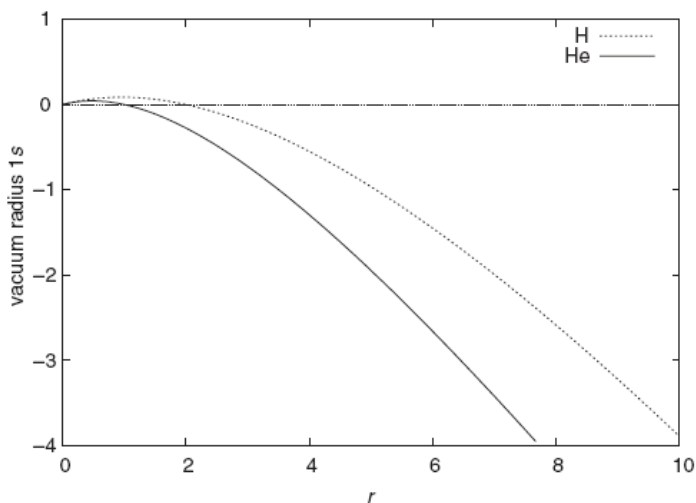


Fig. 3. Vacuum interaction radius $r(\text{vac})$ for H 1s and He 1s orbitals.

$$\begin{aligned}
 J = & \frac{e^2}{4\pi\epsilon_0} \left(\frac{Z^3}{\pi a_0^3} \right)^2 (4\pi)^2 \int_0^\infty (r_2 + r(\text{vac})(r_2)) e^{-\frac{2Zr_2}{a_0}} \\
 & \cdot \left(\int_0^{r_2+r(\text{vac})(r_1)} \frac{(r_1 + r(\text{vac})(r_1))^2}{r_2 + r(\text{vac})(r_2)} e^{-\frac{2Zr_1}{a_0}} dr_1 \right. \\
 & \left. + \int_{r_2+r(\text{vac})(r_1)}^\infty (r_1 + r(\text{vac})(r_2)) e^{-\frac{2Zr_1}{a_0}} dr_1 \right) dr_2
 \end{aligned} \tag{50}$$

This is a complicated function with integration limits depending on $r(\text{vac})$. Evaluation of J is feasible for a constant $r(\text{vac})$. The result is graphed in Fig. 4. For relatively small values of $r(\text{vac})$, J grows nearly exponentially over several orders of magnitude. Another feasible form of $r(\text{vac})$ is

$$r(\text{vac})(r_i) = r_i. \tag{51}$$

Since this is a single function, we do not obtain a curve but a single value for J in this case:

$$J = \frac{152}{27} \frac{e^2}{4\pi\epsilon_0} \frac{Z}{a_0}. \tag{52}$$

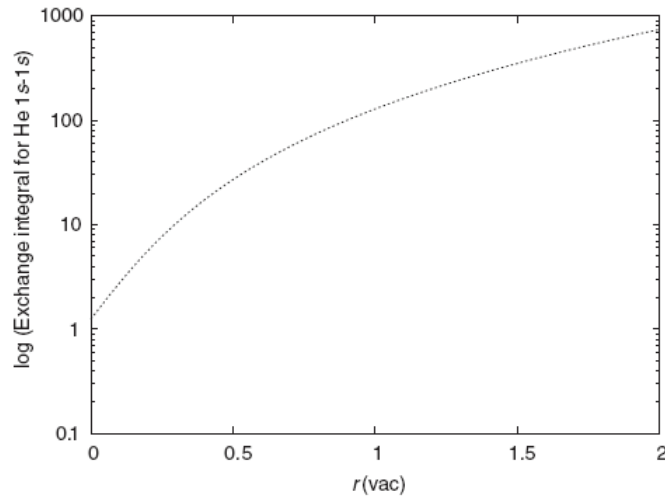


Fig. 4. $r(\text{vac})$ dependence of exchange integral J from for He $1s - 1s$.

Again this value is much greater than the original value of Eq. (20):

$$J = \frac{5}{8} \frac{e^2}{4\pi\epsilon_0} \frac{Z}{a_0}. \quad (53)$$

These results indicate that the interaction of atoms with the quantum background of spacetime has an enormous potential. The interaction should be exploitable via spin connection resonance in order to deliver energy from spacetime.

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