### RELATIVISTIC QUANTUM m THEORY

by

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## ABSTRACT

The relativistic quantization of m theory is developed along the lines of Dirac quantization. Using the H atom as an example it is shown that the nature of m space profoundly affects all aspects of the quantization: the main energy levels; the Zeeman effect, the second order effect in the vector potential, and the fine structure due to spin orbit interaction. Therefore the spectrum of an H atom in a strong gravitational field will be changed, and in general there will be radiative corrections due to energy from m space.

Keywords: ECE theory, relativistic quantum m theory.

4FT 428

In recent papers of this series  $\{1 - 41\}$  the ECE unified field theory in the most general spherically symmetric space (m theory) has been developed on the classical level using the three classical and complete dynamical systems: Euler Lagrange, Hamilton and Hamilton Jacobi. These systems have been augmented by a new system of classical dynamics, the Evans Eckardt system based on the constancy of the hamiltonian and the angular momentum. In Section 2 the relativistic quantization of m theory is developed along the lines of the well known Dirac quantization in flat or Minkowski spacetime. In the general, spherically symmetric, spacetime it is expected that radiative corrections will appear, because m theory is known on the classical level to produce a new source of energy which is not present in flat spacetime or its Newtonian limit. In preceding papers it has been shown that the energy from m space is precisely the same in the Euler Lagrange and Hamilton systems. On the classical level it gives rise to the spin connection and is a property of m space. One example of m space is the space of Einsteinian general relativity (EGR), in which the m (r) function is the so called Schwarzschild function. This shows that the albeit obsolete EGR gives energy from a well defined m space. This energy gives rise to the well known effects of general relativity, for example light deflection by gravitation, orbital precession and so on. It also gives rise to the well known radiative corrections such as the anomalous g factor of the electron and the Lamb shift. The ECE theory greatly improves and extends EGR in approximately seven hundred papers and books produced since 2003 and greatly develops the theory of the radiative corrections. Section 3 is an analysis and graphical development by co author Horst Eckardt.

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This paper is a short synopsis of extensive calculations in the notes accompanying UFT427 and UFT428 on <u>www.aias.us.</u> Only a small part of these calculations are used in the final paper and the notes should be regarded as being part of the complete paper. Note 427(1)

has been used for UFT427 on the Hamilton Jacobi formulation of m theory on the classical level. Note 427(2) is a comparison of the Hamilton Jacobi and Schroedinger equations of m theory. Note 427(3) develops quantization schemes for m theory. Note 427(4) reviews the well known Dirac theory and initiates its development into relativistic quantum m theory. Note 427(5) develops the hamiltonians of relativistic quantum m theory in preparation for future work.

#### 2. RELATIVISTIC QUANTIZATION of m THEORY

First review the Dirac theory, which applies in Minkowski sapce or in the space of ECE2 with finite torsion and curvature. The hamiltonian is:

$$H = E + T - (1)$$

where E is the total relativistic energy and U the potential energy. Here:  $E = \chi_{mc}^{2} = (c p + m c +)^{1/2} - (3)$ 

where the Lorentz factor is:

$$\gamma = \left( \begin{array}{c} 1 - \sqrt{N} \\ - \sqrt{2} \end{array} \right)^{-1/2} - (3)$$

and where v is the Newtonian linear velocity. The relativistic momentum is:

$$p = \Im n \vee N = (4)$$

In the H atom the potential energy between electron and proton is:

$$V = -\frac{e}{4\pi \epsilon_0 \epsilon} - (5)$$

where e is the charge on the proton,  $f_{\bullet}$  is the vacuum permittivity and r is the magnitude

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of the distance between the electron and proton. Finally c is the speed of light in vacuo and m the mass of the electron.

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Consider:

$$E^2 = c^2 p + m^2 c^4 - (b)$$

and write it as:

$$(E-mc)(E+mc) = cp^2 - (7)$$

It follows that:

Using:

+mc. - (8)= <u>cp</u> Etmc E E = H - U٩

+mc + U - (10)

The hamiltonian may be expressed as:

$$H = \frac{c p}{H - u + mc^{2}}$$

$$H = Vmc^2 + U - (11)$$

The well known Dirac approximations are:

 $U \langle \langle E - (12) \rangle$  $H \sim E \sim mc^2 - (13)$ 

and:

so the hamiltonian becomes:

hian becomes:  

$$H = \frac{2}{2m} \left( 1 - \frac{U}{2mc^{2}} \right)^{-1} + mc^{2} + W - (14)$$

Assuming that:

it follows that:

$$H \sim \frac{2}{2m} \left( 1 + \frac{U}{2mc^{2}} \right) + mc^{2} + U - (16)$$

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It follows that the hamiltonian minus the rest energy (the reduced hamiltonian) is:

$$H_{o} := H - mc^{2} = \frac{1}{2m} \left( 1 + \frac{U}{2mc} \right) + U - (\Pi)$$

and in the approximation ( 15) reduces to the Newtonian hamiltonian:

$$H_0 = \frac{p}{2n} + U - (18)$$

Dirac introduced the SU(2) basis to find that:

$$H = \frac{1}{2m} \underbrace{\overline{\sigma} \cdot p}_{-} \left( \frac{1 + \overline{U}}{2mc} \right) \underbrace{\overline{\sigma} \cdot p}_{-} + \frac{mc}{19} + \frac{1}{19} \underbrace{\overline{\sigma} \cdot p}_{-} + \frac{1}{19} \underbrace{\overline{\sigma} \cdot p}_$$

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In the presence of a magnetic field the minimal prescription gives:

$$p \rightarrow p - e A - (20)$$

where A is the vector potential, so  

$$H = \frac{1}{2m} \underbrace{\sigma} \cdot \left( p - e \underbrace{A} \right) \left( 1 + \frac{U}{2mc} \right) \underbrace{\sigma} \cdot \left( p - e \underbrace{A} \right) + mc^{2} + U$$

$$- (21)$$

is the well known Dirac hamiltonian. At ths point the theory is quantized using:

$$p q = -i f \nabla q - (22)$$

where  $\checkmark$  is the wave function.

The Dirac theory gives the main energy levels of the H atom, the Zeeman effect, and the fine structure due to spin orbit interaction, but gives an electronic g factor of exactly two and no Lamb shift.

In m theory the hamiltonian ( ) becomes:

$$H = m(r_i) \forall mc^2 + U - (23)$$

where the generalized Lorentz factor is:

$$\chi = \left(m\left(r_{1}\right) - \frac{\sqrt{3}}{c^{3}}\right)^{-1/3} - \left(34\right)$$

in a coordinate system defined by:

$$\Gamma_{1} = \frac{\Gamma}{m(\Gamma_{1})^{1/2}}, \quad \nabla_{1N} = \frac{V_{N}}{m(\Gamma_{1})^{1/2}} - (25)$$

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 $+m(r_{1})^{1/2}m_{1}^{2}+U_{-}^{2}$ 

It has been shown in immediately preceding papers that m theory produces:

$$E_{j} = w(L_{j})(c_{j}b_{j}^{j} + w_{j}c_{j}^{k}) - (q_{j}p_{j})$$

so:

$$(E-m(r_1)^{1/3}mc)(E+m(r_1)^{1/3}mc) = m(r_1)cp_1 - (27)$$

It follows as in Note 427(4) that the hamiltonian is:

$$H = \frac{m(r_i)c}{H - U + m(r_i)^{1/3} mc^3}$$

Now apply the Dirac type approximations:

 $U \langle \zeta E - (29) \rangle$ 

and

if

$$H \sim E \sim n(r_i) \forall nc^2 - (30)$$

From Eqs. (
$$\mathcal{M}_{+}$$
) and ( $\mathcal{S}_{0}$ )  
 $\mathcal{H} \sim \mathcal{M}(\mathcal{G}) \stackrel{1/3}{\longrightarrow} \left( \frac{1 - \mathcal{N}_{i\mathcal{M}}}{\mathcal{C}_{i\mathcal{M}}(\mathcal{G})} \right)^{-1/3} \mathcal{M}_{c} \rightarrow \mathcal{M}(\mathcal{G}) \stackrel{1/3}{\longrightarrow} \mathcal{M}(\mathcal{G}) \stackrel$ 

It follows from the Dirac type approximation that:

from the Dirac type approximation that:  

$$H = \frac{n(r_1)c}{2n(r_1)^{1/3}nc^2 - U} + n(r_1)^{1/3}nc^2 + U.$$

$$-(33)$$

2

The potential energy in frame  $(\Pi, \varphi)$  is:  $U = -m(r) \frac{e}{4\pi \epsilon r} := -m(r) \frac{1}{3} \overline{U}_{0}$   $U = -m(r) \frac{e}{4\pi \epsilon r} := -m(r) \frac{1}{3} \overline{U}_{0}$ 

so in frame 
$$(r, \phi)$$
 the hamiltonian is  

$$H = \frac{C p}{m(r)^{1/2}(2mc^2 - U_0)} + m(r)^{1/2}(nc^2 + U_0)$$
where:

wnere

$$\overline{U_0} = -\frac{e}{4\pi f_0 r} - (36)$$

Therefore the m theory hamiltonian prior to quantization is:  

$$H = \frac{1}{n(r)^{1/3}} \frac{1}{2n} \left( \frac{1 - \overline{U_0}}{2nc^3} - \frac{1}{37} + n(r) \right) \left( \frac{1}{nc^2 + U_0} - \frac{1}{37} \right)$$

$$U_0 \left( \left( \frac{1 - \overline{U_0}}{2nc^2} - \frac{1}{38} \right) \right)$$

:

and if

the hamiltonian is approximated by:  

$$\begin{aligned}
H \sim \frac{1}{n(r)} \frac{1}{r} \frac{p}{2m} \left( 1 + \frac{U_0}{2mc^2} + m(r) \frac{1}{r} \left( nc^2 + U_0 \right) - (3q) \right) \\
&= \frac{1}{n(r)} \frac{\sigma}{r} \frac{r}{r} \frac{\rho}{r} \left( \frac{1}{n(r)} \frac{1}{r} \left( 1 + \frac{U_0}{2mc^2} \right) \frac{\sigma}{r} \frac{\rho}{r} + m(r) \frac{1}{r} \frac{r}{r} \frac{r}{r} \frac{r}{r} \frac{1}{r} \\
&= \frac{1}{n(r)} \frac{\sigma}{r} \frac{r}{r} \frac{\rho}{r} \left( \frac{1}{n(r)} \frac{1}{r} \frac{\sigma}{r} \frac{r}{r} \frac{\rho}{r} \frac{1}{r} \frac{r}{r} \frac{r}{r} \frac{r}{r} \frac{r}{r} \right) \\
&= \frac{1}{n(r)} \frac{\sigma}{r} \frac{r}{r} \frac{r}{r}$$

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In Dirac theory the hamiltonian can be analyzed as the sum:  

$$H = nc + U_0 + H_1 + H_3 + H_4 - (4)$$
where:  

$$H_1 = \frac{1}{2n} - \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} - \frac{1}{2} \cdot \frac{1}{2} - \frac{1}{2} \cdot \frac{1}{2} - \frac{1}{2} \cdot \frac{1}{2} - \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} - \frac{1}{2} \cdot \frac{1}{2} \cdot$$

$$H_{2} = -\frac{e}{2m} \left( \overrightarrow{\sigma} \cdot \overrightarrow{A} \overrightarrow{\sigma} \cdot \overrightarrow{p} + \overrightarrow{\sigma} \cdot \overrightarrow{p} \overrightarrow{\sigma} \cdot \overrightarrow{A} \right) - (44)$$

$$H_{3} = \frac{e}{2m} \left( \overrightarrow{\sigma} \cdot \overrightarrow{A} \overrightarrow{\sigma} \cdot \overrightarrow{A} - (45) \right)$$

$$H_{3} = \frac{e}{2m} \left( \overrightarrow{\sigma} \cdot \overrightarrow{A} \overrightarrow{\sigma} \cdot \overrightarrow{A} - (45) \right)$$

$$H_{4} = \frac{1}{2m} \left( \overrightarrow{\rho} - e\overrightarrow{A} \right) \frac{V_{6}}{2mc^{2}} \overrightarrow{\sigma} \cdot \left( \overrightarrow{\rho} - e\overrightarrow{A} \right) - (46)$$

Eq. (43) is the free particle kinetic energy; Eq. (444) is the Zeeman effect hamiltonian; Eq. (45) is the second order hamiltonian and Eq. (46) is the fine structure hamiltonian.

In m theory these well known hamiltonians of Dirac theory ae developed as in Note

427(5) into:  $H = m(r)^{1/3} (mc^{2} + U_{0}) + H_{1} + H_{3} + H_{4} - (47)$ where:  $H_{1} = \frac{1}{2m} \stackrel{\sigma}{=} \cdot \stackrel{\rho}{P} \frac{1}{m(r)^{1/3}} \stackrel{\sigma}{=} \cdot \stackrel{\rho}{P} - (48)$   $H_{2} = -\frac{\varrho}{2m} \left( \stackrel{\sigma}{=} \cdot \stackrel{A}{A} \frac{1}{m(r)^{1/3}} \stackrel{\sigma}{=} \cdot \stackrel{\rho}{P} + \stackrel{\sigma}{=} \cdot \stackrel{\rho}{P} \frac{1}{m(r)^{1/3}} \stackrel{\sigma}{=} \cdot \stackrel{A}{A} \right)$   $H_{3} = \frac{\varrho}{2m} \stackrel{\sigma}{=} \cdot \stackrel{A}{A} \frac{1}{m(r)^{1/3}} \stackrel{\sigma}{=} \cdot \stackrel{A}{A} - (53)$   $H_{4} = \frac{1}{2m} \stackrel{\sigma}{=} \cdot (p - e\stackrel{A}{A}) \frac{U_{0}}{2mr^{2}m(r)^{1/3}} \stackrel{\sigma}{=} \cdot (p - e\stackrel{A}{A}) - (51)$ 

It is seen that each hamiltonian is changed by m(r)<sup>1/2</sup>. Therefore the spectral structure of atoms and molecules depends on the m space and the m(r) function. For example the g factor of the electron is no longer exactly two as in the Dirac theory, and there should be a Lamb shift. In a very intense gravitational field surrounded by an atmosphere containing

atoms and molecules, their spectra should be changed in several interesting ways.

For example the energy levels of the H atom are changed by m theory. In the Dirac

theory these energy levels are given by:

where:

So:

$$H_{n}p = E_{n}p - (52)$$
  
 $H = H_{1} + \overline{M}_{0} - (53)$ 

$$H = \frac{e}{2m} - \frac{e}{4\pi \epsilon_0 r} - (51)$$

and the energy levels are the expectation values:

$$E = \langle H \rangle = -\frac{h^2}{2m} \left[ \psi^* \nabla^2 dt - e + \frac{1}{4\pi} e \right] \psi^* \frac{1}{r} dt = -\frac{he}{32\pi^2 6^3 t^2 m} - \frac{1}{(55)} \psi^* \frac{1}{r} \frac{h}{r} dt = -\frac{he}{32\pi^2 6^3 t^2 m} + \frac{1}{4\pi} \frac{h}{r} \frac{h}{r}$$

where n is the principal quantum number:

$$n = 1, 3, 3, 4, \dots - (56)$$

and where the reduced mass if electron (m  $_{1}$  ) and proton (m  $_{2}$  ) is:

$$\mu = \frac{m_1 m_2}{m_1 + m_2} - (57)$$

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Here  $\psi$  are the well known hydrogenic wave functions used in several previous UFT papers.

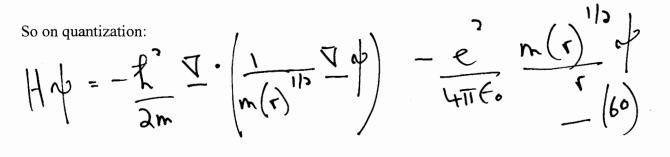
In m theory the energy levels of the H atom are given by:

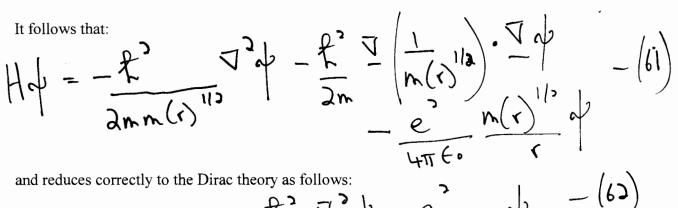
$$Hap = Eap - (58)$$

where:

ere:  

$$H = \frac{1}{2m} \frac{p}{r} \cdot \left( \frac{1}{m(r)^{1/2}} \frac{p}{r} \right) - m(r)^{1/2} U_0 - (59)$$





 $H \psi \xrightarrow{n(i) \rightarrow 1} - \frac{f^2}{2m} \sqrt{\psi} - \frac{e^2}{4\pi f_o r} \psi - \frac{e^2}{62}$ 

The energy levels ae the expectation values:

$$E = -\frac{1}{2m} \int_{-\frac{1}{2m}}^{\infty} \frac{1}{n(r)^{1/3}} \nabla^{2} \phi d\tau$$

$$-\frac{1}{2m} \int_{-\frac{1}{2m}}^{\infty} \frac{1}{r} \frac{1}{n(r)^{1/3}} \cdot \nabla \phi d\tau - (63)$$

$$-\frac{1}{2m} \int_{-\frac{1}{2m}}^{\infty} \frac{1}{r} \phi^{\frac{1}{2m}} \frac{1}{n(r)^{1/3}} \cdot \frac{1}{r} \phi d\tau$$

$$-\frac{1}{4\pi} \int_{-\frac{1}{2m}}^{\infty} \frac{1}{r} \phi^{\frac{1}{2m}} \frac{1}{r} \frac{1}{r} \phi^{\frac{1}{2m}} \frac{1}{r} \phi^{\frac{1}{2m}} \frac{1}{r} \frac$$

so each energy level is changed in a different way, giving rise to a new type of spectroscopy.

The hamiltonians will be developed in future work

# Relativistic quantum m theory

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## 3 Numerical calculations and graphics

The total energy of the Hydrogen atom was evaluated for different kinds of m functions. The total energy of relativistic m theory is given by Eq. (63), consisting of three parts:

$$E = E_1 + E_2 + E_3 \tag{64}$$

with

$$E_1 = -\frac{\hbar^2}{2m} \int \psi^* \frac{1}{\mathrm{m}(r)^{\frac{1}{2}}} \nabla^2 \psi \, d\tau,$$
 (65)

$$E_2 = -\frac{\hbar^2}{2m} \int \psi^* \nabla \left(\frac{1}{\mathrm{m}(r)^{\frac{1}{2}}}\right) \cdot \nabla \psi \, d\tau, \tag{66}$$

$$E_3 = -\frac{e^2}{4\pi\epsilon_0} \int \psi^* \frac{\mathbf{m}(r)^{\frac{1}{2}}}{r} \psi \, d\tau. \tag{67}$$

 $E_1$  and  $E_2$  are the terms of kinetic energy, while  $E_3$  describes the potential energy contribution. We evaluate the integrals in the approximation of using the non-relativistic wave functions of Hydrogen as obtained from the Schrödinger equation. The wave functions are a product of radial functions  $(R_{nl})$  and spherical harmonics  $(Y_{lm})$  as is well known. Omitting the quantum number indices we have

$$\psi(r,\theta,\phi) = R(r) Y(\theta,\phi).$$
(68)

The spherical harmonics obey the normalization condition

$$\int Y^* Y d\omega = 1 \tag{69}$$

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for equal quantum numbers, where  $d\omega$  is the differential two-dimensional surface element. The Laplace operator  $\nabla^2$  in  $E_1$  produces angular derivatives. These can be separated from the radial parts because the Laplacian is additive in its coordinate dependence:

$$\nabla^2 = \nabla_r^2 + \nabla_{\theta,\phi}^2. \tag{70}$$

Therefore  $E_1$  can be written as

$$E_{1} = -\frac{\hbar^{2}}{2m} \int R^{*}Y^{*} \frac{1}{\mathrm{m}(r)^{\frac{1}{2}}} \left(\nabla_{r}^{2} + \nabla_{\theta,\phi}^{2}\right) RY d\tau$$
(71)

$$= -\frac{\hbar^2}{2m} \int R^* \frac{1}{\mathrm{m}(r)^{\frac{1}{2}}} \left(\nabla_r^2 R\right) r^2 dr$$
(72)

$$-\frac{\hbar^2}{2m}\int \left(Y^*\nabla^2_{\theta,\phi}Yd\omega\right) R^*\frac{1}{\mathrm{m}(r)^{\frac{1}{2}}}Rr^2dr$$
(73)

where we have used the normalization condition (69). Thus the angular integration has been separated from the radial integrations. The angular integration can be executed analytically while the radial integration can be performed either analytically (if possible) or numerically.

Concerning  $E_2$  the integral contains a product of two gradients. Because the first gradient is zero in the components for  $\theta$  and  $\phi$ , there is only a product of radial gradients left, giving

$$E_2 = -\frac{\hbar^2}{2m} \int R^* \frac{\partial}{\partial r} \left(\frac{1}{\mathrm{m}(r)^{\frac{1}{2}}}\right) \frac{\partial R}{\partial r} r^2 dr.$$
(74)

The potential energy simplifies to

$$E_3 = -\frac{e^2}{4\pi\epsilon_0} \int R^* \frac{\mathbf{m}(r)^{\frac{1}{2}}}{r} R r^2 dr.$$
 (75)

For a constant m function, the integrals can be solved analytically. Using atomic units, the ground state energy of Hydrogen is -1/2 Hartree units. The factor m(r) := x then leads to expressions of

$$E(n=1) = \frac{1}{2\sqrt{x}} - \sqrt{x},$$
(76)

$$E(n=2) = \frac{1}{8\sqrt{x}} - \frac{\sqrt{x}}{4},$$
(77)

$$E(n=3) = \frac{1}{18\sqrt{x}} - \frac{\sqrt{x}}{9},\tag{78}$$

giving -1/2 for x = 1 as expected for example. The dependence of E from x is graphed in Fig. 1. The curves differ for the principal quantum number n only. Interestingly, all curves have the same zero crossing at x = 0.5 but it has to be kept in mind that the calculation was done with the undistorted wave functions. For large deviations of m from 1, the wave functions will be different.

For a non-constant m, the radial integrals have to be evaluated numerically. We consider two principal cases, where  $m(r) \leq 1$  and  $m(r) \geq 1$ . It turned out that the Schwarzschild-like m function is not suited for these calculations because it reaches to negative m valued near to r = 0, which leads to imaginary parts of the integrands. The exponential function is much better suited. For the two cases we used:

$$m_1(r) = 2 - \exp\left(\log(2)\exp(-\frac{r}{R})\right) \quad \text{for} \quad m(r) \le 1,$$
(79)

$$m_2(r) = \exp\left(\log(2)\exp(-\frac{r}{R})\right) \quad \text{for} \quad m(r) \ge 1.$$
(80)

The m functions of both cases are graphed in Fig. 2. The constant R is used as a parameter for the calculation. The resulting total energies are graphed in Figs. 3 and 4 for the case of  $m(r) \leq 1$ . For R = 0 we have the non-relativistic results of the Schrödinger equation with m(r) = 1. There is a general rise of energies for increasing R, i.e. when m takes values below 1. The changes are most significant for the 1s state. There is a dependence on the angular quantum number l which is not there in the non-relativistic case, producing a finestructure.

For  $m(r) \geq 1$  the effect is reversed, there is a deepening of energy levels, again with a fine structure splitting. This can be seen from Figs. 5 and 6. From relativistic local-density calculations of atomic binding energies it is known that energies of deep core states often come out too small. This could hint to relativistic effects of m theory where m(r) > 1 at the core position. From our experience in astronomy (S2 star) it is more plausible that m(r) < 1 near to the gravitational centre. Further experience has to show if there is such a difference between macro- and microcosm.

In a relativistic calculation with spin-orbit coupling (Dirac theory), the energy levels of  $2s_{1/2}$  and  $2p_{1/2}$  are identical. By m theory, however, there is a splitting of these levels which is generally attributed to vacuum effects (Lamb shift). Therefore m theory seems to produce the most general fine structure of atomic spectra.

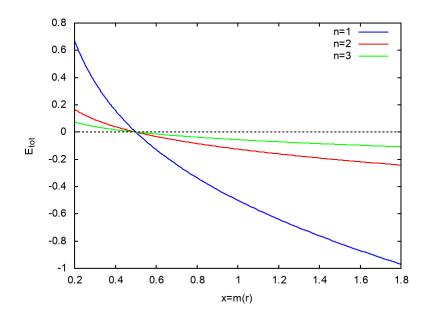


Figure 1: Total energy of H in dependence of m(r)=const.

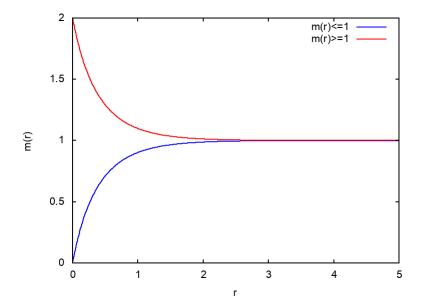


Figure 2: m functions of type  $m(r) \le 1$  and  $m(r) \ge 1$ .

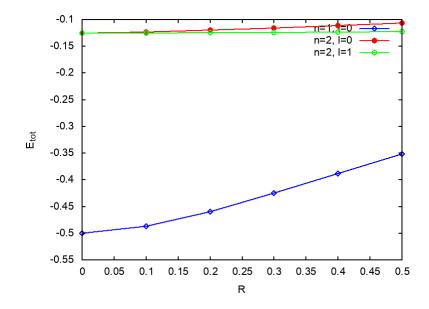


Figure 3: Total energy of H in dependence of R for  $m(r) \le 1, n = 1, 2$ .

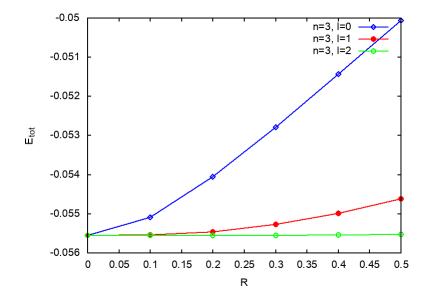


Figure 4: Total energy of H in dependence of R for  $m(r) \leq 1$ , n = 3.

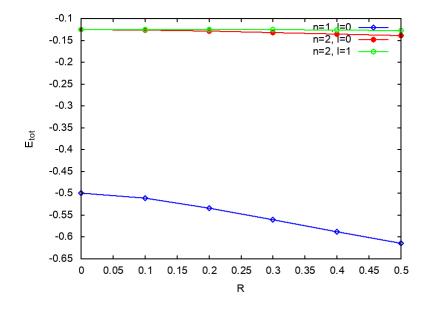


Figure 5: Total energy of H in dependence of R for  $m(r) \ge 1, n = 1, 2$ .

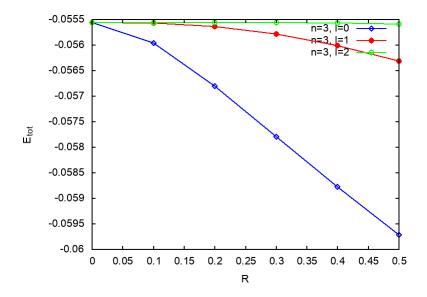


Figure 6: Total energy of H in dependence of R for  $m(r) \ge 1$ , n = 3.

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#### REFERENCES

{1} M. W. Evans, H. Eckardt, D. W. Lindstrom, D. J. Crothers and U. E. Bruchholtz,
 "Principles of ECE Theory, Volume Two" (ePubli, Berlin 2017).

{2} M. W. Evans, H. Eckardt, D. W. Lindstrom and S. J. Crothers, "Principles of ECE Theory, Volume One" (New Generation, London 2016, ePubli Berlin 2017).

{3} M. W. Evans, S. J. Crothers, H. Eckardt and K. Pendergast, "Criticisms of the Einstein Field Equation" (UFT301 on <u>www.aias.us</u> and Cambridge International 2010).

{4} M. W. Evans, H. Eckardt and D. W. Lindstrom "Generally Covariant Unified Field Theory" (Abramis 2005 - 2011, in seven volumes softback, open access in various UFT papers, combined sites <u>www.aias.us</u> and <u>www.upitec.org</u>).

{5} L. Felker, "The Evans Equations of Unified Field Theory" (Abramis 2007, open access as UFT302, Spanish translation by Alex Hill).

{6} H. Eckardt, "The ECE Engineering Model" (Open access as UFT203, collected equations).

{7} M. W. Evans, "Collected Scientometrics" (open access as UFT307, New Generation, London, 2015).

{8} M.W. Evans and L. B. Crowell, "Classical and Quantum Electrodynamics and the B(3) -Field" (World Scientific 2001, open access in the Omnia Opera section of <u>www.aias.us)</u>.

{9} M. W. Evans and S. Kielich, Eds., "Modern Nonlinear Optics" (Wiley Interscience, New York, 1992, 1993, 1997 and 2001) in two editions and six volumes, hardback, softback and e book.

{10} M. W. Evans and J. - P. Vigier, "The Enigmatic Photon" (Kluwer, Dordrecht, 1994 to
1999) in five volumes hardback and five volumes softback, open source in the Omnia Opera
Section of <u>www.aias.us).</u>

{11} M. W. Evans, Ed. "Definitive Refutations of the Einsteinian General Relativity"(Cambridge International Science Publishing, 2012, open access on combined sites).

{12} M. W. Evans, Ed., J. Foundations of Physics and Chemistry (Cambridge International Science Publishing).

{13} M. W. Evans and A. A. Hasanein, "The Photomagneton in Quantum Field Theory (World Scientific 1974).

{14} G. W. Robinson, S. Singh, S. B. Zhu and M. W. Evans, "Water in Biology, Chemistry and Physics" (World Scientific 1996).

{15} W. T. Coffey, M. W. Evans, and P. Grigolini, "Molecular Diffusion and Spectra" (Wiley Interscience 1984).

{16} M. W. Evans, G. J. Evans, W. T. Coffey and P. Grigolini", "Molecular Dynamics and the Theory of Broad Band Spectroscopy (Wiley Interscience 1982).

{17} M. W. Evans, "The Elementary Static Magnetic Field of the Photon", Physica B, 182(3), 227-236 (1992).

{18} M. W. Evans, "The Photon's Magnetic Field: Optical NMR Spectroscopy" (World Scientific 1993).

{19} M. W. Evans, "On the Experimental Measurement of the Photon's Fundamental Static
Magnetic Field Operator, B(3): the Optical Zeeman Effect in Atoms", Physica B, 182(3), 237
- 143 (1982).

{20} M. W. Evans, "Molecular Dynamics Simulation of Induced Anisotropy: I Equilibrium Properties", J. Chem. Phys., 76, 5473 - 5479 (1982).

ani fo

{21} M. W. Evans, "A Generally Covariant Wave Equation for Grand Unified Theory"Found. Phys. Lett., 16, 513 - 547 (2003).

{22} M. W. Evans, P. Grigolini and P. Pastori-Parravicini, Eds., "Memory FunctionApproaches to Stochastic Problems in Condensed Matter" (Wiley Interscience, reprinted2009).

{23} M. W. Evans, "New Phenomenon of the Molecular Liquid State: Interaction of Rotation and Translation", Phys. Rev. Lett., 50, 371, (1983).

{24} M.W. Evans, "Optical Phase Conjugation in Nuclear Magnetic Resonance: Laser NMRSpectroscopy", J. Phys. Chem., 95, 2256-2260 (1991).

{25} M. W. Evans, "New Field induced Axial and Circular Birefringence Effects" Phys. Rev.Lett., 64, 2909 (1990).

{26} M. W. Evans, J. - P. Vigier, S. Roy and S. Jeffers, "Non Abelian Electrodynamics",

"Enigmatic Photon V olume 5" (Kluwer, 1999)

{27} M. W. Evans, reply to L. D. Barron "Charge Conjugation and the Non Existence of the Photon's Static Magnetic Field", Physica B, 190, 310-313 (1993).

{28} M. W. Evans, "A Generally Covariant Field Equation for Gravitation and

Electromagnetism" Found. Phys. Lett., 16, 369 - 378 (2003).

{29} M. W. Evans and D. M. Heyes, "Combined Shear and Elongational Flow by Non Equilibrium Electrodynamics", Mol. Phys., 69, 241 - 263 (1988).

{30} Ref. (22), 1985 printing.

[31] M. W. Evans and D. M. Heyes, "Correlation Functions in Couette Flow from Group Theory and Molecular Dynamics", Mol. Phys., 65, 1441, - 1453 (1988).

{32} M. W. Evans, M. Davies and I. Larkin, Molecular Motion and Molecular Interaction in

the Nematic and Isotropic Phases of a Liquid Crystal Compound", J. Chem. Soc. Faraday II, 69, 1011-1022 (1973).

21 F

{33} M. W. Evans and H. Eckardt, "Spin Connection Resonance in Magnetic Motors",Physica B., 400, 175 - 179 (2007).

{34} M. W. Evans, "Three Principles of Group Theoretical Statistical Mechanics", Phys.Lett. A, 134, 409 - 412 (1989).

{35} M. W. Evans, "On the Symmetry and Molecular Dynamical Origin of Magneto Chiral Dichroism: "Spin Chiral Dichroism in Absolute Asymmetric Synthesis" Chem. Phys. Lett., 152, 33 - 38 (1988).

{36} M. W. Evans, "Spin Connection Resonance in Gravitational General Relativity", ActaPhysica Polonica, 38, 2211 (2007).

{37} M. W. Evans, "Computer Simulation of Liquid Anisotropy, III. Dispersion of the Induced Birefringence with a Strong Alternating Field", J. Chem. Phys., 77, 4632-4635 (1982).

{38} M. W. Evans, "The Objective Laws of Classical Electrodynamics, the Effect of Gravitation on Electromagnetism" J. New Energy Special Issue (2006).

{39} M. W. Evans, G. C. Lie and E. Clementi, "Molecular Dynamics Simulation of Water from 10 K to 1273 K", J. Chem. Phys., 88, 5157 (1988).

{40} M. W. Evans, "The Interaction of Three Fields in ECE Theory: the Inverse FaradayEffect" Physica B, 403, 517 (2008).

{41} M. W. Evans, "Principles of Group Theoretical Statistical Mechanics", Phys. Rev., 39, 6041 (1989).