EFFECT OF VACUUM FLUCTUATIONS ON THE MAGNETIC DIPOLE POTENTIAL

- 1

AND FIELDS.

by

M. W. Evans and H. Eckardt,

Civil List and AIAS / UPITEC

(www.aias.us, , www.upitec.org, www.et3m.net, www.archive.org, www.webarchive.org.uk)

ABSTRACT

Using the macroscopic zitterbewegung (MZ) theory of the two immediately preceding papers of this series, it is shown that the familiar magnetic dipole potential and field develops intricate structures when vacuum fluctuations are considered. It is shown that the isotropically averaged contact term of the magnetic dipole flux density no longer vanishes, and that the isotropically averaged magnetic potential and magnetic dipole flux density develop intricate properties produced by the vacuum fluctuations. These are observable in hyperfine structure.

Keywords: ECE2 theory, MZ theory, vacuum induced structure in magnetic dipole fields.

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1. INTRODUCTION

In the two immediately preceding papers of this series {1 - 41}, UFT392 and UFT393, the well known concept of zitterbewegung (shivering induced by the vacuum) has been developed on the macroscopic level and for the whole of physics. This has been named macroscopic zitterbewegung (MZ) theory. UFT392 considered the Coulomb field and UFT393 considered the electric dipole field and potential. In this paper, MZ theory is extended in section 2 to the familiar magnetic dipole potential and fields used in NMR theory for example. The effect of vacuum fluctuations can be observed in hyperfine structure. In Section 3 the analytical results are evaluated numerically using isotopic averaging as in UFT393, and the graphical results show intricate new structures induced by the vacuum. These structures are expected to exist in all areas of physics, a major advance in understanding.

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This paper is a short synopsis of detailed calculations in the notes accompanying UFT394 on <u>www.aias.us.</u> Note 394(1) defines the magnetic dipole flux density and potential for a current loop. Note 394(2) applies antisymmetry, and Note 394(3) is a preliminary development of antisymmetry in MZ theory. It was decided to revert to the original Lindstrom law of trace antisymmetry, so this note is not used. Note 394(4) is a preliminary development of MZ theory for electrodynamics. Note 394(5) shows that vector antisymmetry is conserved automatically in MZ theory. Notes 394(6) and 394(6a) - 394(8) form the basis for Section 2.

2. MZ THEORY OF THE MAGNETIC DIPOLE POTENTIAL AND FIELDS

It is proposed that vacuum fluctuations in macroscopic physics introduce fluctuations in the frame of reference. So, for example, the position vector:

f = Xi + Yj + 7k - (1)

is changed to

$$\underline{r} + \delta \underline{r} = (x + \delta x) \underline{i} + (y + \delta y) \underline{j} + (z + \delta z) \underline{k} - (3)$$

in which fluctuations of the Cartesian coordinates are induced by the vacuum. This is the same idea as the accurate zitterbewegung theory of the Lamb shift. In the end result of a calculation that is carried through with the position vector \mathbf{r} , the result is modified by the frame change:

$$\underline{\Gamma} \rightarrow \underline{\Gamma} + S\underline{\Gamma}, -(4)$$

The well known magnetic dipole potential used in NMR theory {1 - 41} is:

$$A = \mu_{0} \qquad m \times r \qquad -(5)$$

where <u>m</u> is the magnetic dipole moment and \mathcal{M}_{o} the S. I. permeability in vacuo. Therefore the effect of vacuum fluctuations \mathfrak{S}_{Σ} is as follows:

As in previous work

$$\frac{\Gamma \rightarrow \Gamma + \delta \Gamma - (6)}{\Gamma = |\Gamma| \rightarrow |\Gamma + \delta \Gamma| - (7)}$$
pus work:

$$|\Gamma + \delta \Gamma| = (r^{2} + \lambda \Gamma \cdot \delta \Gamma + \delta \Gamma \cdot \delta \Gamma)^{1/2}$$

$$:= \Gamma (1+x)^{1/2} - (8)$$

where

$$x = \frac{1}{r^{2}} \left(\frac{2r \cdot \delta r}{r} + \frac{\delta r}{s} \cdot \frac{\delta r}{s} \right) - (9)$$

So the magnetic dipole potential in the presence of the vacuum is:

$$\underline{A} = \underbrace{\mu_{o}}_{4\pi} \underbrace{\underline{m} \times (\underline{r} + \underline{s}\underline{r})}_{|\underline{r} + \underline{s}\underline{r}|^{3}} - (10)$$

$$= \frac{\mu_0}{4\pi r^3} m \times (\underline{r} + \delta \underline{r}) (1+2)^{-3/2}$$

Using the well known expansion:

$$(1+3)^{-3/2} = 1-3x + \frac{15}{2}x^{2} + \cdots + \frac{11}{8}x^{2} + \cdots + \frac{11}{8}x^{2}$$

the vector potential is:

$$\underline{A} = \underbrace{\mu_{o}}_{4\pi\pi} \underbrace{m}_{s} \times (\underline{r} + \underline{\delta r}) \left(1 - \frac{3}{2} + \frac{15}{8} \times + ...\right) - (12)$$

and may be averaged as in UFT393 using the isotropy assumptions:

 $\langle S_{\underline{r}} \rangle = 0$; $\langle S_{\underline{r}} \cdot S_{\underline{r}} \rangle \neq 0$. -(13)

The dipole magnetic flux density due to the dipole potential (5) is $\{1 - 41\}$:

$$\underline{B} = \underline{\nabla} \times \underline{A} = -\frac{\mu_0}{4\pi r} \underline{m} \nabla^2 \underline{I} + \frac{\mu_0}{4\pi r^3} \begin{pmatrix} \underline{3m} \cdot \underline{rr} \\ \underline{r}^2 \end{pmatrix} - \begin{pmatrix} \underline{14} \end{pmatrix}$$

The contact magnetic flux density is:

$$B_{c} = -\mu_{o} m \overline{V} \left(\frac{1}{r} \right) - (.15)$$

and the magnetic dipole flux density is:

$$\underline{B}_{D} = \underbrace{\mu_{0}}_{4\pi\tau^{3}} \left(\underbrace{\underline{Sm}}_{r} \cdot \underbrace{\underline{rr}}_{r} - \underline{\underline{m}}_{r} \right) - (16)$$

which has the same structure as the electric dipole flux density of UFT393. In the presence of the vacuum the magnetic dipole flux density becomes:

$$\underline{B} = \underbrace{\mu_{\bullet}}_{4\pi\tau^{5}} \left(\underline{r} + \delta \underline{r} \right) \left(\underline{\rho} \cdot (\underline{r} + \delta \underline{r}) \left(1 - 5\underline{x} + 35\underline{s}\underline{c}^{2} + \cdots \right) \right)$$
$$- \underbrace{\mu_{\bullet}}_{4\pi\tau^{5}} \left(1 - 3\underline{x} + 15\underline{x}^{2} + \cdots \right) - (17)$$
$$- \underbrace{\mu_{\bullet}}_{4\pi\tau^{5}} \left(1 - 3\underline{x} + 15\underline{x}^{2} + \cdots \right) \right)$$

and may be isotropically averaged in the same was as in UFT393 for the electric dipole field strength

The effect of vacuum fluctuations on the contact field (15) may be developed using the result:

$$\sqrt[n]{1}{r} = -\frac{3}{r^3} + \frac{3}{r} = 0 - (18)$$

In the presence of vacuum fluctuations, Eq. (18) becomes:

$$\nabla^{3}\left(\frac{1}{\varsigma_{1}}\right) = -\frac{3}{\varsigma^{3}\left(1+\varsigma_{2}\right)^{3/2}} + \frac{3\left(\frac{\varsigma}{\varsigma}+\delta_{1}\right)\cdot\left(\frac{\varsigma}{\varsigma}+\delta_{1}\right)\cdot\left(\frac{\varsigma}{\varsigma}+\delta_{1}\right)}{r^{5}\left(1+\kappa\right)^{5/2}} - (19)$$

$$= -\frac{3}{\varsigma^{3}}\left(1-3\kappa+\frac{15}{8}\kappa^{2}+\cdots\right) + \frac{3}{\varsigma}\left(\frac{\varsigma}{\varsigma}+\delta_{1}\right)\cdot\left(\frac{\varsigma}{\varsigma}+\delta_{1}\right)\left(1-\frac{5\kappa}{2}+\frac{35}{8}\kappa^{2}+\cdots\right) + \frac{3}{\varsigma}\left(\frac{\varsigma}{\varsigma}+\delta_{1}\right)\cdot\left(\frac{\varsigma}{\varsigma}+\delta_{1}\right)\left(1-\frac{5\kappa}{2}+\frac{35}{8}\kappa^{2}+\cdots\right) + \frac{3}{\varsigma}\left(\frac{\varsigma}{\varsigma}+\delta_{1}\right)\cdot\left(\frac{\varsigma}{\varsigma}+\delta_{2}\right)\left(1-\frac{5\kappa}{2}+\frac{35}{8}\kappa^{2}+\cdots\right) + \frac{3}{\varsigma}\left(\frac{\varsigma}{\varsigma}+\delta_{1}\right)\cdot\left(\frac{\varsigma}{\varsigma}+\delta_{2}\right)\left(1-\frac{5\kappa}{2}+\frac{35}{8}\kappa^{2}+\frac{35}{8}\kappa^{2}+\cdots\right) + \frac{1}{\varsigma}\left(1-\frac{5\kappa}{2}+\frac{35}{8}\kappa^{2}+\frac{35}{8$$

and the contact term is no longer zero. Its isotopic average is worked out by computer in

Section (3):

$$\left\langle \begin{array}{c} B\left(c_{a}Jad\right)\right\rangle = -\mu_{o} m \left\langle J\left(\frac{1}{r_{1}}\right)\right\rangle^{-(2)}$$

$$\left\langle H\left(\frac{1}{r_{1}}\right)\right\rangle^{-(2)}$$

where:

$$\underline{\Gamma}_1 = \underline{\Gamma} + S \underline{\Gamma} - (2i)$$

Eq. ($\mathbf{\lambda}\mathbf{0}$) can be computed to any order in x.

As shown in section 3 these procedures lead to intricate structures induced by vacuum fluctuations, structures which are absent completely from standard physics, but which are nevertheless obtained with the same type of shivering motion as considered in the accurate zitterbewegung theory of the Lamb shift.

The MZ theory removes the contradiction inherent in standard physics, which

asserts that:

$$\underline{B}_{c} = \mu_{o} \underline{m} \delta_{0}(r) - (22)$$

where $\delta_{\mathbf{y}}$ is the Dirac delta function. However, direct differentiation using computer algebra gives the result:

$$\underline{B}_{c} = \underbrace{\mu_{o} \underline{m}}_{4\pi} \sqrt[2]{\frac{1}{r}} = \underbrace{0}_{r} - (23)$$

This is why mathematicians contemporary with Dirac rejected the Dirac delta function as pure nonsense. In MZ theory, the Dirac delta function is not used and is not needed.

Finally, the spin connection for any magnetic flux density B in the presence of the vacuum is defined by:

$$\underline{B} = \overline{2} \times \underline{A}_{\bullet} - \underline{\Theta} \times \underline{A}_{\bullet} - (24)$$

where A is the vector potential in the hypothetical absence of the vacuum. So the vector spin

3. NUMERICAL AND GRAPHICAL ANALYSIS.

Section by Horst Eckardt.

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M. W. Evans, H. Eckardt[†] Civil List, A.I.A.S. and UPITEC

(www.webarchive.org.uk, www.aias.us, www.atomicprecision.com, www.upitec.org)

3 Numerical and graphical analysis

The magnetic dipole field is formally identical to the electrical dipole field discussed in UFT 393. The same holds when zitterbewegung is added. The magnetic dipole field (16) is in linear approximation of x which is defined in Eq. (9):

$$\langle \mathbf{B}_D \rangle^{(2)} = \frac{\mu_0}{4\pi r^3} \left(\frac{3\mathbf{r}(\mathbf{m} \cdot \mathbf{r})}{r^2} - \mathbf{m} \right)$$

$$- \frac{\mu_0}{4\pi r^5} \left\langle \delta \mathbf{r} \cdot \delta \mathbf{r} \right\rangle \left(\frac{35\,\mathbf{r}(\mathbf{p} \cdot \mathbf{r})}{2\,r^2} - \frac{5}{2}\mathbf{p} \right) - \frac{5\mu_0}{8\pi r^7} \left\langle (\delta \mathbf{r} \cdot \delta \mathbf{r})^2 \right\rangle \mathbf{m}.$$

$$(25)$$

This gives quadratic $\delta \mathbf{r}$ terms, i.e. in proportion to $\langle \delta \mathbf{r} \cdot \delta \mathbf{r} \rangle$. In additon, a fourth-order term appears which is not complete as discussed in UFT 393. The quadratic approximation in x gives correct fourth-order terms and a sixth-order term in $\delta \mathbf{r}$:

$$\langle \mathbf{B}_D \rangle^{(4)} = \frac{\mu_0}{4\pi r^3} \left(\frac{3\mathbf{r}(\mathbf{m} \cdot \mathbf{r})}{r^2} - \mathbf{m} \right)$$

$$+ \frac{\mu_0}{4\pi r^7} \left\langle (\delta \mathbf{r} \cdot \delta \mathbf{r})^2 \right\rangle \left(\frac{1435 \, \mathbf{r}(\mathbf{m} \cdot \mathbf{r})}{24 \, r^2} + \frac{35}{24} \mathbf{m} - \frac{35}{3r^2} \begin{bmatrix} X^2 & 0 & 0\\ 0 & Y^2 & 0\\ 0 & 0 & Z^2 \end{bmatrix} \mathbf{m} \right)$$

$$+ \frac{\mu_0}{4\pi r^9} \left\langle (\delta \mathbf{r} \cdot \delta \mathbf{r})^3 \right\rangle \frac{35}{8} \mathbf{m}.$$

$$(26)$$

For the magnetic dipole the contact term vanishes but there are shivering contributions. Using the same methods as for Eqs. (25, 26) we obtain for the

^{*}email: emyrone@aol.com

[†]email: mail@horst-eckardt.de

contact term given by (15):

$$\left\langle \mathbf{B}_{C} \right\rangle^{(2)} = \frac{\mu_{0}}{4\pi} \mathbf{m} \left(\frac{10 \left\langle \left(\delta \mathbf{r} \cdot \delta \mathbf{r} \right) \right\rangle}{r_{0}^{5}} + \frac{15 \left\langle \left(\delta \mathbf{r} \cdot \delta \mathbf{r} \right)^{2} \right\rangle}{2r_{0}^{7}} \right), \tag{27}$$

$$\left\langle \mathbf{B}_{C}\right\rangle^{(4)} = -\frac{\mu_{0}}{4\pi} \mathbf{m} \left(\frac{105 \left\langle (\delta \mathbf{r} \cdot \delta \mathbf{r})^{2} \right\rangle}{2r_{0}^{7}} + \frac{105 \left\langle (\delta \mathbf{r} \cdot \delta \mathbf{r})^{3} \right\rangle}{8r_{0}^{9}} \right).$$
(28)

Figs. 1 and 2 show the dipole fields of UFT 393 again in 2nd and 4th order $\delta \mathbf{r}$ approximation. There are additional central structures appearing which were discussed in UFT 393. Figs. 3 and 4 show the magnetic contact terms in both approximations. Outside the centre, these are constant in direction of the magnetic dipole \mathbf{m} which was chosen in vertical direction. Near to the centre there is a strong shear strain in form of a rotation. The directions of both approximations differ in sign but are quite similar else.

When these contact terms are added to the dipole fields of Figs. 1 and 2, the structures of Figs. 5 and 6 result. The contact terms break symmetry of the magnetic dipoles. In Fig. 5 (quadratic approximation) four spiraling field structures appear in addition to the upper and lower divergence regions of the undistorted case. The inner spirals are not symmetric to the vertical symmetry axis. When the intersecting plane is rotated around Z, this gives an oblique ring which is tilted against the outer more symmetrically positioned ring. In the 4thorder approximation (Fig. 6) only two spirals remain but rotated against the symmetry plane so that these represent a tilted ring in 3D. The rings could represent currents in counter direction. There are indeed certain models of elementary particles which assume such a structure.

The divergence and curl of the field of Fig. 6 has been graphed in Figs. 7 and 8 in the same way as described in UFT 393. There is a significant curl of the field. In this case it is a magnetic field. For the static case follows from the ECE2 field equations:

$$\boldsymbol{\nabla} \times \mathbf{B} = \mu_0 \mathbf{J} \neq \mathbf{0}. \tag{29}$$

This means that an electric vacuum current is induced by the zitterbewegung. This is a consequence of the contact term which is zero in average but not at each instance of time. If we had a contact term in case of the electric dipole, this would mean that there is a magnetic monopole current.

The spiralling structure appearing in Figs. 5 and 6 is a mixture between a source field and a rotational field, reminding to the Lense-Thirring effect in astronomy. The source part of the magnetic field leads to

$$\boldsymbol{\nabla} \cdot \mathbf{B} \neq \mathbf{0}. \tag{30}$$

This means there are fluctuating magnetic charges which average out over time. The shivering of dipoles gives a lot of interesting insights to the vacuum.



Figure 1: Dipole field with variable shivering radius, quadratic terms.



Figure 2: Dipole field with variable shivering radius, 4th-order terms.



Figure 3: Contact term, quadratic approximation.



Figure 4: Contact term, 4th-order approximation.



Figure 5: Total field with contact term, quadratic approximation.



Figure 6: Total field with contact term, 4th-order approximation.



Figure 7: Divergence plot of Fig. 6 in the (r_X, r_Y) plane.



Figure 8: Curl **B** plot of Fig. 6 perpendicular to the (r_X, r_Y) plane.

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