THE CLASSICAL GYROSCOPE AS A THREE DIMENSIONAL ORBIT.

by

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ABSTRACT

The motion of the classical gyroscope is described in terms of spherical polar coordinates and it is shown that the gravitational attraction between its centre of mass and the earth's mass is countered by three dimensional centrifugal and Coriolis forces. The overall motion of the centre of mass is intricate and is governed by the expression for acceleration n spherical polars. The gyroscope's point of contact with the earth's surface may be elevated by an additional force or torque. ECE2 fluid dynamics is used to propose one such origin.

Keywords: ECE2 theory, Gyroscope in classical dynamics, force due to convective derivative.

UFT367

1. INTRODUCTION

In recent papers of this series {1 - 12} ECE2 fluid dynamics has been developed systematically, the basic concept being that spacetime, aether or vacuum is developed as a fluid governed by the laws of fluid dynamics in ECE2 format. In Section 2 the method is extended to rotational motion, the gyroscope being used as an example. This paper is a summary of detailed calculations found in the notes accompanying UFT367 on www.aias.us. Note 367(1) summarizes motion in plane polar coordinates and introduces the effect of the fluid spacetime through the convective derivative of a velocity field. Note 367(2) introduces the three dimensional Euler equations and discusses the basic concept of motion in a rotating frame - the dynamics of the axes themselves. Note 367(3) calculates the effect of adding the convective derivative of angular momentum to the laboratory frame torque on a gyro. The convective derivative represents the effect of a fluid spacetime on the gyro. Note 367(4) considers gyroscope theory in classical dynamics, with the addition of the convective derivative. Note 365(5) is a force based evaluation of the gyroscope with considerations of the extra force due to the convective derivative, so the complete derivative of velocity becomes the convective derivative. Note 367(6) is the lagrangian development of the motion of a symmetric top with one point fixed. The geometry of this note can be adjusted to describe the well known experiment by Laithwaite in which the gyroscope is horizontal. Note 367(7) defines the linear velocity and acceleration in spherical polar coordinates. Note 367(8) defines moving frame forces in terms of spherical polar coordinates and is the basis of Section 2 of this paper.

2. THE GYROSCOPE IN SPHERICAL POLAR COORDINATES

The fundamental concept of the moving frame (1, 2, 3) is the motion of its unit

vectors:

$$\frac{d\underline{\ell}_i}{dt} = \underline{\omega} \times \underline{\ell}_i \quad , \quad i = 1, 2, 3 \quad -(1)$$

where $\underline{\omega}$ is the angular velocity:

$$\underline{\omega} = \omega_1 \underline{\ell}_1 + \omega_2 \underline{\ell}_2 + \omega_3 \underline{\ell}_3. \quad -(2)$$

Therefore the force in frame (1, 2, 3) is defined {1 - 12} by:

$$\underline{F} = m \left(\frac{d\underline{v}}{dt} + \underline{\omega} \times \underline{v} \right) \quad -(3)$$

where the velocity is defined by:

$$\underline{v} = \frac{d\underline{r}}{dt} + \underline{\omega} \times \underline{r} \quad -(4)$$

Fig. (1) defines frame (1, 2, 3) in terms of spherical polar coordinates:

$$\begin{aligned} r_1 &= r \sin\theta \cos\phi \\ r_2 &= r \sin\theta \sin\phi \\ r_3 &= r \cos\theta \end{aligned} \quad -(5)$$
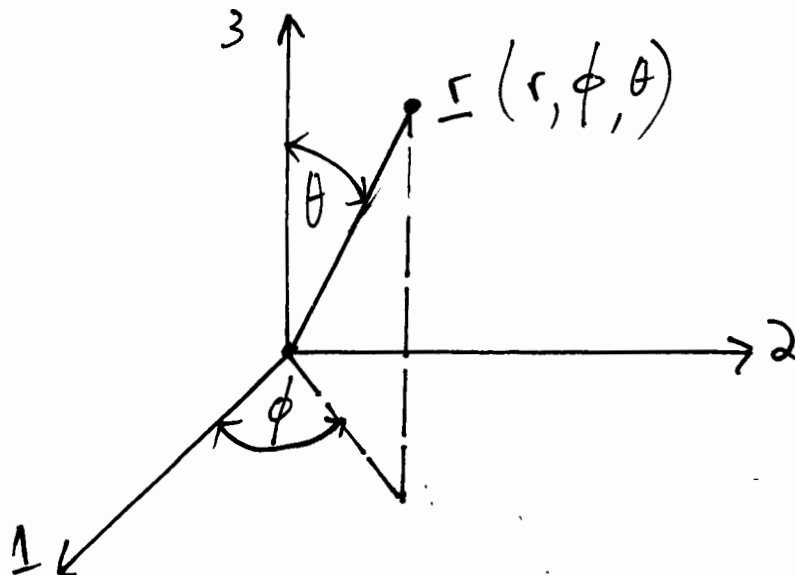

Fig. (1)

The force in the moving frame is:

$$\underline{F} = F_1 \underline{e}_1 + F_2 \underline{e}_2 + F_3 \underline{e}_3 \qquad - (6)$$

$$= F_r \underline{e}_r + F_\theta \underline{e}_\theta + F_\phi \underline{e}_\phi$$

where:

$$F_1 = m\left(\frac{dV_1}{dt} + (\omega_2 V_3 - \omega_3 V_2)\right)$$

$$F_2 = m\left(\frac{dV_2}{dt} + (\omega_3 V_1 - \omega_1 V_3)\right) \qquad - (7)$$

$$F_3 = m\left(\frac{dV_3}{dt} + (\omega_1 V_2 - \omega_2 V_1)\right)$$

and:

$$F_r = m\left(\ddot{r} - r\dot{\theta}^2 - r\sin^2\theta \, \dot{\phi}^2\right)$$

$$F_\theta = m\left(2\dot{r}\dot{\theta} + r\ddot{\theta} - r\sin\theta\cos\theta \, \dot{\phi}^2\right) \qquad - (8)$$

$$F_\phi = m\left(2\dot{r}\dot{\phi}\sin\theta + 2r\dot{\theta}\dot{\phi}\cos\theta + r\sin\theta \, \ddot{\phi}\right)$$

The force in the laboratory frame (X, Y, Z) is:

$$\underline{F} = F_X \underline{i} + F_Y \underline{j} + F_Z \underline{k} . \qquad - (9)$$

By galilean invariance in classical dynamics:

$$F_1^2 + F_2^2 + F_3^2 = F_r^2 + F_\theta^2 + F_\phi^2 \qquad - (10)$$

$$= F_X^2 + F_Y^2 + F_Z^2 .$$

The force in the laboratory frame is the gravitational force on the centre of mass of

the gyro of mass m:

$$F = mg\,\underline{k} = m\frac{d^2\underline{r}}{dt^2} \quad - (11)$$

where the acceleration due to the earth's gravity is:

$$g = -\frac{mG}{R^2} \quad - (12)$$

where M and R are the mass and radius of the earth and G is Newton's constant. In spherical

polar coordinates the radius vector is defined as:

$$\underline{r} = r\,\underline{\ell}_r \quad - (13)$$

where $\underline{\ell}_r$ is the radial unit vector of the spherical polar coordinates. Note that the gyro is

always governed by Eq. (11) and its point of contact cannot be elevated. This is every day

experience as in a spinning top. In general:

$$\begin{aligned}
\underline{F} &= F_x\,\underline{i} + F_y\,\underline{j} + F_z\,\underline{k} \quad - (14)\\
&= F_r\,\underline{\ell}_r + F_\theta\,\underline{\ell}_\theta + F_\phi\,\underline{\ell}_\phi\\
&= F_1\,\underline{\ell}_1 + F_2\,\underline{\ell}_2 + F_3\,\underline{\ell}_3.
\end{aligned}$$

In the gyro:

$$\underline{F} = mg\,\underline{k} = F_r\,\underline{\ell}_r + F_\theta\,\underline{\ell}_\theta + F_\phi\,\underline{\ell}_\phi. \quad -(15)$$

Note that if:

$$\dot{\theta} = \dot{\phi} = 0 \quad - (16)$$

then:

$$\underline{F} = F_r \, \underline{e}_r = m\ddot{r} \, \underline{e}_r \qquad -(17)$$

and:

$$\underline{F} = mg \, \underline{k} = m\ddot{r} \, \underline{e}_r. \qquad -(18)$$

This describes a non spinning gyro, its motion is pure gravitational attraction between m and M. When the gyro is spun, Eq. (15) applies and the force of gravitation is counterbalanced as follows.

In Eq. (15):

$$\underline{e}_r = \sin\theta\cos\phi \, \underline{i} + \sin\theta\sin\phi \, \underline{j} + \cos\theta \, \underline{k}. \qquad -(19)$$

$$\underline{e}_\theta = \cos\theta\cos\phi \, \underline{i} + \cos\theta\sin\phi \, \underline{j} - \sin\theta \, \underline{k} \qquad -(20)$$

$$\underline{e}_\phi = -\sin\phi \, \underline{i} + \cos\phi \, \underline{j} \qquad -(21)$$

so equating k components:

$$\underline{F} = mg \, \underline{k} = \left(F_r \cos\theta - F_\theta \sin\theta \right) \underline{k}. \qquad -(22)$$

It follows that:

$$\ddot{r}\cos\theta = g + \left(r\dot{\theta}^2 + r\sin^2\theta \, \dot{\phi}^2 \right)\cos\theta \qquad -(23)$$
$$+ \left(2\dot{r}\dot{\theta} + r\ddot{\theta} - r\sin\theta\cos\theta \, \dot{\phi}^2 \right)\sin\theta.$$

In the absence of spin:

$$\dot{\theta} = \dot{\phi} = 0 \qquad -(24)$$

and:

$$g = \ddot{r}\cos\theta \qquad -(25)$$
$$d = -mG/R^3$$

Therefore the dynamics of the gyroscope are described by:

$$\ddot{r}\cos\theta = -\frac{mG}{R^2} + \left(r\dot{\theta}^2 + r\sin^2\theta\,\dot{\phi}^2\right)\cos\theta \qquad -(26)$$
$$+\left(2\dot{r}\dot{\theta} + r\ddot{\theta} - r\sin\theta\cos\theta\,\dot{\phi}^2\right)\sin\theta$$

and the force of attraction is counterbalanced by three dimensional centrifugal and Coriolis forces.

In the special case:

$$\theta = 0, \quad R = r \qquad -(27)$$

Eq. (26) reduces to the Leibnitz equation of planar orbits:

$$\ddot{r} = -\frac{mG}{r^2} + r\dot{\theta}^2 \qquad -(28)$$

Therefore the gyro is a three dimensional orbit of its centre of mass about its point of contact with the earth's surface, QED.

As described in UFT270, Eq. (26) contains constants of motion which will be developed in the next paper, and which simplify the problem of solving Eq. (26) . In ECE2 fluid dynamics the complete force is the convective derivative:

$$\underline{F} = m\left(\frac{d\underline{v}}{dt} + \underline{\omega}\times\underline{v}\right)_{123} + m\left(\underline{v}\cdot\underline{\nabla}\right)\underline{v} \qquad -(29)$$

and there is present in general a force that can elevate the point of the gyro:

$$F_Z = \left(V_x \frac{\partial}{\partial x} + V_y \frac{\partial}{\partial y} + V_z \frac{\partial}{\partial z} \right) V_Z. \quad -(30)$$

In ECE2 fluid dynamics the velocity of classical dynamics:

$$\underline{v} = \underline{v}(t) \quad -(31)$$

is replaced by the velocity field:

$$\underline{v} = \underline{v}\left(\underline{r}(t), t \right) \quad -(32)$$

and the convective derivative of velocity is used:

$$\frac{D\underline{v}}{Dt} = \frac{d\underline{v}}{dt} + \left(\underline{v} \cdot \underline{\nabla} \right) \underline{v} \quad -(33)$$

where:

$$\frac{d\underline{v}}{dt} = \left(\frac{d\underline{v}}{dt} + \underline{\omega} \times \underline{v} \right)_{123} \quad -(34)$$

for rotational motion.

ECE2 fluid dynamics is capable of providing an explanation for well known experiments by Laithwaite and Shipov in which the point of the gyro is elevated. It may also be elevated, of course, by an applied mechanical force in the laboratory frame.

3. NUMERICAL AND GRAPHICAL ANALYSIS

Section by Dr. Horst Eckardt

The classical gyroscope as a three dimensional orbit

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3 Numerical and graphical analysis

According to Eq. (29), the complete fluid dynamics force is

$$\mathbf{F} = \frac{D\mathbf{v}}{Dt} = \frac{d\mathbf{v}}{dt} + \boldsymbol{\omega} \times \mathbf{v} + \mathbf{F}_c \qquad (35)$$

with the convective force

$$\mathbf{F}_c = (\mathbf{v} \cdot \boldsymbol{\nabla})\mathbf{v}. \qquad (36)$$

In this section we give some examples for the convective force and observe if this will produce a lifting force in Z direction for the gyroscope. Some analytical expressions for the velocity field $\mathbf{v}$ are given in Table 1, together with the resulting convective term $\mathbf{F}_c$.

The first example is a longitudinal harmonic wave in Z direction. The graphs of $\mathbf{v}$ and $\mathbf{F}_c$ in Fig. 1 (for $t = 0$) show that the resulting fluid dynamics force has the same direction but doubled frequency, i.e. the force oscillates twice as fast as the velocity field. A similar result is obtained for a circularly polarized wave, see case 2. The spatial distribution of the velocity vector in the XY plane is graphed in Fig. 2. There are virtual sources and sinks that vary over time (we only consider the case $t = 0$ again). The pattern of the corresponding force (Fig. 3) shows an oscillating pattern with doubled space frequency.

A longitudinal spherical wave (case 3) gives the same results as case 1 and is not extra graphed. An interesting case are polynomially varying velocities in Z direction (case 4). The exponent a is changed into $2a - 1$ which gives force enhancements with higher exponents for $a > 1$, see examples in Fig. 4. When $a < 1$ is chosen, then root-like velocities result. The force has a singularity at $z = 0$ so that it is possible to create very strong fluid forces near to this point (Fig. 5). An exponentially growing velocity (case 6) generates a force with doubled exponent so that this is also a possibility for enhancing the force significantly.

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No.	Type	$\mathbf{v}$	$\mathbf{F}_c$
1	longitudinal wave in Z direction	$\begin{bmatrix} 0 \\ 0 \\ v_3\cos(k_Z Z - \omega t) \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0 \\ -v_3{}^2 k_Z \sin\left(2k_Z Z - 2\omega t\right) \end{bmatrix}$
2	circularly polarized plane wave	$\begin{bmatrix} v_1\cos(k_X X - \omega t) \\ v_2\sin(k_Y Y - \omega t) \\ 0 \end{bmatrix}$	$\begin{bmatrix} -\frac{v_1{}^2}{2} k_X \sin\left(2k_X X - 2\omega t\right) \\ \frac{v_2{}^2}{2} k_Y \sin\left(2k_Y Y - 2\omega t\right) \\ 0 \end{bmatrix}$
3	spherical longitudinal wave	$\begin{bmatrix} v_1\cos(k_r r - \omega t) \\ 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} -v_1{}^2 k_r \sin\left(2k_r r - 2\omega t\right) \\ 0 \\ 0 \end{bmatrix}$
4	polynomial velocity characteristics	$\begin{bmatrix} 0 \\ 0 \\ v_3 Z^a \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0 \\ a v_3{}^2 Z^{2a-1} \end{bmatrix}$
5	exponential velocity characteristics	$\begin{bmatrix} 0 \\ 0 \\ v_3 \exp(aZ) \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0 \\ a v_3{}^2 \exp(2aZ) \end{bmatrix}$

Table 1: Examples for velocity $\mathbf{v}$ and convective force term $\mathbf{F}_c$.
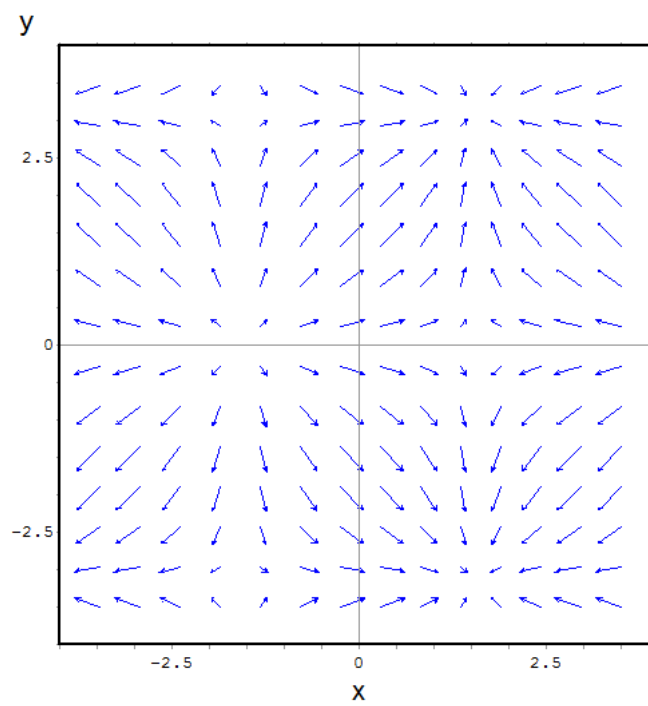

Figure 1: Longitudinal wave in Z direction.

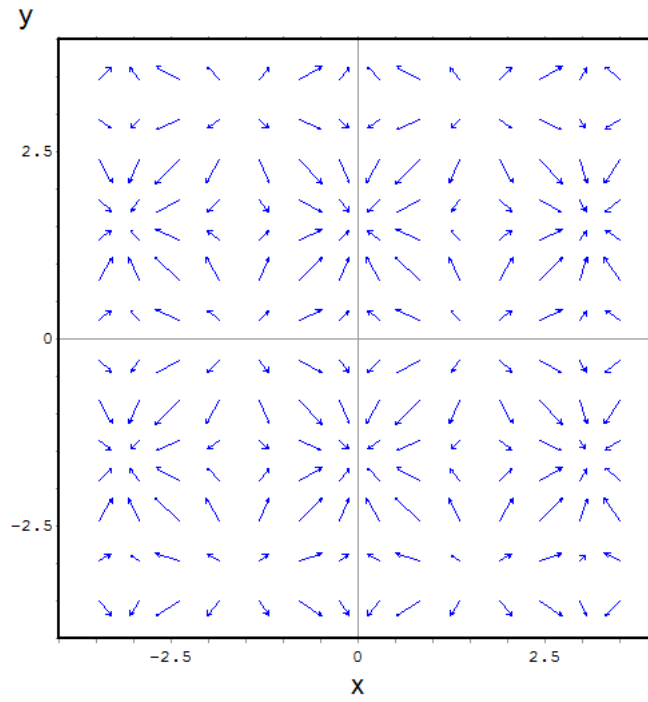Figure 2: Circularly polarized wave, **v** distribution.

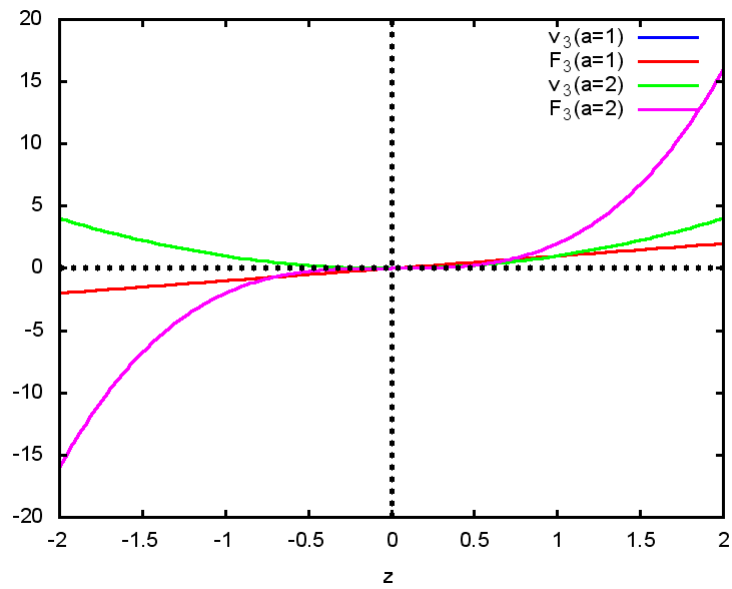Figure 3: Circularly polarized wave, $\mathbf{F}_c$ distribution.
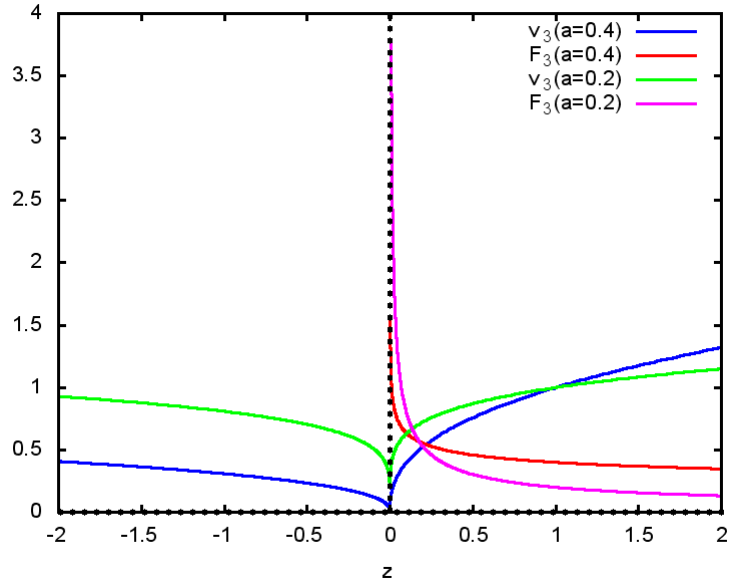

Figure 4: Polynomial velocity, $a \geq 1$.

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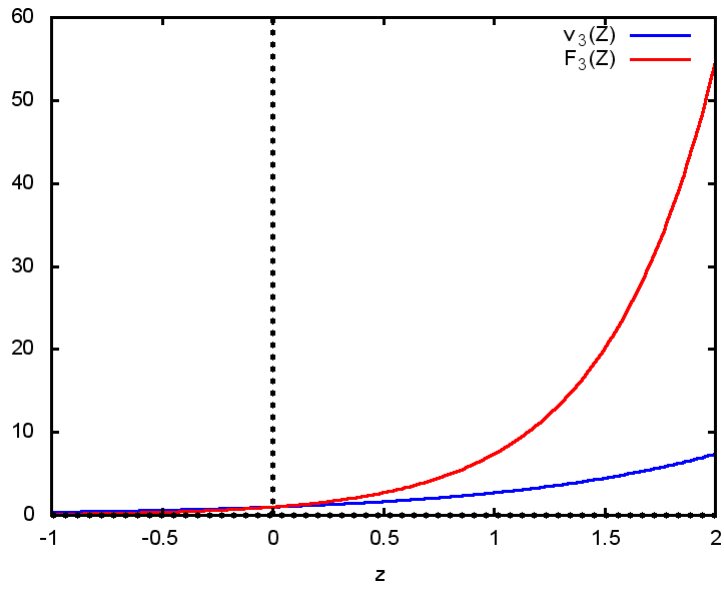Figure 5: Polynomial velocity, $a < 1$.

Figure 6: Exponential velocity.

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ACKNOWLEDGMENTS

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