THE GENERALLY COVARIANT INVERSE SQUARE LAW OF ALL ORBITS.

by

M. W. Evans and H. Eckardt

Civil List and AIAS / UPITEC

(www.aias.us, www.upitec.org, www.et3m.net, www.archive.org, www.webarchive.org.uk)

ABSTRACT

Using the theory of fluid gravitation it is shown that all observable orbits can be expressed as a generally covariant inverse square law of universal gravitation. The law can be expressed in several ways, notably that the acceleration due to gravity between a mass m orbiting a mass M is the derivative of the orbital velocity of m in a moving frame of reference. This is the Lagrange derivative, and in fluid gravitation becomes the convective derivative.

Keywords: ECE2, fluid gravitation, generally covariant inverse square law.

4FT 360

1. INTRODUCTION

In immediately preceding papers of this series {1-12} the subject of fluid gravitation has been developed by unifying the ECE2 field equations of gravitation and fluid dynamics. Similarly, in earlier papers, the subject of fluid electrodynamics was developed by unifying the ECE2 field equation of electrodynamics and fluid dynamics. In the preceding paper, fluid dynamics led to a new inverse square law of attraction between a mass m orbiting a mass M. In Section 2 this law is developed in various ways and applied to three examples of planar orbits: the conic section, the precessing ellipse and the hyperbolic spiral. The law can also be applied in three dimensions as shown in the preceding paper.

This paper is a brief synopsis of detailed calculations given in the notes accompanying UFT360 on www.aias.us. Note 360(1) gives a detailed self consistency check for the elliptical planar orbit. Notes 360(2) gives the generally covariant inverse square law of all orbits. Note 360(3) derives the new force law for a whirlpool galaxy. Note 360(4) gives the new force law for a precessing elliptical orbit, and Note 360(5) gives new general expressions for acceleration due to gravity.

Section 3 is a numerical and graphical analysis of the new law.

2. DEVELOPMENT OF THE NEW LAW

The law can be expressed as follows:

$$\underline{g} = (\underline{v} \cdot \underline{\nabla})\underline{v} \qquad -(1)$$

in two or three dimensions. Here $\underline{g}$ is the acceleration due to gravity between a mass m orbiting a mass M. The orbital velocity of the mass m is $\underline{v}$. Therefore $\underline{g}$ is the derivative of $\underline{v}$ in a moving frame of reference - the Lagrange derivative. In fluid gravitation it is referred to as the convective derivative using the traditional terminology. For planar orbits, the inverse square law is:

$$\underline{g} = \frac{-mG}{x^2 + y^2}\,\underline{e}_r. \quad -(2)$$

From Eq. (1), the orbital velocity is:

$$\underline{v} = (mG)^{1/2}\frac{(-X\underline{i} + Y\underline{j})}{(x^2+y^2)^{3/4}}. \quad -(3)$$

The law (1) is generally covariant because it is derived from a generally covariant unified field theory. For a circular orbit:

$$x^2 + y^2 = r^2 \quad -(4)$$

and the law (1) reduces to the form of the Hooke Newton inverse square law of universal gravitation. However, note carefully that for all orbits, including the circular orbit, the new law is generally covariant. The Hooke Newton law is empirical, non relativistic and galilean covariant.

In plane polar coordinates (r, θ):

$$v^2 = \frac{mG}{(x^2+y^2)^{1/2}} = \dot{r}^2 + r^2\dot{\theta}^2. \quad -(5)$$

For the elliptical orbit it is well known that:

$$v^2 = mG\left(\frac{2}{r} - \frac{1}{a}\right). \quad -(6)$$

where a is the semi major axis, so for the elliptical orbit:

$$\frac{1}{(x^2+y^2)^{1/2}} = \frac{2}{r} - \frac{1}{a} = \frac{1}{mG}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) \quad -(7)$$

and the acceleration due to gravity in fluid gravitation is:

$$\underline{g} = -\frac{mG}{x^2+y^2}\,\underline{e}_r = -mG\left(\frac{2}{r} - \frac{1}{a}\right)^2 \underline{e}_r = -\frac{1}{mG}\left(\dot{r}^2 + r^2\dot{\theta}^2\right)^2 \underline{e}_r \quad -(8)$$

being defined by a generally covariant inverse square law. In ECE fluid dynamics (an extension of Kambe fluid dynamics as described in previous papers):

$$\underline{g} = (\underline{v}\cdot\underline{\nabla})\underline{v} = -\frac{\partial \underline{v}}{\partial t} - \underline{\nabla}\phi \quad -(9)$$

where $\underline{v}$ plays the role of the vector potential and where

$$\phi = h \quad -(10)$$

is the gravitational scalar potential. Here h is the enthalpy per unit mass m.

Therefore Eq. (7) shows that the acceleration due to gravity is the Lagrange derivative of the orbital velocity and can be expressed as the following law of general relativity, the generally covariant law::

$$\underline{g} = -\frac{v^4}{mG}\,\underline{e}_r \quad -(11)$$

where $\underline{e}_r$ is the radial unit vector.

In the galilean covariant Newtonian theory:

$$\phi = -\frac{mG}{r} \quad -(12)$$

and

$$H = \frac{1}{2} m v^2 - \frac{m M G}{r} \quad -(13)$$

where H is the hamiltonian.

The new law applies to all orbits, in the sense that all orbits can be described by the generally covariant inverse square law (2). It is well known that the Newtonian law applies only to conic sections. The hyperbolic orbit of a whirlpool galaxy is considered in Note 360(3) and the precessing elliptical orbit in Note 360(4).

For all planar orbits X and Y are defined as:

$$X = mG \frac{(\dot{r} \sin\theta + r\dot{\theta}\cos\theta)}{(\dot{r}^2 + r^2\dot{\theta}^2)^{3/2}} \quad -(14)$$

and

$$Y = mG \frac{(r\dot{\theta}\sin\theta - \dot{r}\cos\theta)}{(\dot{r}^2 + r^2\dot{\theta}^2)^{3/2}} \quad -(15)$$

and this is the general definition of the moving frame of reference. Astronomical observation of any planar orbit defines:

$$\dot{r} = \frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt} \quad -(16)$$

and

$$\dot{\theta} = \frac{d\theta}{dt} = \frac{L}{mr^2} \quad -(17)$$

by establishing the functional dependence of r on θ .

1) Whirlpool Galaxy

In this case:

$$r = \frac{r_0}{\theta} \quad -(18)$$

where r_0 is a constant. It follows that:

$$\dot{r}^2 + r^2 \dot{\theta}^2 = \frac{L^2}{m^2} \left(\frac{1}{r^2} + \frac{1}{r_0^2} \right) \quad -(19)$$

where L is the angular momentum of the system defined by m orbiting M. It is a constant of motion worked out from the relevant Euler Lagrange equation (see Note 360(4)). It follows as in Note 360(3) that:

$$X = mG \frac{\left(\frac{L}{mr} \cos\left(\frac{r_0}{r}\right) - \frac{L}{mr_0} \sin\left(\frac{r_0}{r}\right) \right)}{\left(\frac{L^2}{m^2} \left(\frac{1}{r^2} + \frac{1}{r_0^2} \right) \right)^{3/2}} \quad -(20)$$

and

$$Y = mG \frac{\left(\frac{L}{mr} \sin\left(\frac{r_0}{r}\right) - \frac{L}{mr_0} \cos\left(\frac{r_0}{r}\right) \right)}{\left(\frac{L^2}{m^2} \left(\frac{1}{r^2} + \frac{1}{r_0^2} \right) \right)^{3/2}} \quad -(21)$$

define the relevant moving frame of Eq. (1) for the whirlpool galaxy and hyperbolic spiral orbit of m about. These coordinates X and Y are graphed in Section 3.

2) Precessing Elliptical Orbit in a Plane

It is observed astronomically to high precision that all solar system objects of mass m orbit the sun of mass M in a planar orbit defined by:

$$r = \frac{\alpha}{1 + \epsilon \cos(x\theta)} \quad -(22)$$

where:
$$x = 1 + \frac{3MG}{c^2 \alpha} \quad - (23)$$

in which α is the half right latitude. This experimental or empirical result is also true in binary objects outside the solar system, in which x is very close to unity. It follows as in Note 360(4) that:

$$\dot{r} = \frac{dr}{dt} = \frac{x \epsilon L}{m \alpha} \left(1 + \frac{1}{\epsilon^2} \left(\frac{\alpha}{r} - 1 \right)^2 \right)^{1/2} \quad - (24)$$

and

$$\dot{\theta} = \frac{L}{m r^2} \quad - (25)$$

So X and Y can be graphed using Eqs. (14), (15), (24) and (25). This procedure is carried out in Section 3 and defines the moving frame of Eq. (1), which is the generally covariant inverse square law of the precessing elliptical orbit observed astronomically.

As shown in Note 350(5):

$$\underline{g} = - \frac{v^4}{MG} \underline{e}_r \quad - (26)$$

for the precessing elliptical orbit, so its generally covariant acceleration due to gravity can be expressed as:

$$\underline{g} = - \frac{L^4}{m^4 MG} \left(\frac{x^2 \epsilon^2}{\alpha^2} \left(1 - \frac{1}{\epsilon^2} \left(\frac{\alpha}{r} - 1 \right)^2 \right) + \frac{1}{r^2} \right)^2 \quad - (27)$$

The same equation applies to the generally covariant acceleration due to gravity between a star of mass m orbiting a central mass M in a whirlpool galaxy. These results are also graphed in Section 3. The precessing ellipse reduces to the static ellipse when:

$$x = 1. \quad -(28)$$

Therefore:

$$\underline{F} = -\frac{mMG}{x^2 + y^2}\,\underline{e}_r \quad -(29)$$

is the generally covariant inverse square law for any planar orbit.

3. NUMERICAL RESULTS AND GRAPHICS.

Section by Dr. Horst Eckardt

The generally covariant inverse square law of all orbits

M. W. Evans*, H. Eckardt†
Civil List, A.I.A.S. and UPITEC

(www.webarchive.org.uk, www.aias.us,
www.atomicprecision.com, www.upitec.org)

3 Numerical results and graphics

We give some examples for the cartesian coordinates and acceleration of several types of planar orbits. The general formulae for the X and Y coordinates are given in Eqs. (14, 15). For a hyperbolic spiral defined by Eqs. (18-21) the functions $X(r)$ and$Y(r)$ are plotted in Fig. 1. A logarithmic r scale has been used to make the oscillations visible. Their exponential growth can be observed.

The coordinates of a precessing ellipse according to x theory are graphed in Fig. 2 for $\epsilon = 0.5$. Two values of x have been used: $x = 1$ (normal ellipse) and $x = 0.9$ (strongly precessing ellipse). As can be seen, the X and Y values start at the same point at $r = r_{\text{min}}$ but $x = 0.9$ overshoots the normal ellipse at $r = r_{\text{max}}$ as expected. There is a cross-over point in the Y coordinates. In case of parabolic and hyperbolic orbits ($\epsilon \geq 1$) the same formulae lead to unbound states, see Fig. 3, with asymptotes.

The second subject of graphical representation is the acceleration, generally given by Eq. (11). For a hyperbolic spiral this is

$$g = -\frac{L^4}{GM\,m^4}\left(\frac{1}{r^2} + \frac{1}{{r_0}^2}\right)^2 \tag{30}$$

which is graphed in Fig. 4, leading to a negative asymptotic behaviour for $r \to \infty$. For the precessing and non-precessing ellipses, we find the well known $1/r^2$ behaviour for $x = 1$ and a curve of small deviations of higher order for $x > 1$ (Fig. 5). In case of other conic sections (Fig. 6) there are differing asymptotes for large r values.

*email: emyrone@aol.com
†email: mail@horst-eckardt.de

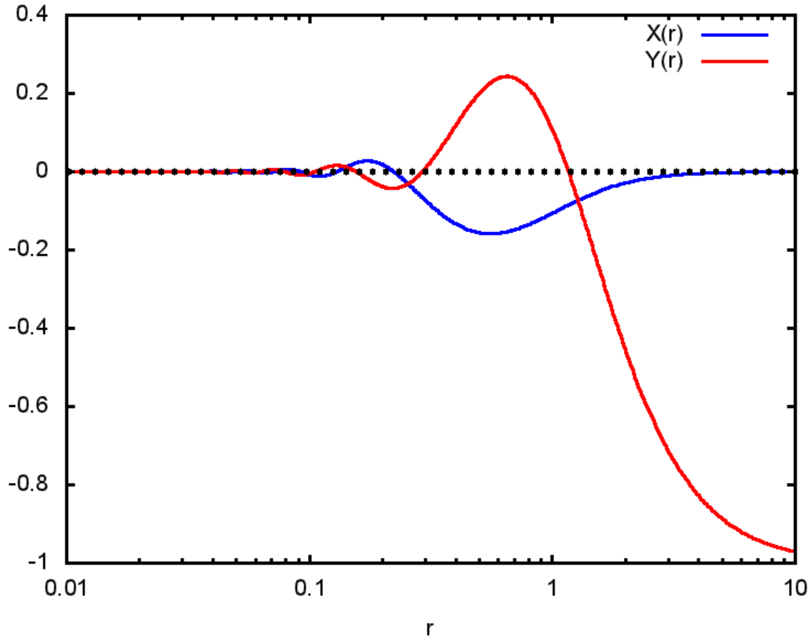

Figure 1: Orbit coordinates $X(r)$ and $Y(r)$ for a hyperbolic spiral.

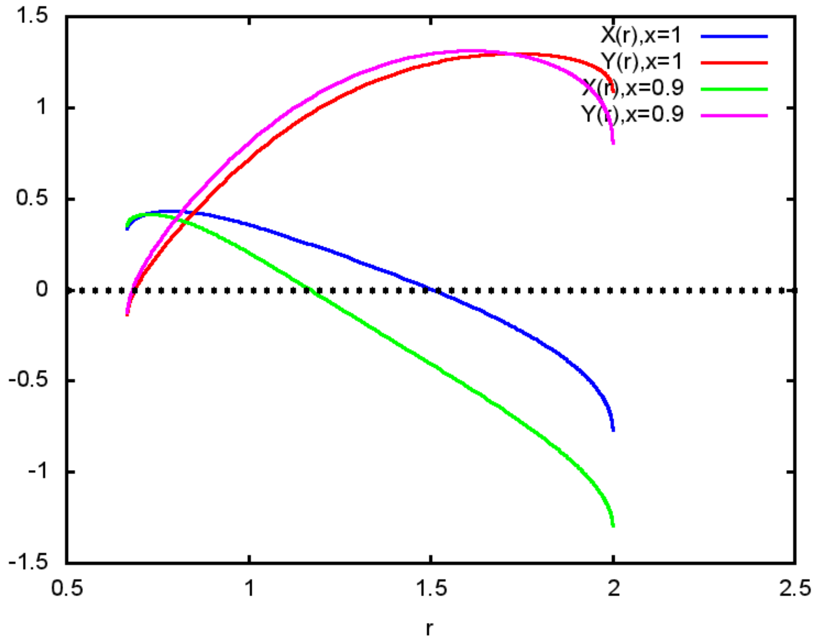

Figure 2: Orbit coordinates $X(r)$ and $Y(r)$ for precessing and non-precessing ellipses.

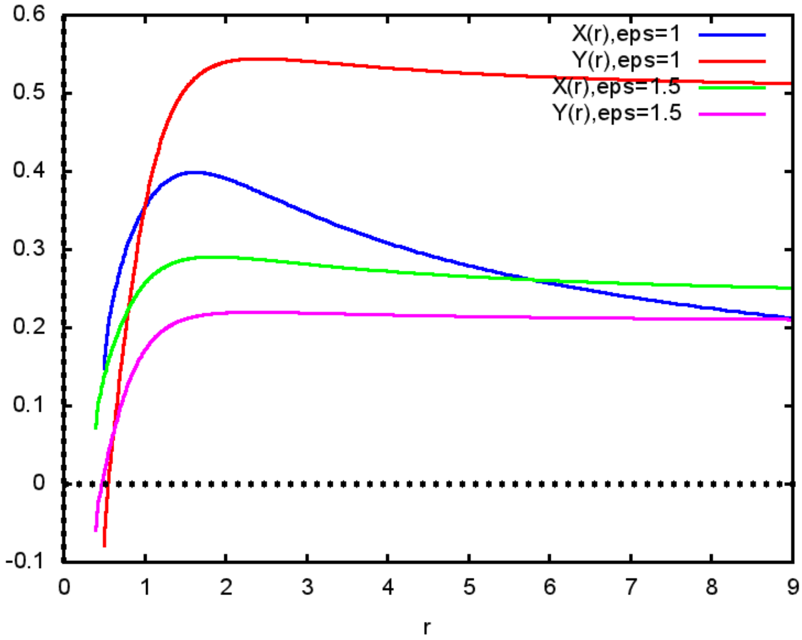

Figure 3: Orbit coordinates $X(r)$ and $Y(r)$ for conic sections.

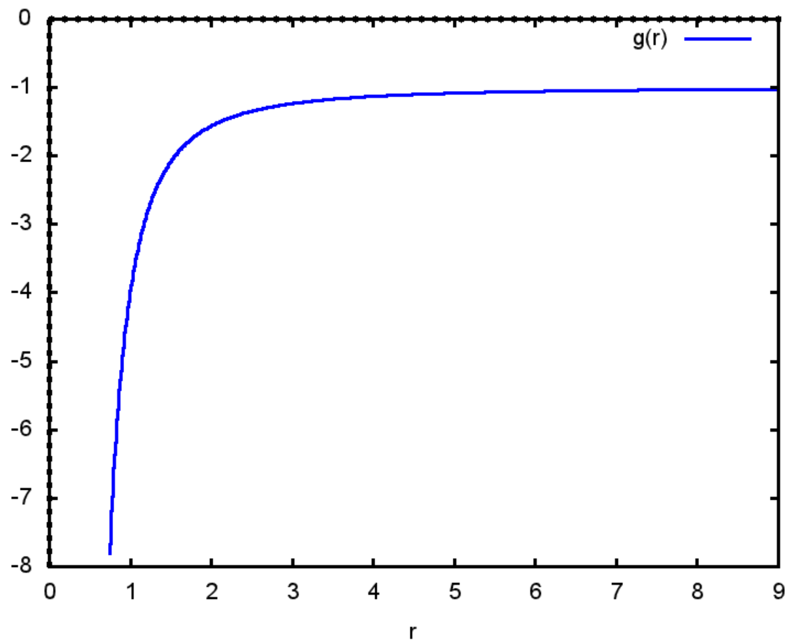

Figure 4: Acceleration for a hyperbolic spiral.

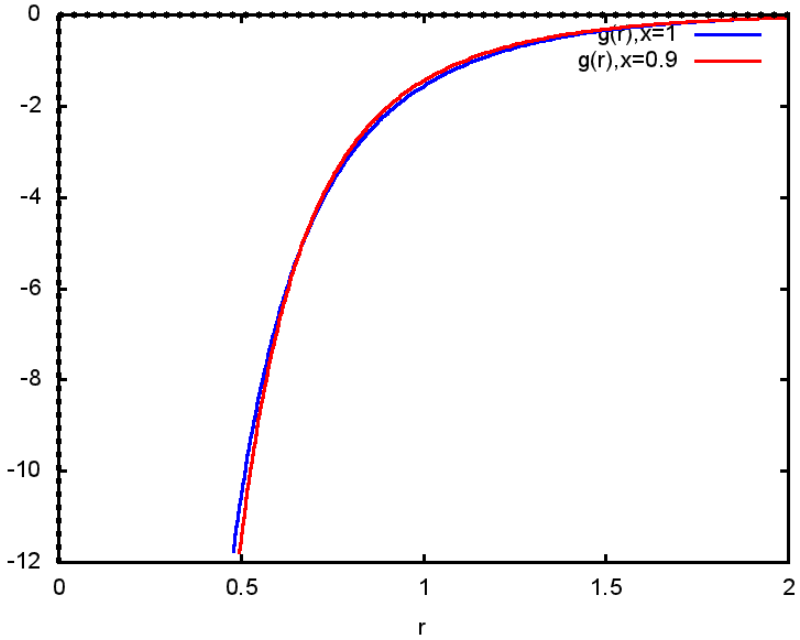

Figure 5: Acceleration for precessing and non-precessing ellipses.

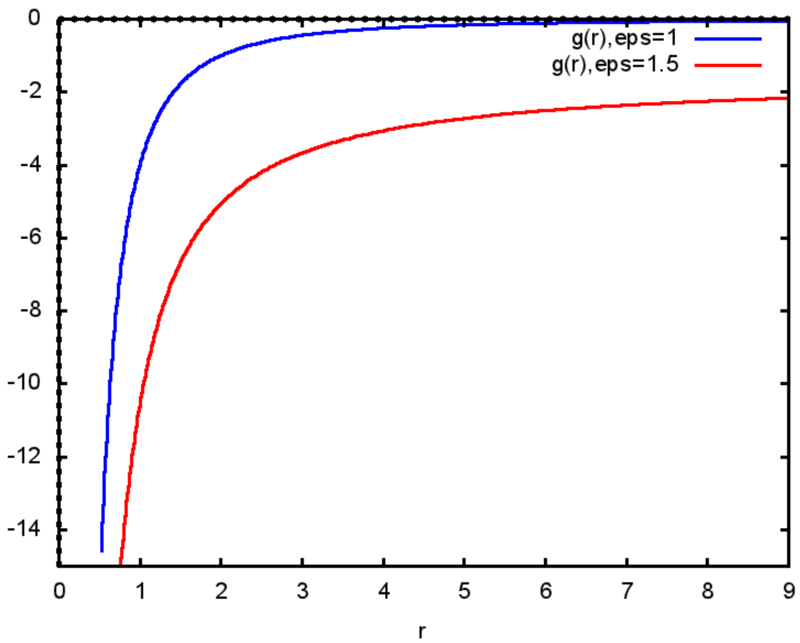

Figure 6: Acceleration for conic sections.

ACKNOWLEDGMENTS

The British Government is thanked for a Civil List pension and the staff of AIAS and others for many interesting discussions. Dave Burleigh, CEO of Annexa Inc., is thanked for hosting www.aias.us, site and software maintenance and posting, Alex Hill is thanked for translation and broadcasting, and Robert Cheshire for broadcasting.

REFERENCES

{1} M. W. Evans, H. Eckardt, D. W. Lindstrom and S. J. Crothers, "Principles of ECE" (open source on www.aias.us and www.upitec.org, hardback epubli Berlin 2016, softback New Generation London 2016, Spanish translation by Alex Hill)

{2} M. W. Evans, S. J. Crothers, H. Eckardt and K. Pendergast,"Criticisms of the Einstein Field Equation" (Cambridge International Science Publishing, CISP, 2010, open source as UFT301).

{3} L. Felker, "The Evans Equations of Unified Field Theory" (Abramis, softback 2007, open source as UFT302, Spanish translation by Alex Hill).

{4} H. Eckardt, "The ECE Engineering Model" (UFT303).

{5} M. W. Evans, H. Eckardt and D. W. Lindstrom, "Generally Covariant Unified Field Theory" (Abramis 2005 to 2011 in seven volumes, open source in relevant UFT papers).

{6} M .W. Evans, "Collected Scientometrics" (UFT307, New Generation 2015).

{7} M. W. Evans, Ed., J. Found. Phys. Chem., (CISP 2011, open source relevant UFT papers).

{8} M. W. Evans, Ed., "Definitive Refutations of the Einstein Field Equation" (CISP 2012, open source www.aias.us and www.upitec,org.)

{9} M. W. Evans and L. B. Crowell, "Classical and Qauntum Electrodynamics and the B(3)

Field" (World Scientific 2001, open source Omnia Opera section of www.aias.us).

{10} M. W. Evans and S. Kielich, Eds., "Modern Nonlinear Optics" (Wiley Interscience, New York, 1992, 1993, 1997 and 2001), in two editions and six volumes.

{11} M. W. Evans and J.-P. Vigier, "The Enigmatic Photon" (Kluwer, Dordrecht, 1994 - 2002) in five volumes hardback and softback.

{12} M. W. Evans and A. A. Hasanein, "The Photomagneton in Quantum Field Theory" (World Sientific, 1994).