THE GRAVITOMAGNETIC LORENTZ TRANSFORM IN ECE2

by

M. W. Evans and H. Eckardt

Civil List, AIAS and UPITEC

(www.webarchive.org.uk, , www.aias.us, www.upitec.org, www.atomicprecision.com ,

www.et3m.net)

ABSTRACT

The ECE2 field equations have a pseudo Lorentz covariance, and a Lorentz transform applied to the field tensor produces the Lorentz force equation for gravitomagnetism and the gravitomagnetic Biot Savart and Ampère laws. These laws are applied to planar orbits to find the gravitomagnetic field of the orbit and the current of mass density of the planar orbit. The method is generally valid and can be used on all scales.

Keywords, ECE2, frame transformation of the gravitomagnetic field tensor, gravitomagnetic Biot Savart and Ampère laws for planar orbits.

UFT 320

1. INTRODUCTION

In recent papers of this series {1 - 12} the development has been initiated of ECE2 theory, which was inferred following the correction in UFT313 of the second Bianchi identity of 1902 for torsion. In UFT314 to UFT319 ECE2 has been developed in vector notation. In this paper the Lorentz transformation of the gravitomagnetic field equations is developed to give the Lorentz force equation of gravitomagnetism and the Biot Savart and Ampère laws for planar orbits. The gravitomagnetic field can therefore be calculated for planar orbits, together with the current of mass density for planar orbits.

As usual this paper should be read together with its background notes posted with UFT320 on www.aias.us. Note 320(1) defines the field tensors of gravitomagnetism in ECE2 and defines the Lorentz transformation. Notes 320(2) to 320(5) begin the development of the theory, the final form of which is reached in Notes 320(6) and 320(7).

In Section 2 the gravitomagnetic field responsible for the centrifugal force of planar orbits is calculated straightforwardly, and the Biot Savart and Ampère laws defined. The gravitomagnetic field responsible for planar orbits in general is calculated from the Biot Savart law. The Ampère law is used to calculate the current of mass density for any planar orbit. The results are graphed and discussed in Section 3.

2. FIELD TENSORS AND LORENTZ TRANSFORMATION

The ECE2 field equations of gravitomagnetism are:

$$\partial_\mu \tilde{G}^{\mu\nu} = 0 \qquad -(1)$$

and

$$\partial_\mu G^{\mu\nu} = J^\nu \qquad -(2)$$

where, in S. I. Units:

$$\tilde{G}^{\mu\nu} = \begin{bmatrix} 0 & -c\Omega^1 & -c\Omega^2 & -c\Omega^3 \\ c\Omega^1 & 0 & g^3 & -g^2 \\ c\Omega^2 & -g^3 & 0 & g^1 \\ c\Omega^3 & g^2 & -g^1 & 0 \end{bmatrix} \quad -(3)$$

and

$$G^{\mu\nu} = \begin{bmatrix} 0 & -g^1 & -g^2 & -g^3 \\ g^1 & 0 & -c\Omega^3 & c\Omega^2 \\ g^2 & c\Omega^3 & 0 & -c\Omega^1 \\ g^3 & -c\Omega^2 & c\Omega^1 & 0 \end{bmatrix} \quad -(4)$$

In these equations $\underline{g}$ denotes the gravitational field and $\underline{\Omega}$ the gravitomagnetic field. These equations have the same structure as those of the electromagnetic field in ECE2 theory with the assumption that the magnetic charge / current density vanishes. Therefore it has been assumed in Eqs. (1) to (4) that the gravitomagnetic charge / current density vanishes. The contravariant index notation means that:

$$g^1 = g_X, \quad g^2 = g_Y, \quad g^3 = g_Z \quad -(5)$$
$$\Omega^1 = \Omega_X, \quad \Omega^2 = \Omega_Y, \quad \Omega^3 = \Omega_Z.$$

Lorentz transformation of the field tensors gives the result:

$$\underline{g}' = \gamma\left(\underline{g} + \underline{v} \times \underline{\Omega}\right) - \frac{\gamma^2}{\gamma+1}\frac{\underline{v}}{c}\left(\frac{\underline{v}}{c}\cdot\underline{g}\right) \quad -(6)$$

$$\underline{\Omega}' = \gamma\left(\underline{\Omega} - \frac{1}{c^2}\underline{v}\times\underline{g}\right) - \frac{\gamma^2}{\gamma+1}\frac{\underline{v}}{c}\left(\frac{\underline{v}}{c}\cdot\underline{\Omega}\right) \quad -(7)$$

where γ is the Lorentz factor:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(8)$$

The primed index indicates the frame in which the particle is moving, so in orbital theory it is the frame of the observer. For example a conical section orbit is observed for the Hooke / Newton inverse square law as is well known. In the rest frame

$$\underline{v} = \underline{0}. \quad -(9)$$

Eqs. (6) and (7) have exact parallels in electrodynamics:

$$\underline{E}' = \gamma\left(\underline{E} + \underline{v} \times \underline{B}\right) - \frac{\gamma^2}{1+\gamma}\frac{\underline{v}}{c}\left(\frac{\underline{v}}{c} \cdot \underline{E}\right) \quad -(10)$$

and

$$\underline{B}' = \gamma\left(\underline{B} - \frac{1}{c^2}\underline{v} \times \underline{E}\right) - \frac{\gamma^2}{1+\gamma}\frac{\underline{v}}{c}\left(\frac{\underline{v}}{c} \cdot \underline{B}\right) \quad -(11)$$

where $\underline{E}$ is the electric field strength in volts per meter and $\underline{B}$ is the magnetic flux density in tesla.

In the non relativistic limit:

$$v \ll c, \quad \gamma \rightarrow 1 \quad -(12)$$

and the gravitational Lorentz force is:

$$\underline{F} = m\left(\underline{g} + \underline{v} \times \underline{\Omega}\right). \quad -(13)$$

As discussed in detail in Notes 320(1) and 320(7), the orbital velocity in cylindrical polar coordinates of a mass m attracted to a mass M is in general:

$$\underline{v} = \dot{r}\,\underline{e}_r + \underline{\omega} \times \underline{r} = \dot{r}\,\underline{e}_r + \omega r\,\underline{e}_\theta \quad -(14)$$

where the unit vectors of the cylindrical polar system are defined {1 - 12} by:

$$\begin{aligned} \underline{e}_r &= \underline{e}_\theta \times \underline{k} \\ \underline{e}_\theta &= \underline{k} \times \underline{e}_r \\ \underline{k} &= \underline{e}_r \times \underline{e}_\theta . \end{aligned} \quad -(15)$$

For a planar orbit {1 - 12}, the acceleration in general is:

$$\underline{a} = \ddot{r}\,\underline{e}_r - \omega^2 r\,\underline{e}_r = \ddot{r}\,\underline{e}_r - \underline{\omega} \times (\underline{\omega} \times \underline{r}). \quad -(16)$$

Here $\underline{r}$ is the radial vector defined by:

$$\underline{r} = r\,\underline{e}_r \quad -(17)$$

and

$$\underline{\omega} = \omega\,\underline{k} \quad -(18)$$

is the angular velocity vector perpendicular to the plane:

$$\underline{\omega} = \omega\,\underline{k} = \frac{d\theta}{dt}\,\underline{k}. \quad -(19)$$

This assumption is made for two dimensional orbital theory but in three dimensional orbital theory a richer structure emerges {1 - 12}. The planar orbital force is therefore:

$$\underline{F} = m\underline{g} - \underline{\omega} \times (\underline{\omega} \times \underline{r}) = -\frac{mMG}{r^2}\,\underline{e}_r \quad -(20)$$

where G is Newton's constant. This is the 1689 Leibnitz equation of orbits still used today in a Newtonian context. The orbital force equation can be written as:

$$\underline{F} = m\underline{g} + \underline{v}_{rot} \times \underline{\omega} \quad -(21)$$

where:

$$\underline{v}_{rot} = \underline{\omega} \times \underline{r}. \quad -(22)$$

It is clear that the orbital force equation is the Lorentz force equation (13) if:

$$\underline{\Omega} = \underline{\omega} = \frac{d\theta}{dt}\,\underline{k} \quad -(23)$$

and

$$\underline{v} = \underline{v}_{rot} = \underline{\omega} \times \underline{r} \quad -(24)$$

which is the gravitomagnetic field responsible for the centrifugal force of any planar orbit.

The velocity of the rotating frame with respect to the fixed frame is the linear orbital velocity, the well known equation:

$$\underline{v}_{rot} = \underline{\omega} \times \underline{r}. \quad -(25)$$

In the non relativistic limit the electromagnetic Lorentz transforms are:

$$\underline{E}' = \underline{E} + \underline{v} \times \underline{B} \quad -(26)$$

$$\underline{B}' = \underline{B} - \frac{1}{c^2}\underline{v} \times \underline{E} \quad -(27)$$

and the gravitomagnetic Lorentz transforms are:

$$\underline{g}' = \underline{g} + \underline{v} \times \underline{\Omega} \quad -(28)$$

$$\underline{\Omega}' = \underline{\Omega} - \frac{1}{c^2}\underline{v}\times\underline{g} \quad -(29)$$

In these equations the primes indicate the field in the observer frame in which the velocity of a charge or a mass is non-zero.

It is well known {1 - 12} that the fundamental definition of magnetic flux density $\underline{B}$, the Biot Savart law, is obtained from Eq. (27) with:

$$\underline{B} = \underline{0} \quad -(30)$$

an equation which means that there is no magnetic field in the rest frame, the frame in which the electric charge does not move. The electromagnetic Biot Savart law is therefore:

$$\underline{B}' = -\frac{1}{c^2}\underline{v}\times\underline{E} \quad -(31)$$

in S. I. Units. The prime in Eq. (31) means that the law is written in the observer frame, the frame in which the velocity $\underline{v}$ of the electric charge is non-zero. In electrodynamics textbooks the prime is omitted, and the Biot Savart law becomes:

$$\underline{B} = -\frac{1}{c^2}\underline{v}\times\underline{E} \quad -(32)$$

It si well known {1 - 12} that the Biot Savart law can be written as:

$$\underline{\nabla}\times\underline{B} = \mu_0\underline{J} \quad -(33)$$

which is the Ampère Law of magnetostatics, describing the magnetic flux density generated by a current loop of any shape. It follows that:

$$\underline{\nabla}\times\underline{B} = -\frac{1}{c^2}\underline{\nabla}\times(\underline{v}\times\underline{E}) = \mu_0\underline{J} \quad -(34)$$

so the current density of electrodynamics is:

$$\underline{J} = -\frac{1}{\mu_0 c^2} \underline{\nabla} \times (\underline{v} \times \underline{E}) \quad -(35)$$
$$= -\epsilon_0 \underline{\nabla} \times (\underline{v} \times \underline{E}).$$

Here:

$$\underline{\nabla} \times (\underline{v} \times \underline{E}) = \underline{v}(\underline{\nabla} \cdot \underline{E}) - (\underline{\nabla} \cdot \underline{v})\underline{E} + (\underline{E} \cdot \underline{\nabla})\underline{v} - (\underline{v} \cdot \underline{\nabla})\underline{E}. \quad -(36)$$

The electromagnetic charge current four density is:

$$J^\mu = (\rho, \underline{J}). \quad -(37)$$

In a precisely analogous manner the gravitatomagnetic Biot Savart law is:

$$\underline{\Omega} = -\frac{1}{c^2} \underline{v} \times \underline{g} \quad -(38)$$

and is equivalent to:

$$\underline{\nabla} \times \underline{\Omega} = \frac{4\pi G}{c^2} \underline{J}_m \quad -(39)$$

where the gravitomagnetic mass / current density is:

$$\underline{J}_m = -\frac{1}{4\pi G} \underline{\nabla} \times (\underline{v} \times \underline{g}). \quad -(40)$$

Therefore:

$$\underline{\nabla} \times \underline{\Omega} = -\frac{1}{c^2} \underline{\nabla} \times (\underline{v} \times \underline{g}) = \frac{4\pi G}{c^2} \underline{J}_m \quad -(41)$$

and the current of mass density is:

$$\underline{J}_m = -\frac{1}{4\pi G} \left(\underline{v}(\underline{\nabla} \cdot \underline{g}) - (\underline{\nabla} \cdot \underline{v})\underline{g} + (\underline{g} \cdot \underline{\nabla})\underline{v} - (\underline{v} \cdot \underline{\nabla})\underline{g} \right) \quad -(42)$$

$$= \frac{c^2}{4\pi G} \, \underline{\nabla} \times \underline{\Omega} .$$

Now use:

$$\Omega^2 = \frac{1}{c^4} \, \underline{v} \times \underline{g} \cdot \underline{v} \times \underline{g} = \frac{1}{c^4} \left(v^2 g^2 - \left(\underline{v} \cdot \underline{g} \right)^2 \right). \quad -(43)$$

Eqs. (38) and (43) can be used with any type of orbital theory, and $\underline{J}_m$ evaluated for any orbit.

For the inverse square law:

$$\underline{g} = - \frac{MG}{r^2} \underline{e}_r \quad -(44)$$

the orbit in plane polar coordinates is the conic section:

$$r = \frac{\alpha}{1 + \epsilon \cos \theta} \quad -(45)$$

and the orbital velocity is:

$$\underline{v} = \frac{dr}{dt} \underline{e}_r + \omega r \, \underline{e}_\theta . \quad -(46)$$

The square of the orbital velocity is:

$$v^2 = \left(\frac{dr}{dt} \right)^2 + r^2 \left(\frac{d\theta}{dt} \right)^2 = MG \left(\frac{2}{r} - \frac{1}{a} \right) \quad -(47)$$

where the semi major axis of an ellipse for example is:

$$a = \frac{\alpha}{1 - \epsilon^2} . \quad -(48)$$

Here α is the half right latitude and ϵ is the eccentricity. Some examples of the orbital gravitomagnetic field are given in Section 3.

The gravitomagnetic Lorentz transform in ECE2

M. W. Evans*, H. Eckardt†
Civil List, A.I.A.S. and UPITEC

(www.webarchive.org.uk, www.aias.us,
www.atomicprecision.com, www.upitec.org)

3 Graphical analysis of orbital gravitomagnetic fields

The gravitomagnetic field $\mathbf{\Omega}$ and the gravitomagnetic current density $\mathbf{J}_m$ have been computed for an elliptic orbit in a plane. From Eqs.(38, 40) we have

$$\mathbf{\Omega} = -\frac{1}{c^2}\mathbf{v} \times \mathbf{g} \tag{49}$$

$$\mathbf{J}_m = \frac{1}{4\pi G}\boldsymbol{\nabla} \times (\mathbf{v} \times \mathbf{g})\,. \tag{50}$$

Using cylindrical coordinates (r, θ, Z) in a X-Y plane, we have for the Newton law of gravitation:

$$\mathbf{v} = \begin{pmatrix} \frac{dr}{dt} \\ \omega r \\ 0 \end{pmatrix} \tag{51}$$

$$\mathbf{g} = \begin{pmatrix} -\frac{MG}{r^2} \\ 0 \\ 0 \end{pmatrix} \tag{52}$$

as described in section 2. The anglular velocity ω is not constant but depends on the coordinates, too. Therefore we use

$$\omega = \frac{L}{m\,r^2} \tag{53}$$

as known from earlier work for conical sections with angular momentum

$$L = m\sqrt{\alpha\,G\,M}. \tag{54}$$

The time derivative of r can be expressed by

$$\frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt} = \frac{dr}{d\theta}\,\omega. \tag{55}$$

*email: emyrone@aol.com
†email: mail@horst-eckardt.de

The elliptic orbit is given by

$$r = \frac{\alpha}{1 + \epsilon \cos(\theta)} \tag{56}$$

from which we get

$$\frac{dr}{d\theta} = \frac{\epsilon\, r^2 \sin(\theta)}{\alpha}. \tag{57}$$

Inserting this into Eq.(51) gives

$$\mathbf{v} = \begin{pmatrix} \frac{\epsilon\, \omega\, r^2 \sin(\theta)}{\alpha} \\ \omega r \\ 0 \end{pmatrix} \tag{58}$$

and

$$\mathbf{v} \times \mathbf{g} = \begin{pmatrix} 0 \\ 0 \\ \frac{\omega\, G\, M}{r} \end{pmatrix}. \tag{59}$$

From (49) and (56) follows

$$\begin{aligned} \mathbf{\Omega} = -\frac{1}{c^2} \mathbf{v} \times \mathbf{g} = \begin{pmatrix} 0 \\ 0 \\ -\frac{\omega\, G\, M}{c^2\, r} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -\frac{\sqrt{\alpha}\, G^{\frac{3}{2}}\, M^{\frac{3}{2}}}{c^2\, r^3} \end{pmatrix} \\ = \begin{pmatrix} 0 \\ 0 \\ -\frac{(\epsilon \cos(\theta)+1)^3\, G^{\frac{3}{2}}\, M^{\frac{3}{2}}}{\alpha^{\frac{5}{2}}\, c^2} \end{pmatrix} \end{aligned} \tag{60}$$

and from (39,50)

$$\begin{aligned} \mathbf{J}_m = \frac{c^2}{4\pi G} \boldsymbol{\nabla} \times \mathbf{\Omega} = \begin{pmatrix} 0 \\ -\frac{\omega\, M}{4\,\pi\, r^2} \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{3\,\sqrt{\alpha}\, G\, M^{\frac{3}{2}}}{4\,\pi\, r^4} \\ 0 \end{pmatrix} \\ = \begin{pmatrix} 0 \\ -\frac{3\,(\epsilon \cos(\theta)+1)^4\, \sqrt{G}\, M^{\frac{3}{2}}}{4\,\pi\, \alpha^{\frac{7}{2}}} \\ 0 \end{pmatrix}. \end{aligned} \tag{61}$$

In Fig. 1 the velocity components including the modulus $|\mathbf{v}|$ are graphed for an elliptic orbit with unit parameters and $\epsilon = 0.3$. As expected, the radial part of the linear velocity oscillates and the angular component varies. The modulus is determined mainly by the angular component and is minimal at aphelion ($\theta = \pi$).

Both $\mathbf{\Omega}_Z$ and $\mathbf{J}_{m\theta}$ are plotted in Fig. 2. Their maximum and minimum absolute values correspond to those of the linear velocity (note that $\mathbf{\Omega}_Z$ and $\mathbf{J}_{m\theta}$ are both negative). For direct comparison, $\mathbf{J}_{m\theta}$ has been stretched so that it fits to the end points of $\mathbf{\Omega}_Z$. It can be seen that the current density has a greater angular variation because its radial dependence is $1/r^4$ while $\mathbf{\Omega}_Z$ exhibits only a $1/r^3$ dependence.

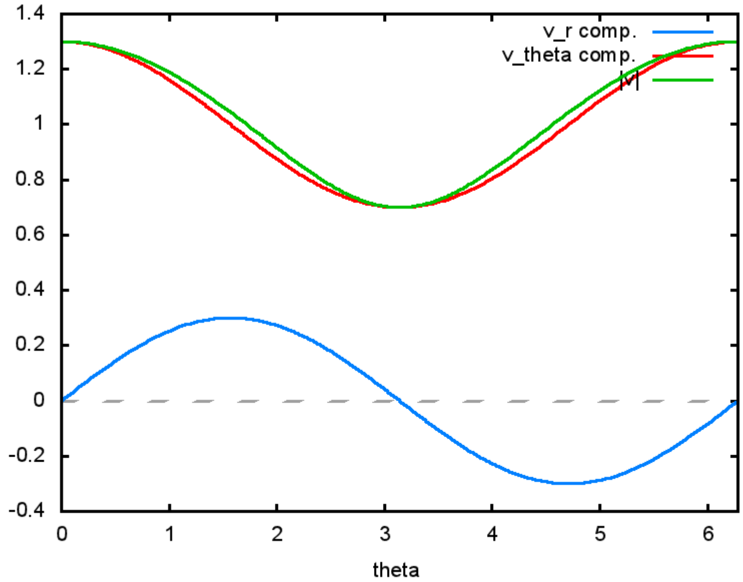

Figure 1: Velocity components v_r, v_θ and $|\mathbf{v}|$ for an elliptic orbit.

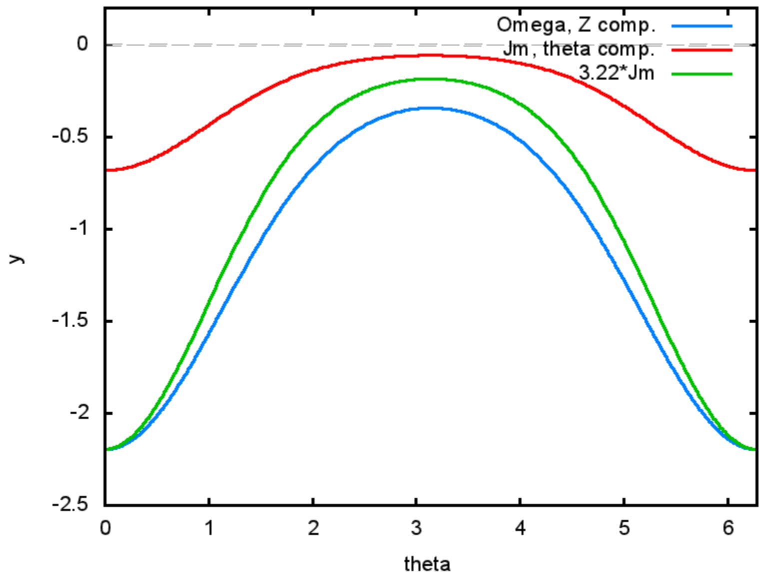

Figure 2: Lorentz force component $\mathbf{\Omega}_Z$, current component $\mathbf{J}_{m\theta}$ and re-scaled current for an elliptic orbit.

3. GRAPHICAL ANALYSIS OF ORBITAL GRAVITOMAGNETIC FIELDS

Section by Dr. Horst Eckardt.

ACKNOWLEDGMENTS

The British Government is thanked for a Civil List Pension and the staff of AIAS and others for many interesting discussions. Dave Burleigh is thanked site maintenance, feedback software and posting, Alex Hill for translation and broadcasting, and Robert Cheshire for broadcasting.

REFERENCES

{1} M .W. Evans, H. Eckardt, D. W. Lindstrom and S. J. Crothers, "Principles of ECE Theory" (UFT281 to UFT288 and New Generation, London, 2015, in prep.).

{2} M. W. Evans, Ed., J. Found. Phys. Chem., (open source on www.aias.us, Cambridge International Science Publishing, CISP, 2011).

{3} M .W. Evans, Ed., "Definitive Refutations of the Einstein Field Equation" (CISP 2012 and open source, www.aias.us).

{4} M .W. Evans, "Collected Scientometrics" (UFT307 and New Generation Publishing, London, 2015).

{5} H. Eckardt, "Engineering Model" (UFT303 on www.aias.us).

{6} M. W. Evans, S. J. Crothers, H. Eckardt and K. Pendergast, "Definitive Criticisms of the Einstein Field Equation" (UFT301 and CISP 2010).

{7} M .W. Evans, H. Eckardt and D. W. Lindstrom, "Generally Covariant Unified Field Theory" (Abramis Academic, Suffolk, 2005 to 2011 and open source on www.aias.us), in seven volumes.

{8} L. Felker, "The Evans Equations of Unified Field Theory" (Abramis Academic 2007 and UFT302).

{9} M .W. Evans and L. B. Crowell, "Classical and Quantum Electrodynamics and the B(3) Field" (World Scientific, 2001, open source Omnia Opera section of www.aias.us)

{10} M .W. Evans and S. Kielich, Eds., "Modern Nonlinear Optics" (Wiley Interscience, New York, 1992, 1993, 1997, 2001) in two editions and six volumes.

{11} M . W. Evans and J.-P. Vigier, "The Enigmatic Photon" (Kluwer, Dordrecht, 1994 to 2002 and open source Omnia Opera section of www.aias.us) in five volumes softback and hardback.

{12} M. W. Evans and A. A. Hasanein, "The Photomagneton in Quantum Field Theory" (World Scientific, 1994).