SOLAR SYSTEM ORBITS FROM THE ANTISYMMETRIC CONNECTION.

by

M .W. Evans,

Civil List and AIAS

(www.aias.us. , www.webarchive.org.uk, www.atomicprecision.com, www.upitec.org, www.et3m.net,)

and

H. Eckardt,

UPITEC and AIAS

ABSTRACT

The commutator equation, metric compatibility equation and Evans identity are used to derive straightforwardly a metric for solar system orbits. One solution for the metric is shown to have a closely similar r dependence to the empirical metric incorrectly known as the Schwarzschild metric. In general the metric is valid for spherically symmetric spacetime for any observed orbit, thus forging a new cosmology without the use of dark matter.

Keywords: ECE theory, antisymmetric metric, metric compatibility equation, Evans identity, solar system orbits, spherically symmetric spacetime.

UFT 189

1. INTRODUCTION

Recently in this series of papers {1 -10} the fundamental theorem of Riemann geometry has been shown to be one based on an antisymmetric Christoffel connection. The fundamental theorem is the equation that derives the antisymmetric connection from the metric. In Section 2 it is shown that the metric must be redefined fundamentally for dimensional self consistency, the metric in the standard physics {11} is ill defined, so there are dimensional problems in the definition of the various connections. The new definition of the metric is self consistent and free of dimensional irregularities. In Section 3 It is shown that for a static metric (one independent of time), there is only one antisymmetric connection, a major step forward in the understanding of cosmology. This single connection must obey the Evans identity relating torsion and curvature, and this allows it to be deduced analytically. In Section 4 a computational and graphical analysis of the new metric is given, and it is shown that various solutions are possible, on of which is closely similar to the empirical attempt at understanding solar system orbits known misleadingly and incorrectly as the Schwarzschild metric.

2. SELF CONSISTENT DEFINITION OF THE METRIC

With reference to note 189(1) accompanying this paper on www.aias.us, consider firstly the three dimensional Euclidean space {12}. Its metric elements are habitually defined as:

$$g_{11} = h_1^2 , \; g_{22} = h_2^2 , \; g_{33} = h_3^2 \quad -(1)$$

where h_1, h_2 and h_3 are the scale factors. In the cylindrical polar system of coordinates

$$h_1 = 1 , \; h_2 = r , \; h_3 = 1 \quad -(2)$$

where r is the radial coordinate, so the habitual definition of the metric is:

$$\gamma_{ij} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad -(3)$$

In the Cartesian representation on the other hand:

$$\gamma_{ij} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad -(4)$$

It is not consistent to have two different metrics representing the same mathematical space, and the presence of r^2 causes dimensional self inconsistency because r^2 has the units of square metres and the other metric elements in (3) are unitless. In consequence of the use of the habitual definition (3) extended to four dimensional Minkowski spacetime, some Christoffel symbols of the so called Schwarzschild metric do not vanish in a flat spacetime, in which all connections must be zero. This is a self contradiction that has gone uncorrected for a century.

The self inconsistency is eliminated by using the definition of the metric in terms of the unit vectors $\underline{e}_i$ of a curvilinear coordinate system, for example:

$$\gamma_{ij} = \underline{e}_i \cdot \underline{e}_j \quad -(5)$$

for a diagonal metric. So in both the Cartesian and cylindrical polar system, or any system of coordinates, the metric is always the unit diagonal and dimensionally self consistent:

$$\gamma_{ij} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad -(6)$$

The Minkowski metric in any coordinate system is also coordinate free in the manner of

Cartan's geometry:

$$g_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}. \quad -(7)$$

The metric of the spherical spacetime {11} is for all coordinate systems

$$g_{\mu\nu} = \begin{bmatrix} m(r) & 0 & 0 & 0 \\ 0 & -1/m(r) & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}. \quad -(8)$$

In consequence there is only one antisymmetric connection in the spherically symmetric spacetime, a discovery of the utmost import. This connection is:

$$\Gamma^{0}_{10} = -\Gamma^{0}_{01} = \frac{1}{2m}\frac{\partial m}{\partial r} \quad -(9)$$

and is constrained by the Evans identity {1 - 10}

$$D_\mu T^{\kappa\mu\nu} = R^{\kappa}{}_{\mu}{}^{\mu\nu}. \quad -(10)$$

In the habitually self inconsistent system the Minkowski metric was sometimes written in cylindrical polar coordinates as:

$$g_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin^2\theta \end{bmatrix} \quad -(11)$$

which is dimensionally incorrect and also contains the error:

$$h_3 = ?\ r\sin\theta \quad -(12)$$

The correct expression for the third scale factor of the cylindrical polar system is well known {12} and should be:

$$h_3 = 1. \quad -(13)$$

3. DERIVATION OF THE METRIC FACTOR $m(r)$

The fundamental theorem of Riemann geometry for a diagonal metric of type (8) is {1 - 10}:

$$\Gamma^{\mu}_{\nu\mu} = \frac{1}{2g_{\mu\mu}} \partial_\nu g_{\mu\mu}, \quad \nu \neq \mu, \quad -(14)$$

where there is no summation over the repeated μ index. The only connection that need be considered is

$$\Gamma^{0}_{10} = -\Gamma^{0}_{01}. \quad -(15)$$

The torsion tensor is twice the connection:

$$T^{\kappa\mu\nu} = 2\Gamma^{\kappa\mu\nu} \quad -(16)$$

and indices are raised and lowered as usual {11} using the metric and inverse metric:

$$\Gamma^{\kappa\mu\nu} = g^{\mu\alpha} g^{\nu\beta} \Gamma^{\kappa}_{\alpha\beta}. \quad -(17)$$

From eqs. (15) and (17):

$$\Gamma^{010} = g^{11} g^{00} \Gamma^{0}_{10}. \quad -(18)$$

From Eqs. (10) and (15):

$$D_1 T^{010} = R^{0}{}_{1}{}^{10} \quad -(19)$$

i.e.

$$D_1 \left(g^{11} g^{00} T^{0}{}_{10} \right) = g^{11} g^{00} R^{0}{}_{110}. \quad -(20)$$

By metric compatibility:

$$D_\mu g^{\nu\rho} = 0 \quad -(21)$$

so

$$D_1 \left(g^{11} g^{00} \right) = g^{11} D_1 g^{00} + g^{00} D_1 g^{11} = 0 \quad -(22)$$

i.e.

$$D_1 T^{0}{}_{10} = R^{0}{}_{110}. \quad -(23)$$

As in UFT 186 ff. on www.aias.us:

$$D_1 T^{0}{}_{10} = \partial_1 T^{0}{}_{10} \quad -(24)$$

so the Evans identity for the single connection (15) reduces to

$$\partial_1 T^{0}{}_{10} = R^{0}{}_{110}. \quad -(25)$$

The curvature tensor is defined {1 - 11} as:

$$R^{\rho}{}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda} \Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda} \Gamma^{\lambda}_{\mu\sigma} \quad -(26)$$

and is known as the Riemann tensor, although it was not derived by Riemann because the idea of the geometrical connection was not known to Riemann. The connection was first inferred by Christoffel in about 1867. The relevant curvature tensor is:

$$R^{0}{}_{110} = \partial_1 \Gamma^{0}_{01} - \partial_0 \Gamma^{0}_{11} + \Gamma^{0}_{1\lambda}\Gamma^{\lambda}_{01} - \Gamma^{0}_{0\lambda}\Gamma^{\lambda}_{11} \quad -(27)$$

in which the following definition must hold:

$$\lambda = 0. \quad -(28)$$

Therefore:

$$R^{0}{}_{110} = \partial_1 \Gamma^{0}_{01} + \Gamma^{0}_{10}\Gamma^{0}_{01}. \quad -(29)$$

It follows that:

$$3\partial_1 \Gamma^{0}_{10} + \left(\Gamma^{0}_{10}\right)^2 = 0. \quad -(30)$$

Using Eq. (9):

$$3\frac{\partial}{\partial r}\left(\frac{1}{m}\frac{\partial m}{\partial r}\right) + \frac{1}{2}\frac{1}{m^2}\left(\frac{\partial m}{\partial r}\right)^2 = 0 \quad -(31)$$

which is a differential equation for m.

Use the change of variable:

$$m = \exp(2\alpha) \quad -(32)$$

to obtain

$$3\frac{\partial^2 \alpha}{\partial r^2} + \frac{1}{2}\left(\frac{\partial \alpha}{\partial r}\right)^2 = 0. \quad -(33)$$

Using the further change of variable:

$$\frac{1}{2}\left(\frac{\partial \alpha}{\partial r}\right)^2 = \frac{\partial \alpha}{\partial r} - f(r). \quad -(34)$$

where f is a function of r, produces:

$$3\frac{\partial^2 \alpha}{\partial r^2} + \frac{\partial \alpha}{\partial r} = f(r). \quad -(35)$$

The reduced equation {13, 14} of Eq. (35) is:

$$3\frac{\partial^2 \alpha}{\partial r^2} + \frac{\partial \alpha}{\partial r} = 0. \quad -(36)$$

The solution of the reduced equation (36) is the complementary function, and for dimensional correctness the complementary function must be:

$$\alpha = \pm \exp\left(-\frac{r}{3R}\right) \quad -(37)$$

where R is a universal constant in units of metres. Therefore the complementary function is:

$$m_c(r) = \pm \exp\left(2\exp\left(-\frac{r}{3R}\right)\right) \quad -(38)$$

and may be positive or negative valued. The particular integral of Eq. (35) is by definition the same as the particular integral of Eq. (33). The general solution of Eq. (33) is the sum of the particular integral and the complementary function {13, 14}. The general solution must be compared with experimental data in the solar system, these data are the orbits of planets, meteorites, and so forth, known to be precessing ellipses to an excellent approximation. In order to describe these data the following general solution is obtained:

$$m(r) = 2 - \exp\left(2\exp\left(-\frac{r}{3R}\right)\right) \quad -(39)$$

in which the particular integral is:

$$m_p(r) = 2 \quad -(40)$$

and in which the complementary function is:

$$m_c(r) = -\exp\left(2\exp\left(-\frac{r}{R}\right)\right). \quad -(41)$$

From purely empirical considerations it is known that a metrical function of the type:

$$m_{emp}(r) = 1 - \frac{r_0}{r} \quad -(42)$$

produces solar system orbits very accurately. Here:

$$r_0 = \frac{2MG}{c^2} \quad -(43)$$

where G is Newton's constant, M the mass of the sun and c the universal constant known as the vacuum speed of light. In the next section it is shown that the correctly deduced metrical function (39)

can be approximated accurately by metric (42). The metric (39) is the first metric in cosmology that has been correctly derived from geometry. It is well known {1 - 10} that the older Einsteinian attempts incorrectly discarded torsion and for this reason are meaningless.

4. COMPUTATIONAL ANALYSIS Of THE METRICAL FUNCTION.

ACKNOWLEDGMENTS

The British Government is thanked for a Civil List Pension and the staff of AIAS for many interesting discussions. David Burleigh is thanked for voluntary posting, Alex Hill for translation and broadcasting and Robert Cheshire and Simon Clifford for broadcasting. The staff of the National Library of Wales and British Library are thanked for archiving the www.aias.us site and its weblog ("blog").

REFERENCES

{1} M. W. Evans, Ed., Journal of Foundations of Physics and Chemistry (Cambridge International Science Publishing, bimonthly from June 2011, www.cisp-publishing.com).

{2} M. W. Evans, S. Crothers, H. Eckardt and K. Pendergast, "Criticisms of the Einstein Field Equation" (Cambridge International Science Publishing, Spring 2011).

{3} K. Pendergast, "The Life of Myron Evans" (Cambridge International Science Publsishing, Spring 2011).

{4} M .W. Evans, H. Eckardt and D. W. Lindstrom, "Generally Covariant Unified Field Theory" (Abramis Academic, 2005 to 2011), in seven volumes.

{5} L. Felker, "The Evans Equations of Unified Field Theory" (Abramis Academic, 2007, available on www.aias.us in English and Spanish).

{6} M .W. Evans, D. W. Lindstrom and H. Eckardt, "ECE Theory Applied to H Bonding" (Serbian Academy of Sciences, 2010).

{7} The ECE open source websites: www.webarchive.org.uk, www.aias.us, www.atomicprecision.com, www.upitec.org, www.et3m.net

{8} M. W. Evans and S. Kielich, (Eds.), "Modern Nonlinear Optics" (Wiley, 1992, 1993, 1997, 2001), in six volumes and two editions.

{9} M. W. Evans and L. B. Crowell, "Classical and Quantum Electrodynamics and the B(3) Field" (World Scientific 2001).

{10} M .W. Evans and J.-P. Vigier, "The Enigmatic Photon" (Kluwer 1994 to 2002, hardback and softback), in ten volumes; M. W. Evans and A. A. Hasanein, "The Photomagneton in Quantum Field Theory" (World Scientific, 1994).

{11} S. P. Carroll, "Spacetime and Geometry: an Introduction to General Relativity" (Addison Wesley, New York, 2004).

{12} E. G. Milewski (Chief Ed.), "The Vector Analysis Problem Solver" (Res. Ed. Assoc., New York, 1987).

{13} J. B. Marion and S. T. Thornton, "Classical Dynamics of Particles and Systems" (Harcourt Brace, New York, 1988, 3rd. ed.).

{14} G. Stephenson, "Mathematical Methods for Science Students" (Longmans, 1968, 5th impression).