SELF CONSISTENT METRIC FROM TORSIONAL COSMOLOGY.

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ABSTRACT

It is argued that the basic theorem of Riemann geometry must be the commutator theorem that establishes the antisymmetry of the Christoffel connection. It is shown that for a diagonal metric all the elements of the Riemann tensor vanish, so that in the solar system for example the motions of the planets are governed by torsion alone. The antisymmetric connection is shown to be the only one that leads to self consistency between metric compatibility and the Evans identity. Using these rigorous criteria severely limits the number of self consistent metrics available for the development of cosmology.

Keywords: Torsional cosmology, self consistent metric, the fundamental commutator theorem.

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1. INTRODUCTION

Recently in this series of papers {1 - 10} it has been shown that the Christoffel connection can be deduced straightforwardly by use of a single metric compatibility equation. The metric compatibility equation is one of the fundamental equations of Riemann geometry but the commutator equation is used to deduce the structure of the Riemann curvature and torsion. The correct use of the commutator equation {11} shows that the connection is always antisymmetric. Therefore the only valid equation of metric compatibility is one in which the connection is antisymmetric. By use of the Evans identity {12} it is shown in Section 2 that the use of an incorrectly non-zero symmetric connection leads to a self contradiction in Riemann geometry itself. This simple reasoning overturns 110 years of received opinion based on the symmetric connection and for a diagonal metric, makes cosmology into a subject based entirely on spacetime torsion. A self consistent metric is derived based on a combination of the metric compatibility condition and Evans identity in a spherically symmetric spacetime. The "Schwarzschild" metric can be self consistent only in the limit of infinite separation between gravitating objects and was not in fact derived by Schwarzschild. It is unclear why it has been misattributed to him for nearly a century.

The concept of metric was introduced by Riemann and the concept of connection by Christoffel. In about 1900 Levi-Civita developed the idea of symmetric connection without use of the commutator equation, which shows that the connection is antisymmetric. So from the outset the received opinion of Riemann geometry was based incorrectly on the symmetric connection, or equivalently the neglect of torsion. In consequence of the erroneous use of a symmetric connection, three metric compatibility conditions were used to obtain what was asserted to be a uniquely defined symmetric connection. This was known as the fundamental theorem of Riemann geometry. The very fundamental commutator method on the other hand shows that the connection is always antisymmetric, so the fundamental theorem of Riemann geometry is incorrect and self inconsistent. The commutator method uses the commutator of covariant derivatives in any spacetime of any dimension to produce the components of the torsion and curvature. If the connection is symmetric the commutator vanishes, meaning that there is no torsion or curvature, and so a zero commutator implies flat spacetime in which the connection vanishes. A hypothetically symmetric connection is therefore zero. Therefore when applying the metric compatibility condition only antisymmetric connections can be used. The next section shows that if this rule is not followed, a self contradiction arises within Riemann geometry.

2. METRIC COMPATIBILITY AND EVANS IDENTITY.

The most fundamental equation of cosmology is the commutator equation:

$$[D_\mu, D_\nu]V^\rho = -T^\lambda_{\mu\nu} D_\lambda V^\rho + R^\rho{}_{\sigma\mu\nu} V^\sigma \quad -(1)$$

in which the Riemann torsion:

$$T^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu} \quad -(2)$$

and Riemann curvature:

$$R^\rho{}_{\sigma\mu\nu} = \partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma} \quad -(3)$$

are deduced from the commutator of covariant derivatives. The covariant derivative of a vector V^σ is defined by:

$$D_\mu V^\sigma = \partial_\mu V^\sigma + \Gamma^\sigma_{\mu\lambda} V^\lambda \quad -(4)$$

where $\Gamma^\sigma_{\mu\lambda}$ is the Christoffel connection. Both the torsion and the curvature are always antisymmetric in the indices of the commutator, μ and ν. Therefore the connection is also antisymmetric in the same indices, μ and ν. If:

$$\mu = \nu \quad -(5)$$

then the commutator, connection, torsion and curvature all vanish, in which case the spacetime becomes a spacetime that can be described by the ordinary four derivative. The commutator of such four derivatives is zero.

The metric compatibility equation {11} is:

$$D_\rho g_{\mu\nu} = \partial_\rho g_{\mu\nu} - \Gamma^\lambda_{\rho\mu} g_{\lambda\nu} - \Gamma^\lambda_{\rho\nu} g_{\mu\lambda} = 0 \quad -(6)$$

If the connection is symmetric, it is zero, as just argued, so in this case the metric compatibility equation is:

$$\partial_\rho g_{\mu\nu} = 0 \quad -(7)$$

and contains no connection. Therefore it is incorrect to use Eq. (6) with a symmetric connection, it can only be used with an antisymmetric connection.

The Evans identity {1 - 10} is:

$$D_\mu T^{\kappa\mu\nu} := R^{\kappa\ \ \mu\nu}_{\ \mu} \quad -(8)$$

and is essentially the Hodge dual identity of the Cartan identity:

$$D_\mu \tilde{T}^{\kappa\mu\nu} := \tilde{R}^{\kappa\ \ \mu\nu}_{\ \mu} \quad -(9)$$

where the tilde denotes Hodge duality. Therefore Eqs. (1), (6) and (8) are the fundamental equations of torsional cosmology. The Einstein field equation is incorrect and thoroughly obsolete because of its use of a symmetric connection. So in ECE type theory, the Einstein field equation is no longer used. The metrics are deduced from the orbital theorem of

UFT 111 (www.aias.us). Given the antisymmetry of the connection therefore, the fundamental equations of torsional cosmology are:

$$D_\rho g_{\mu\nu} = 0 \quad -(10)$$

and

$$D_\mu T^{\kappa\mu\nu} := R^{\kappa}{}_{\mu}{}^{\mu\nu} . \quad -(11)$$

As shown in the preceding paper UFT 186 (www.aias.us) all elements of the curvature tensor $R^{\kappa}{}_{\mu}{}^{\mu\nu}$ vanish for the following metric:

$$g_{00} = 1 - \frac{r_0}{r} \quad -(12)$$

$$g_{11} = -\left(1 - \frac{r_0}{r}\right)^{-1} \quad -(13)$$

$$g_{22} = -r^2 \quad -(14)$$

$$g_{33} = -r^2 \sin^2\phi \quad -(15)$$

where

$$r_0 = \frac{2MG}{c^2} . \quad -(16)$$

Here M is the mass of an attracting object, G is Newton's constant, and c the vacuum speed of light. This metric should be regarded as an allowed solution of the orbital theorem of UFT 111, based on a spacetime of spherical symmetry. The appellation "Schwarzschild metric" is incorrect, because in his only two extant papers of relevance (published in 1916) Schwarzschild did not derive the metric. What he actually did was to solve the Einstein field equation for two examples. The Einstein equation is now known to be flawed irretrievably because of its use of a symmetric connection, tantamount to neglect of torsion. The metric (12) – (16) is a fortuitously accurate description of a precessing elliptical orbit, and this lucky strike is what catalysed Einstein to fame, together with the flawed Eddington experiment. The metric fails completely for whirlpool galaxies, and the meaningless idea of dark matter introduced in a confused attempt to save the Einstein equation from obsolescence.

Written out in full, Eqs. (10) and (11) are:

$$\partial_\rho g_{\mu\nu} - \Gamma^\lambda_{\rho\mu} g_{\lambda\nu} - \Gamma^\lambda_{\rho\nu} g_{\mu\lambda} = 0 \quad -(17)$$

and

$$\partial_\mu T^\kappa_{\ \nu\sigma} + \Gamma^\kappa_{\mu\lambda} T^\lambda_{\ \nu\sigma} - \Gamma^\lambda_{\mu\nu} T^\kappa_{\ \lambda\sigma} - \Gamma^\lambda_{\mu\sigma} T^\kappa_{\ \nu\lambda} = 0 \quad -(18)$$

respectively using the rules {11} for the covariant derivative of a rank two and rank three tensor. The symmetries within these equations are:

$$g_{\mu\nu} = g_{\nu\mu} \quad -(19)$$

$$T^\kappa_{\ \nu\sigma} = 2\Gamma^\kappa_{\nu\sigma} = -T^\kappa_{\ \sigma\nu} \quad -(20)$$

For a metric of type (12) – (16) the preceding paper UFT 186 (www.aias.us) shows that the use of metric compatibility with antisymmetric connection produces:

$$\Gamma^0_{10} = \frac{r_0}{2r(r-r_0)}, \quad \Gamma^2_{12} = \Gamma^3_{13} = \frac{1}{r}, \quad \Gamma^3_{23} = \frac{\cos\phi}{\sin\phi}. \quad -(21)$$

If an erroneous attempt is made to use a symmetric connection in Eq. (17) results such as the following are obtained as in the preceding paper, UFT 186 (www.aias.us):

$$\Gamma^1_{11} = \frac{1}{2g_{11}} \partial_1 g_{11}. \quad -(22).$$

This result gives a non-zero symmetric connection. This is not consistent with the commutator equation (1), which shows that all symmetric connections are zero. So the commutator equation must be used to define the antisymmetry of the connection, and the connection must be antisymmetric in the metric compatibility equation. The latter must be solved simultaneously with the Evans identity. Note that the commutator equation does not use the concept of metric at all, it uses only the connection, and thus defines the antisymmetry of the connection and the antisymmetry of torsion and curvature. The latter two tensors are the only two fundamental tensors of any spacetime in any dimension {11}. The Evans identity relates these two tensors, so is equally as fundamental as the commutator equation. The metric compatibility condition on the other hand asserts only that the covariant derivative of the metric vanishes and does not define curvature or torsion, nor does it define their antisymmetry. Using the metric compatibility condition without any other information, it is not possible therefore to define the antisymmetry of the connection. In logic therefore the antisymmetry of the connection can be defined only by the commutator equation, and so this antisymmetry property must be used in the metric compatibility equation.

If one tries to argue that the connection may have any symmetry (the obsolete point of view) then it may be decomposed into a sum of symmetric and anstisymmetric parts as follows:

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2}\left(\Gamma^{\lambda}_{\mu\nu} + \Gamma^{\lambda}_{\nu\mu}\right) + \frac{1}{2}\left(\Gamma^{\lambda}_{\mu\nu} - \Gamma^{\lambda}_{\nu\mu}\right) \quad -(23)$$

using a well known theorem of matrices. If the connection is symmetric then:

$$\Gamma^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\nu\mu}. \quad -(24)$$

For example:

$$\mu = \nu = 1 \quad -(25)$$

in which case:

$$[D_1, D_1] V^\rho = R^\rho{}_{\sigma 11} V^\sigma = 0 \quad -(26)$$

and both the torsion and curvature vanish. This means that:

$$[D_\mu, D_\nu] V^\rho = [\partial_\mu, \partial_\nu] V^\rho = 0 \quad -(27)$$

i.e. it means that the covariant derivative D_μ can be replaced by the ordinary derivative ∂_μ. The connection, however, is defined by {11}:

$$D_\mu V^\sigma = \partial_\mu V^\sigma + \Gamma^\sigma_{\mu\lambda} V^\lambda \quad -(28)$$

so if

$$D_\mu V^\sigma = \partial_\mu V^\sigma \quad -(29)$$

then

$$\Gamma^\sigma_{\mu\lambda} = 0. \quad -(30)$$

The Christoffel connection vanishes if

$$\mu = \lambda \quad -(31)$$

and so a symmetric connection is zero, Q. E. D.

Unfortunately this simple proof was not known in about 1900, when Levi-Civita proposed a symmetric connection. It was not known because the commutator equation was not known, and so the wrong connection symmetry was used uncritically for 110 years in a fog of dogma, or fogma, with catastrophic results for standard cosmology.

By use of computer algebra the following Riemann tensor elements were found to be non zero for the connections in Eq. (21):

$$R^{0}{}_{101} = -\frac{r_0(4r-3r_0)}{4r^2(r-r_0)^2}, \quad -(32)$$

$$R^{3}{}_{123} = -\frac{2\cos\phi}{r\sin\phi}, \quad R^{3}{}_{223} = 1.$$

Compatibility with the Evans identity means:

$$D_\mu T^{\kappa\mu\nu} = R^{\kappa}{}_{\mu}{}^{\mu\nu}. \quad -(33)$$

From Eq. (32) and using the methods of the preceding paper it is found that the identity reduces to:

$$-2\,\partial_1 \Gamma^{0}{}_{10} = R^{0}{}_{1}{}^{10}. \quad -(34)$$

The left hand side is:

$$-2\,\partial_1 \Gamma^{0}{}_{10} = \frac{(2r-r_0)\,r_0}{r^2(r-r_0)^2} \quad -(35)$$

and is not the same as the right hand side in general with the exception of the limit:

$$r \rightarrow \infty. \quad -(36)$$

The so called Schwarzschild metric is successful in the solar system only in this limit. This alone is enough to refute all claims to Big Bang and the existence of black holes.

The metric of a spherically symmetric spacetime {11} may be written as:

$$\left.\begin{array}{l} g_{00} = e^{2\alpha}, \; g_{11} = -e^{2\beta}, \\ g_{22} = -r^2, \quad g_{33} = -r^2\sin^2\phi \end{array}\right]. \quad -(37)$$

where α and β are in general functions of r and t. Computer algebra and hand calculations show that for such a metric the antisymmetric connections are:

$$\left.\begin{array}{l} \Gamma^0_{\ 10} = \partial\alpha / \partial r, \ \Gamma^1_{\ 01} = \partial\beta / \partial t, \\ \Gamma^2_{\ 12} = \Gamma^3_{\ 13} = \frac{1}{r}, \ \Gamma^3_{\ 23} = \frac{\cos\phi}{\sin\phi}, \end{array}\right\} \quad -(38)$$

and the non vanishing Riemann tensor elements are:

$$R^0_{\ 001} = \frac{\partial\alpha}{\partial r}\frac{\partial\beta}{\partial t} + \frac{\partial^2\alpha}{\partial r\,\partial t}, \quad -(39)$$

$$R^0_{\ 101} = \frac{\partial^2\alpha}{\partial r^2} + \left(\frac{\partial\alpha}{\partial r}\right)^2, \quad -(40)$$

$$R^1_{\ 001} = -\left(\frac{\partial^2\beta}{\partial t^2} + \left(\frac{\partial\beta}{\partial t}\right)^2\right), \quad -(41)$$

$$R^1_{\ 101} = -\left(\frac{\partial\alpha}{\partial r}\frac{\partial\beta}{\partial t} + \frac{\partial^2\beta}{\partial r\,\partial t}\right), \quad -(42)$$

$$R^2_{\ 012} = R^3_{\ 013} = -R^3_{\ 103} = \frac{1}{r}\frac{\partial\beta}{\partial t}, \quad -(43)$$

$$R^3_{\ 123} = -\frac{2}{r}\frac{\cos\phi}{\sin\phi}, \quad -(44)$$

$$R^3_{\ 223} = 1. \quad -(45)$$

Applying the Evans identity as in note 187(10) accompanying this paper produces the constraint:

$$3\frac{\partial^2\alpha}{\partial r^2} + \left(\frac{\partial\alpha}{\partial r}\right)^2 = 0 \quad -(46)$$

and if this constraint is obeyed the metric does not violate the equations of torsional cosmology. This metric may therefore be an explanation of the dynamics of th esolar system and of an object such as the whirlpool galaxy provided that an appropriate function can be found. Note carefully that the metric does not depend on the use of the incorrect Einstein field equation.

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