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The Spinning and Curving of Spacetime: The Electromagnetic and Gravitational Fields In The Evans Unified Field Theory

Summary. The unification of the gravitational and electromagnetic fields achieved geometrically in the generally covariant unified field theory of Evans implies that electromagnetism is the spinning of spacetime, gravitation is the curving of spacetime. The homogeneous unified field equation of Evans is a balance of spacetime spin and curvature and governs the influence of electromagnetism on gravitation using the first Bianchi identity of differential geometry. The second Bianchi identity of differential geometry is shown to lead to the conservation law of the Evans unified field, and also to a generalization of the Einstein field equation for the unified field. Rigorous mathematical proofs are given in appendices of the four equations of differential geometry which are the cornerstones of the Evans unified field theory: the first and second Maurer Cartan structure relations and the first and second Bianchi identities. As an example of the theory the origin of wavenumber and frequency is traced to elements of the torsion tensor of spinning spacetime.

Key words: Evans unified field theory, spinning and curving spacetime, origin of the wavenumber and frequency.

17.1 Introduction

From 1925 to 1955 Einstein made various attempts to unify the gravitational and electromagnetic fields within general relativity. These attempts are summarized in updated appendices of various editions of ref. [1] and are all based on geometry. The gravitational sector of the unified field was developed by Einstein and others in terms of Riemann geometry with a symmetric Christoffel connection, $\Gamma^{\kappa}{}_{\mu\nu}$, which implies the first Bianchi identity:

$$R_{\sigma\mu\nu\rho} + R_{\sigma\nu\rho\mu} + R_{\sigma\rho\mu\nu} = 0 \tag{17.1}$$

by symmetry [2]. In Eq (17.1) $R_{\sigma\mu\nu\rho}$ is the Riemann or curvature tensor with lowered indices, defined by:

$$R_{\sigma\mu\nu\rho} = g_{\sigma\kappa} R^{\kappa}{}_{\mu\nu\rho} \tag{17.2}$$

where $g_{\sigma\kappa}$ is the symmetric metric tensor [1]. The Riemann curvature tensor is defined in terms of the gamma connection $\Gamma^{\kappa}{}_{\mu\nu}$ by:

$$R^{\rho}{}_{\sigma\mu\nu} = \partial_{\mu}\Gamma^{\rho}{}_{\nu\sigma} - \partial_{\nu}\Gamma^{\rho}{}_{\mu\sigma} + \Gamma^{\rho}{}_{\mu\lambda}\Gamma^{\lambda}{}_{\nu\sigma} - \Gamma^{\rho}{}_{\nu\lambda}\Gamma^{\lambda}{}_{\mu\sigma} . \tag{17.3}$$

Eq (17.3) is true for any kind of gamma connection, as is the second Bianchi identity:

$$D_{\lambda}R^{\rho}{}_{\sigma\mu\nu} + D_{\rho}R^{\sigma}{}_{\lambda\mu\nu} + D_{\sigma}R^{\lambda}{}_{\rho\mu\nu} := 0 \tag{17.4}$$

where $D\wedge$ is the covariant derivative [2] defined with the general gamma connection of any symmetry. The symmetric Christoffel connection is the special case where the gamma connection is symmetric and defined by:

$$\Gamma^{\kappa}{}_{\mu\nu} = \Gamma^{\kappa}{}_{\nu\mu} . \tag{17.5}$$

Using the metric compatibility postulate [2]:

$$D_{\rho}g^{\mu\nu} = 0 \tag{17.6}$$

the symmetric Christoffel connection can be expressed in terms of the symmetric metric:

$$\Gamma^{\sigma}{}_{\mu\nu} = \frac{1}{2}g^{\sigma\rho}\left(\partial_{\mu}g_{\nu\rho} + \partial_{\nu}g_{\rho\mu} - \partial_{\rho}g_{\mu\nu}\right) . \tag{17.7}$$

The use of Eq. (17.7) automatically implies that the torsion tensor $T^{\kappa}{}_{\mu\nu}$ vanishes:

$$T^{\kappa}{}_{\mu\nu} = \Gamma^{\kappa}{}_{\mu\nu} - \Gamma^{\kappa}{}_{\nu\mu} \tag{17.8}$$

So Einstein's famous gravitational theory is one in which there is no spacetime torsion or spinning. The first Bianchi identity (1) is also a special case therefore, defined by Eq. (17.5). More generally the cyclic sum in Eq. (17.1) is NOT zero if the gamma connection is not symmetric, and this turns out to be of fundamental importance for unified field theory: any mutual influence of gravitation upon electromagnetism depends on the fact that Eq. (17.1) does not hold in general. In contrast, note carefully that the second Bianchi identity (4) is ALWAYS true for any type of connection, because it is fundamentally the cyclic sum of commutators of covariant derivatives [2]:

$$[[D_{\lambda}, D_{\rho}], D_{\sigma}] + [[D_{\rho}, D_{\sigma}], D_{\lambda}] + [[D_{\sigma}, D_{\lambda}], D_{\rho}] := 0. \tag{17.9}$$

The above are the well known geometrical equations that are the cornerstones of Einstein's generally covariant theory of gravitation [1, 2].

The type of Riemann geometry almost always used by Einstein and others [2] for generally covariant gravitational field theory is a special case of the more general Cartan differential geometry [2] in which the connection is no longer symmetric and which the metric is in general the outer or tensor product of two more fundamental tetrads$q^{a}{}_{\mu}$. Thus, in differential geometry the metric tensor is in general an asymmetric matrix. Any asymmetric matrix is always the sum of a symmetric matrix and an antisymmetric matrix [3], so it is possible to construct an antisymmetric metric tensor. The symmetric metric used by Einstein to describe gravitation is therefore the special case defined by the inner or dot product of two tetrads [2]:

$$g_{\mu\nu} = q^a{}_\mu q^b{}_\nu \eta_{ab} \tag{17.10}$$

where η_{ab} is the diagonal, constant metric in the orthonormal or flat spacetime of the tetrad, indexed a:

$$\eta_{ab} = diag\,(-1, 1, 1, 1)\,. \tag{17.11}$$

Any attempt to construct a generally covariant unified field theory of all radiated and matter fields must therefore be based on differential geometry and must be based on the tetrad rather than the metric. The fundamental reason for this is that electromagnetism is known experimentally to be a spin phenomenon, and spin does not enter into Einstein's theory of gravitation because the torsion tensor vanishes as we have argued. Therefore a unified field theory must be based on geometry (such as differential geometry) that considers a non-zero torsion tensor as well as a non-zero curvature tensor. The conclusive advantage of a geometrical theory of fields over a gauge theory of fields is that the tangent bundle spacetime indexed a in the former theory is geometrical and therefore physical from the outset, whereas the fiber bundle spacetime of gauge theory is abstract and is a mathematical construct imposed for convenience on the base manifold without any reference to geometry. Thus gauge theory can never be a valid theory of general relativity because the latter is fundamentally based on geometry and must always be developed logically therefrom. Proceeding on this fundamental geometrical hypothesis of general relativity, therefore, the Evans unified field theory follows straightforwardly by tracing its origins to the fundamental equations that define differential geometry. These fundamental equations of differential geometry become the fundamental equations of the unified field theory through the Evans Ansatz [4]:

$$A^a{}_\mu = A^{(0)} q^a{}_\mu. \tag{17.12}$$

In Eq. (17.4) $A^a{}_\mu$ is the potential field of the electromagnetic sector of the unified field and the tetrad $q^a{}_\mu$ is the fundamental building block of the gravitational sector. Here $A^{(0)}$ denotes a $\hat{C}$ negative scalar originating in the magnetic fluxon $\hbar/e$, a primordial and universal constant of physics. Here $\hbar$ is the reduced Planck constant $h/(2\pi)$ and e the charge on the proton (the negative of the charge on the electron):

$$\hbar = 1.05459 \times 10^{-34} Js, \tag{17.13}$$

$$e = 1.60219 \times 10^{-19} C. \tag{17.14}$$

In Section 17.2 we give the four fundamental equations of differential geometry: the first and second Maurer Cartan structure relations and the first and second Bianchi identities and transform them into the equations of the Evans unified field [5]–[20]using Eq. (17.12). The rigorous mathematical proofs of all four equations are given in Appendices A to D.

In Section 17.3 the Maxwell Heaviside and Einstein limits of the Evans unified field theory are derived and discussed and in Section 17.4 a discussion is given of the implications of the unified field theory in evolution and various new technologies based on the ability of the gravitational and electromagnetic fields to be mutually influential.

17.2 The Fundamental Equations

The fundamental equations of the unified field theory are the fundamental equations of differential geometry [2], namely the two Maurer Cartan structure relations and the two Bianchi identities. There are two fundamental differential forms [2] that together describe any spacetime, the torsion or spin form and Riemann or curvature form. Any radiated or matter field in general relativity is therefore defined in terms of these forms. The structure relations of differential geometry define the spin and curvature forms respectively as the covariant exterior derivatives of the tetrad form and spin connection one-form:

$$T^a = D \wedge q^a = d \wedge q^a + \omega^a{}_b \wedge q^b, \tag{17.15}$$

$$R^a{}_b = D \wedge \omega^a{}_b = d \wedge \omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b \tag{17.16}$$

It is shown rigorously in the Appendices that these definitions are equivalent to the definitions of the spin and curvature tensors in terms of the gamma connection of any symmetry. Differential geometry is valid for any spacetime and any type of connection, and this realization is a key step towards the evolution of the Evans unified field theory, the first successful unified theory of the gravitational and electromagnetic fields.

The other two fundamental equations of differential geometry are the first and second Bianchi identities [2]:

$$D \wedge T^a := R^a{}_b \wedge q^b, \tag{17.17}$$

$$D \wedge R^a{}_b := 0. \tag{17.18}$$

These are written out in tensor notation and rigorously proven in the Appendices. The first Bianchi identity (17) generalizes Eq. (17.1), and the second Bianchi identity, Eq. (17.18), is Eq. (17.4) defined for any type of connection.

Eqs. (17.15) to (17.18) are the four cornerstones of any unified field theory based on geometry, i.e. of any generally covariant unified field theory. They are transformed into equations of the unified field using the Ansatz (17.12), and so Eqs. (17.15) and (17.17) become:

$$F^a = D \wedge A^a = d \wedge A^a + \omega^a{}_b \wedge A^b, \tag{17.19}$$

$$D \wedge F^a := R^a{}_b \wedge A^b. \tag{17.20}$$

Eq. (17.19) defines the field in terms of the potential, Eq. (17.20) is the homogeneous field equation of the electromagnetic sector. Eqs. (17.16) and (17.18) define the gravitational sector for any connection. In general both the spin and curvature forms are non-zero and so Eq. (17.20) demonstrates the way in which the gravitational field may influence the electromagnetic field and vice-versa. The extent to which this occurs must be found experimentally but Eq (17.20) shows that it is possible through a balance of spin and curvature in differential geometry. When both sides of Eq. (17.20) are non-zero the electromagnetic field can be influenced by the gravitational field and the gravitational field can be influenced by the electromagnetic field. In the first instance it then becomes possible to build electric power stations from spacetime curved by mass, and in the second instance it becomes possible to build counter gravitational devices built from electromagnetic technology. The possibility of such technologies must be tested by high precision experiments [21]. It seems likely that the chances of success are maximised by using high intensity femtosecond laser pulses incident on a high precision gravimeter in a high vacuum. The latter is used to remove "ion wind" artifact, i.e. extraneous effects due to atmospheric charging.

If it is found within experimental uncertainty that there is no effect of the gravitational field on the electromagnetic field and vice versa then the primordial Evans field has split entirely during the course of billions of years of evolution into what we term "pure electromagnetism" and "pure gravitation". These independent fields are described by the unified field equation:

$$d \wedge F^a = 0. \tag{17.21}$$

This is evidently an equation of differential geometry in the limit:

$$d \wedge F^a = 0, \tag{17.22}$$

$$R^a{}_b \wedge A^b = \omega^a{}_b \wedge F^b, \tag{17.23}$$

and so is a generally covariant unified field equation. In the following section we discuss this equation further in order to define precisely the Einsteinian and Maxwell - Heaviside limits of the Evans field theory.

17.3 Limiting Forms of the Evans Field

The Einsteinian limit is defined by:

$$T^a = 0, \tag{17.24}$$

so the torsion or spin form vanishes and we recover the equations of the introduction. In the language of differential geometry the Einstein field theory is therefore:

$$D \wedge q^a = 0, \tag{17.25}$$

$$R^a{}_b = D \wedge \omega^a{}_b, \tag{17.26}$$

$$R^a{}_b \wedge q^b = 0, \tag{17.27}$$

$$D \wedge R^a{}_b = 0. \tag{17.28}$$

It is defined by the two Bianchi identities with a symmetric Christoffel symbol, and by the structure relations for zero torsion or spin form. The first structure relation in the Einstein theory gives a differential equation for the tetrad in terms of the spin connection:

$$d \wedge q^a = -\omega^a{}_b \wedge q^b \tag{17.29}$$

which is equivalent to Eq. (17.7) of the introduction. The second Bianchi identity of the Einstein field theory, Eq. (17.28), leads directly [2] to the well known Einstein field equation. In tensor notation this is:

$$G_{\mu\nu} = kT_{\mu\nu} \tag{17.30}$$

where:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \tag{17.31}$$

is the Einstein field tensor. The Ricci tensor $R_{\mu\nu}$ and the metric $g_{\mu\nu}$ are symmetric in the Einstein field theory because the Christoffel connection is symmetric in its lower two indices (Eq. (17.5)). In the more general Evans unified field theory the Ricci tensor and the metric tensor are asymmetric matrices with anti-symmetric components representing spin. In the Einstein theory the tetrad postulate [2] of differential geometry:

$$D_\nu q^a{}_\mu = 0 \tag{17.32}$$

is specialized to the metric compatibility condition [2] for a symmetric metric:

$$D_\nu g^{\mu\rho} = D_\nu g_{\mu\rho} = 0. \tag{17.33}$$

Finally the canonical energy-momentum tensor is symmetric in the Einstein limit and is used in the well known Noether Theorem [2]:

$$D^\mu T_{\mu\nu} = 0. \tag{17.34}$$

More generally $T_{\mu\nu}$ is asymmetric in the Evans theory and therefore has an anti-symmetric component representing canonical angular energy/angular momentum.

So the Einstein limit of the Evans unified field theory is a special case in which the spinning of spacetime is not considered.

The Maxwell Heaviside theory of the electromagnetic field is older than general relativity and is conceptually a different theory in which the field is an abstract, mathematical entity superimposed on flat or Minkowski spacetime. In general relativity on the other hand the field is always non-Euclidean geometry itself and so must be the frame of reference itself. General relativity is simpler (one concept, the frame, instead of two concepts, field and frame) and is therefore the preferred theory by Ockham's Razor. The simplest way of thinking about this conceptual jump is to think of a helix. In general relativity the helix is the spinning and translating baseline, while in Maxwell Heaviside theory the helix is the abstract field superimposed on a static frame. If we restrict attention to three space dimensions the flat frame is the static Cartesian frame. In differential geometry the spinning and translating frame is the base manifold for the electromagnetic field, labelled, μ, and the tangent bundle is described by a Minkowski spacetime labelled a. The tetrad is defined for the electromagnetic field by:

$$V^a = q^a{}_\mu V^\mu \tag{17.35}$$

where V^μ is a vector in the base manifold, and where V^a is a vector in the tangent bundle. The tetrad is therefore the four by four invertible transformation matrix [2] between base manifold and tangent bundle. The tetrad is therefore a geometrical construct as required for general relativity. Circular polarization, discovered experimentally by Arago in 1811, is described geometrically by elements of $A^a{}_\mu$ from Eq. (17.12), i.e. by the following complex valued tetrad elements:

$$A^{(1)}{}_x = \left(A^{(0)}/\sqrt{2}\right) e^{i\phi}, \tag{17.36}$$

$$A^{(1)}{}_y = -i\left(A^{(0)}/\sqrt{2}\right) e^{i\phi}, \tag{17.37}$$

where ϕ is the electromagnetic phase. The complex conjugates of these elements are:

$$A^{(2)}{}_x = \left(A^{(0)}/\sqrt{2}\right) e^{-i\phi}, \tag{17.38}$$

$$A^{(2)}{}_y = i\left(A^{(0)}/\sqrt{2}\right) e^{-i\phi}. \tag{17.39}$$

Therefore these tetrad elements are individual components of the following vectors:

$$\boldsymbol{A}^{(1)} = \left(A^{(0)}/\sqrt{2}\right)(\boldsymbol{i} - i\boldsymbol{j})\, e^{i\phi}, \tag{17.40}$$

$$\boldsymbol{A}^{(2)} = \left(A^{(0)}/\sqrt{2}\right)(\boldsymbol{i} + i\boldsymbol{j})\, e^{-i\phi}, \tag{17.41}$$

representing a spinning and forward moving frame. This frame is multiplied by $A^{(0)}$ to give the generally covariant electromagnetic potential field. In 1992 it was inferred by Evans [22] that these vectors define the Evans spin field, $\boldsymbol{B}^{(3)}$, of electromagnetism:

$$\boldsymbol{B}^{(3)*} = -ig\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}, \tag{17.42}$$

where g is defined by the wavenumber:

$$g = \frac{\kappa}{A^{(0)}}. \tag{17.43}$$

The Evans spin field is a fundamental spin invariant of general relativity and is observed through the fact [23] that circularly polarized electromagnetic radiation of any frequency magnetizes any material. This reproducible and repeatable phenomenon is known as the inverse Faraday effect.

In order to reach the Maxwell Heaviside limit from the Evans field theory we must make the above important conceptual adjustments and consider the limit:

$$R^a{}_b \wedge q^b = \omega^a{}_b \wedge T^b \tag{17.44}$$

i.e. the limit reached when there is no gravitation curvature, but when there is spacetime spin defined by the following differential geometry:

$$T^a = D \wedge q^a, \tag{17.45}$$

$$d \wedge T^a = 0, \tag{17.46}$$

$$D \wedge \omega^a{}_b = 0. \tag{17.47}$$

In this geometry the Riemann or curvature form is zero so the second Bianchi identity becomes a differential equation for the gravitational spin connection:

$$d \wedge \omega^a{}_b = -\omega^a{}_c \wedge \omega^c{}_b. \tag{17.48}$$

This is therefore the underlying differential geometry that defines the pure, generally covariant, electromagnetic field if it has split away completely from the gravitational field during the course of billions of years of evolution. If there is any residual influence of gravitation upon electromagnetism and vice versa to be found experimentally then BOTH the curvature and spin forms are experimentally non-zero, and the Evans unified field is described by the differential geometry of Section 17.2 - the geometry of the primordial Evans field.

In general relativity, once we have found the geometry we understand the physics.

For pure electromagnetism the geometry translates into its field equations using the Ansatz (17.12) to give:

$$F^a = D \wedge A^a, \tag{17.49}$$

$$d \wedge F^a = 0, \tag{17.50}$$

and the gravitational equation

$$d \wedge \omega^a{}_b = -\omega^a{}_c \wedge \omega^c{}_b. \tag{17.51}$$

The electromagnetic field is thus always defined through the spin connection by:

$$F^a = D \wedge A^a = d \wedge A^a + \omega^a{}_b \wedge A^b. \tag{17.52}$$

Generally covariant non-linear optics is therefore generated by expanding the spin connection in terms of the tetrad or potential as follows:

$$\begin{aligned} \omega^a{}_b \wedge A^b = -gA^b \wedge \big(A^c + g\epsilon^{cde} A^d A^e \\ + g^2 \epsilon^{cfgh} A^f A^g A^h + \cdots \big) . \end{aligned} \tag{17.53}$$

All observable non-linear optical phenomena are therefore phenomena of spinning spacetime always describable by a well defined spin connection. The Maxwell Heaviside theory is the weak field limit or linear limit described by:

$$F^a = d \wedge A^a. \tag{17.54}$$

The Maxwell Heaviside theory is further restricted by the fact that it implicitly suppresses the index a, meaning that only one unwritten scalar component of the tangent bundle spacetime is considered, and then only implicitly. In other words the tangent bundle is ill defined in the Maxwell Heaviside field theory. More generally, in the Evans unified field theory, there are four physical components of the Minkowski spacetime of the tangent bundle: (ct, X, Y, Z), although a can represent any suitable set of basis elements such as unit vectors or Pauli matrices [2] in any well defined mathematical representation space of the physical tangent bundle spacetime). Suppressing the index a gives the familiar equations:

$$F = d \wedge A, \tag{17.55}$$

$$d \wedge F = 0. \tag{17.56}$$

These are the Maxwell Heaviside equations in differential geometric notation. The second equation is a combination of the Gauss Law and the Faraday Law of induction. Therefore in order to construct a unified field theory with differential geometry it is necessary to recognise that the Maxwell Heaviside structure is incomplete.

The inference of the Evans spin field in 1992 [21] was the first step towards this recognition of the incompleteness of the Maxwell Heaviside theory, and therefore towards the the unified field theory long sought after by Einstein[1] and others. The Evans spin field is the torsion or spin form component defined by the wedge product:

$$B^{(3)*} = -igA^{(1)} \wedge A^{(2)}. \tag{17.57}$$

The inference of $\boldsymbol{B}^{(3)}$ led to the development [5]–[20] of O(3) electrodynamics, in which the field is defined by:

$$\begin{aligned} F^{(3)*} = d \wedge A^{(3)*} - igA^{(1)} \wedge A^{(2)} \\ et\ \ cyclicum \end{aligned} \tag{17.58}$$

and so O(3) electrodynamics is a special case of Eqns. (17.49) to (17.53). The Gauss and Faraday Laws in O(3) electrodynamics are given by:

$$d \wedge F^a = R^a{}_b \wedge A^b - \omega^a{}_b \wedge F^b = 0. \tag{17.59}$$

Experimentally, it is found [5]–[20] that Eq. (17.59) must split into the particular solution:

$$d \wedge F^a = 0, \tag{17.60}$$

$$\omega^a{}_b \wedge F^b = R^a{}_b \wedge A^b. \tag{17.61}$$

These equations are obeyed by the circularly polarized electromagnetic potential field described in Eqs (17.40) and (17.41). Eqs. (17.60)and (17.61)also imply:

$$\boldsymbol{\nabla} \cdot \boldsymbol{B}^{(3)} = 0, \tag{17.62}$$

$$\frac{\partial \boldsymbol{B}^{(3)}}{\partial t} = \boldsymbol{\nabla} \times \boldsymbol{B}^{(3)} = \mathbf{0}. \tag{17.63}$$

for the Evans spin field. Therefore the latter does not give rise to Faraday induction, but gives rise to the magnetization observed in the inverse Faraday effect. Once we recognise the existence of the index a the inverse Faraday effect and all of non-linear optics follows logically. In the linear Maxwell Heaviside theory these non-linear optical effects have to be described [5]–[20] with additional ad hoc and non-linear constitutive relations which are obviously extraneous to the original linear Maxwell Heaviside structure. This original linear structure was inferred in the nineteenth century, long before the advent of non-linear optics. The latter never became available to Einstein, who never realized its significance to unified field theory. All sectors of a generally covariant unified field theory must be non-linear, because geometry is non-linear. Self consistently therefore, the linear Maxwell Heaviside structure can be obtained only if the spin connection vanishes, in which case spacetime becomes the Minkowski spacetime of special relativity, and Maxwell Heaviside theory was the first theory of special relativity. The frame covariance of the latter was first inferred (circa 1900 - 1904) by Poincare and Lorentz. Only later, in 1905, did Einstein finally extend the concept of special relativity to all of physics from electromagnetism.

The Evans unified field theory is therefore much more powerful than the earlier Maxwell Heaviside field theory, being a generally covariant theory of all radiated and matter fields. One example out of many possible examples is given to end this section, the description of the class of all Aharonov Bohm effects [5]–[20] for all fields. This class of phenomena can be defined within the context of Evans' field when F^a is zero but the potential A^a is non-zero. For the electromagnetic sector this means that:

$$d \wedge A^a = 0, \tag{17.64}$$

$$F^a = d \wedge A^a + \omega^a{}_b \wedge A^b \neq 0, \tag{17.65}$$

and for the gravitational sector it means that:

$$d \wedge q^a = -\omega^a{}_b \wedge q^b, \tag{17.66}$$

$$R^a{}_b = 0, \tag{17.67}$$

The potential field $A^a$of electromagnetism for example can interact with matter fields such as electrons when F^a is zero. The first experimental evidence for this inference was given by Chambers using a static magnetic field, but the Evans unified field theory shows that there is also an optical or electromagnetic Aharonov Bohm effect [5]–[20] and also a gravitational Aharonov Bohm effect. The latter has been observed precisely but the electromagnetic Aharonov Bohm effect has not been observed experimentally yet. However the theory of the latter effect has been given in considerable detail [5]–[20] and has major technological consequences if observed. The class of Aharonov Bohm effects is therefore explained straightforwardly as spacetime phenomena in experimental situations when F^a or T^a is zero but when q^a is non-zero. It is also possible to explain them when $R^a{}_b$ is zero and when the spin connection is non-zero. This will be the subject of a future paper. They are simply the consequence of geometry in general relativity

17.4 Consequences for Evolutionary Theory and New Technology

Before proceeding to a discussion of the implications of the Evans unified field theory we note the generally covariant wave equation:

$$D \wedge (D \wedge F^a) = R^a{}_b \wedge F^b \tag{17.68}$$

which can be derived from the two Bianchi identities written as:

$$D \wedge (D \wedge q^a) = \left(D \wedge \omega^a{}_b\right) \wedge q^b, \tag{17.69}$$

$$D \wedge \left(D \wedge \omega^a{}_b\right) := 0. \tag{17.70}$$

Using the Ansatz (17.12) Eqs. (17.69) and (17.70) become:

$$D \wedge (D \wedge A^a) := \left(D \wedge \omega^a{}_b\right) \wedge A^b, \tag{17.71}$$

$$D \wedge \left(D \wedge \omega^a{}_b\right) := 0. \tag{17.72}$$

Differentiating the right hand side of Eq (17.68) and using the Leibnitz Theorem [2]:

$$D \wedge \left(\left(D \wedge \omega^a{}_b\right) \wedge q^b\right) := \left(D \wedge \omega^a{}_b\right) \wedge \left(D \wedge q^b\right) \tag{17.73}$$

Using Eq (17.16), Eq (17.73) becomes:

$$D \wedge \left(\left(D \wedge \omega^a{}_b\right) \wedge q^b\right) := R^a{}_b \wedge T^b. \tag{17.74}$$

Use Eq (17.17) in Eq (17.74) to give:

$$D \wedge (D \wedge T^a) := R^a{}_b \wedge T^b, \tag{17.75}$$

which using the Ansatz (17.12) translates into Eq (17.68). Using the latter the condition for independent fields (no mutual interaction of gravitation and electromagnetism) becomes:

$$D \wedge (D \wedge F^a) := R^a{}_b \wedge F^b = 0 \tag{17.76}$$

which means that the fields are independent when the wedge product of the Riemann tensor and electromagnetic field tensor vanishes. Conversely if this wedge product is non-zero the fields can influence each other. This influence, if found to be non-zero experimentally with precise, well designed experiments, implies major new technology as discussed briefly already. This type of technology is governed in general by Eq. (17.68) which is a wave equation or quantum equation with the field F^a as eigenfunction or wave-function and the field $R^a{}_b$ as eigenvalues or quantum values. These new technologies would therefore depend on the fact that the quantum values of F^a are $R^a{}_b$ within a factor $A^{(0)}$, in other words it would depend on a generally covariant quantum mechanics of the unified Evans field.

Using the tetrad postulate [2]:

$$\partial_\mu q^a{}_\nu + \omega^a{}_{\mu b} q^b{}_\nu = \Gamma^\lambda{}_{\mu\nu}\, q^a{}_\lambda \tag{17.77}$$

the torsion form becomes the equivalent torsion tensor (see Appendix One):

$$T^a{}_{\mu\nu} = \left(\Gamma^\lambda{}_{\mu\nu} - \Gamma^\lambda{}_{\nu\mu}\right) q^a{}_\lambda, \tag{17.78}$$

$$T^\lambda{}_{\mu\nu} = q^a{}_\lambda T^a{}_{\mu\nu} = \Gamma^\lambda{}_{\mu\nu} - \Gamma^\lambda{}_{\nu\mu}\,. \tag{17.79}$$

Therefore, using the Ansatz (17.12) the magnetic field is defined by:

$$B^a{}_{\mu\nu} = T^\lambda{}_{\mu\nu}\, A^a{}_\lambda \tag{17.80}$$

and it becomes particularly clear from Eq (17.80) that spacetime spinning gives rise to electromagnetism.

The Maxwell Heaviside limit has the mathematical structure:

$$\boldsymbol{B}^{(1)} = \boldsymbol{\nabla} \times \boldsymbol{A}^{(1)} \tag{17.81}$$

where $A^{(1)}$ is given by Eq (17.40). So the magnetic field from Eqs (17.40) and (17.81) is:

$$\boldsymbol{B}^{(1)} = \frac{B^{(0)}}{\sqrt{2}}\,(i\boldsymbol{i} + \boldsymbol{j})\, e^{i\phi}, \tag{17.82}$$

and is the limit of Eq. (17.80) when the spin connection vanishes. Taking components of Eqs. (17.39) and (17.82)

$$B^{(1)}{}_x = -B^{(1)}{}_1 = -B^{(1)}{}_{23} = B^{(1)}{}_{32} = i\frac{B^{(0)}}{\sqrt{2}}e^{i\phi}, \tag{17.83}$$

$$B^{(1)}{}_y = -B^{(1)}{}_2 = -B^{(1)}{}_{31} = B^{(1)}{}_{13} = \frac{B^{(0)}}{\sqrt{2}}e^{i\phi}, \tag{17.84}$$

$$A^{(1)}{}_0 = A^{(1)}{}_3 = B^{(1)}{}_0 = B^{(1)}{}_3 = 0. \tag{17.85}$$

Therefore simple algebra gives:

$$iB^{(0)} = A^{(0)}\left(T^1{}_{23} - iT^2{}_{23}\right). \tag{17.86}$$

As in any paradigm shift Eq (17.86) gives new insight into known things. Eq (17.86) shows that the Maxwell Heaviside field theory defines the magnetic field as a complex sum of torsion tensor components. From Eq (17.86):

$$B^{(0)} = A^{(0)}\left(T^2{}_{32} + iT^1{}_{32}\right). \tag{17.87}$$

In the Maxwell Heaviside limit we know however that [5]–[20]:

$$B^{(0)} = \kappa A^{(0)} \tag{17.88}$$

where

$$\kappa = \frac{\omega}{c}. \tag{17.89}$$

Here ω is the angular frequency and c the speed of light. From Eqs. (17.87) and (17.88) therefore

$$\kappa = T^2{}_{32} + iT^1{}_{32}, \tag{17.90}$$

a result which traces the origin of wave-number in the Maxwell Heaviside limit to a complex sum of scalar valued torsion tensor components.

It is then possible to define the scalar curvature in the Maxwell Heaviside limit:

$$R = \kappa\kappa^* = \left(T^2{}_{32}\right)^2 - \left(T^1{}_{32}\right)^2, \tag{17.91}$$

a result which illustrates the fact that we are now thinking of the electromagnetic field as a spinning of spacetime, and not as an abstract mathematical field superimposed on a static frame of reference in Minkowski (flat) spacetime. In flat spacetime the scalar curvature is zero, in spinning spacetime it is non-zero. In flat spacetime initially parallel lines remain parallel, in spinning spacetime they become geodesics [2]of the electromagnetic field. The dielectric permittivity and the absorption coefficient [24] are defined in terms of a complex wavenumber, so these fundamental spectroscopic properties are traced to a geometrical origin. Photon mass is defined by the Evans Principle of Least Curvature [5]–[20] in which is subsumed the Hamilton Principle of Least Action in dynamics and the Fermat Principle of Least Time in optics. The Evans Principle asserts that in the limit of Minkowski spacetime:

$$\kappa \to \frac{2\pi}{\lambda_0} = 2\pi\frac{mc}{\hbar}, \tag{17.92}$$

where λ_0 is the Compton wavelength of any particle of mass m, including the photon. Therefore mass is also identified has having a geometrical origin, a least or minimized curvature in the Minkowski limit. Indeed, we see from Eqs. (17.90) and (17.92) that within a factor $2\pi c/\hbar$, mass is a torsion element for any matter field or radiated field:

$$T^2{}_{32} \rightarrow 2\pi \frac{mc}{\hbar}, \tag{17.93}$$

Therefore the origin of all fundamental physical concepts in the Evans unified field theory is differential geometry. Recently Pinter [25], in a remarkable multi-disciplinary work, has extended this basic insight to infer in a rigorously logical sequence of arguments that the origin of life is also differential geometry. Live organisms are extensions of general relativity itself and have evolved to their present condition through a sequence of transitions brought about by the theory of general relativity. The laws of physics, chemistry, biology, geology and genetics for example, are the laws of differential geometry. Within the context of the Evans unified field theory it is now recognized that electromagnetism as well as gravitation, originates in differential geometry, i.e. in the primordial or unified Evans field. Electromagnetism and gravitation are two parts of the same thing, and both are essential for the evolution of life. This inference justifies the fundamental and closely argued hypothesis used by Pinter [25], that gravitational effects evolve into effects driven by electrodynamics, for example photosynthesis in the early planet Earth. In the older Maxwell Heaviside field theory electrodynamics appears "out of the blue" but in the Evans unified field theory its origin is the same as that of gravitation. Similarly the origin of the weak and strong fields is also differential geometry in the Evans unified field theory. There are no longer any abstract internal gauge spaces or strings in nature.

The technological implications of the Evans field theory depend on its inference that one type of field may affect another, as briefly discussed already. If it were possible to obtain electromagnetic power from spacetime curved, for example, by the Earth's mass or by the mass of an electron in a circuit, the problem of burning fossil fuel would be obviated. Conversely, if it were possible for electromagnetic devices to counter or enhance the gravitational force, great technological strides would be made in any future aerospace industry. The Evans unified field theory shows that this is indeed possible but very careful, very precise, experiments are needed to measure the extent of the interaction (if any) between the sectors of the Evans field.

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A

The First Maurer-Cartan Structure Relation

The first structure relation [2] defines the torsion or spin form as the exterior covariant derivative of the tetrad form:

$$T^a{}_{\mu\nu} = (D \wedge q^a)_{\mu\nu} = (d \wedge q^a)_{\mu\nu} + \omega^a{}_{\mu b} q^b{}_\nu - \omega^a{}_{\nu b} q^b{}_\mu \tag{A.1}$$

where $\omega^a{}_{\mu b}$ is the spin connection. The torsion tensor is therefore:

$$T^\lambda{}_{\mu\nu} = q^\lambda{}_a T^a{}_{\mu\nu} \tag{A.2}$$

and using the tetrad postulate:

$$T^\lambda{}_{\mu\nu} = q^\lambda{}_a \left(\partial_\mu q^a{}_\nu - \partial_\nu q^a{}_\mu + \omega^a{}_{\mu b} q^b{}_\nu - \omega^a{}_{\nu b} q^b{}_\mu \right) \tag{A.3}$$

we obtain:

$$T^\lambda{}_{\mu\nu} = \Gamma^\lambda{}_{\mu\nu} - \Gamma^\lambda{}_{\nu\mu} \,. \tag{A.4}$$

This is an expression for the torsion tensor in terms of the gamma connection of any symmetry. If the gamma connection is the symmetric Christoffel symbol:

$$\Gamma^\lambda{}_{\mu\nu} = \Gamma^\lambda{}_{\nu\mu} \tag{A.5}$$

then the torsion tensor vanishes.

B

The Second Maurer-Cartan Structure Relation

The second structure relation defines the Riemann or curvature form as the exterior covariant derivative of the spin connection, regarded as a one-form:

$$R^a{}_b = D \wedge \omega^a{}_b \tag{B.1}$$

i.e

$$R^a{}_{b\nu\mu} = \partial_\nu \omega^a{}_{\mu b} - \partial_\mu \omega^a{}_{\nu b} + \omega^a{}_{\nu c}\omega^c{}_{\mu b} - \omega^a{}_{\mu c}\omega^c{}_{\nu b}. \tag{B.2}$$

It is proven in this appendix that the second structure relation is equivalent to the definition of the Riemann tensor for a gamma connection of any symmetry.

The proof starts with the tetrad postulate expressed as:

$$\omega^a{}_{\mu b} = q^a{}_\nu q^\lambda{}_b \Gamma^\nu{}_{\mu\lambda} - q^\lambda{}_b \partial_\mu q^a{}_\lambda. \tag{B.3}$$

Multiplying both sides of Eq (B.3) by $q^b{}_\lambda$ and using:

$$q^b{}_\lambda q^\lambda{}_b = 1 \tag{B.4}$$

the tetrad postulate can be expressed as:

$$\partial_\mu q^a{}_\lambda = q^a{}_\nu \Gamma^\nu{}_{\mu\lambda} - q^b{}_\lambda \omega^a{}_{\mu b}. \tag{B.5}$$

Differentiating Eq (B.3) and using the Leibnitz Theorem:

$$\begin{aligned}\partial_\nu \omega^a{}_{\mu b} =& \partial_\nu \left(q^a{}_\sigma q^\lambda{}_b \Gamma^\sigma{}_{\mu\lambda} \right) - \partial_\nu \left(q^\lambda{}_b \partial_\mu q^a{}_\lambda \right) \\ =& \partial_\nu \left(q^a{}_\sigma q^\lambda{}_b \right) \Gamma^\sigma{}_{\mu\lambda} + q^a{}_\sigma q^\lambda{}_b \partial_\nu \Gamma^\sigma{}_{\mu\lambda} \\ & - \left(\partial_\nu q^\lambda{}_b \right) \left(\partial_\mu q^a{}_\lambda \right) - q^\lambda{}_b \left(\partial_\nu \partial_\mu \left(q^a{}_\lambda \right) \right).\end{aligned} \tag{B.6}$$

Now use the Leibnitz Theorem again:

$$\partial_\nu \left(q^\lambda{}_b q^a{}_\sigma \right) = q^a{}_\sigma \partial_\nu q^\lambda{}_b + q^\lambda{}_b \partial_\nu q^a{}_\sigma \tag{B.7}$$

to obtain:

$$\partial_\nu \omega^a{}_{\mu b} = \left(q^a{}_\sigma \Gamma^\sigma{}_{\mu\lambda} - \partial_\mu q^a{}_\lambda \right) \partial_\nu q^\lambda{}_b + q^\lambda{}_b \Gamma^\sigma{}_{\mu\lambda} \partial_\nu q^a{}_\sigma + q^a{}_\sigma q^\lambda{}_b \partial_\nu \Gamma^\sigma{}_{\mu\lambda} - q^\lambda{}_b \left(\partial_\nu \partial_\mu \left(q^a{}_\lambda \right) \right) \tag{B.8}$$

Now use Eq. (B.5)in Eq. (B.8):

$$\partial_\nu \omega^a{}_{\mu b} = q^b{}_\lambda \omega^a{}_{\mu b} \partial_\nu q^\lambda{}_b + q^\lambda{}_b \Gamma^\sigma{}_{\mu\lambda} \partial_\nu q^a{}_\sigma + q^\lambda{}_b q^a{}_\sigma \partial_\nu \Gamma^\sigma{}_{\mu\lambda} - q^\lambda{}_b \left(\partial_\nu \partial_\mu \left(q^a{}_\lambda \right) \right). \tag{B.9}$$

Switching the μ and ν indices gives:

$$\partial_\mu \omega^a{}_{\nu b} = q^b{}_\lambda \omega^a{}_{\nu b} \partial_\mu q^\lambda{}_b + q^\lambda{}_b \Gamma^\sigma{}_{\nu\lambda} \partial_\mu q^a{}_\sigma + q^\lambda{}_b q^a{}_\sigma \partial_\mu \Gamma^\sigma{}_{\nu\lambda} - q^\lambda{}_b \left(\partial_\mu \partial_\nu \left(q^a{}_\lambda \right) \right) \tag{B.10}$$

which implies:

$$\partial_\nu \omega^a{}_{\mu b} - \partial_\mu \omega^a{}_{\nu b} = q^b{}_\lambda \left(\omega^a{}_{\mu b} \partial_\nu q^\lambda{}_b - \omega^a{}_{\nu b} \partial_\mu q^\lambda{}_b \right) + q^b{}_\lambda \left(\Gamma^\sigma{}_{\mu\lambda} \partial_\nu q^a{}_\sigma - \Gamma^\sigma{}_{\nu\lambda} \partial_\mu q^a{}_\sigma \right) + q^\lambda{}_b q^a{}_\sigma \left(\partial_\nu \Gamma^\sigma{}_{\mu\lambda} - \partial_\mu \Gamma^\sigma{}_{\nu\lambda} \right) \tag{B.11}$$

because

$$\left(\partial_\nu \partial_\mu - \partial_\nu \partial_\mu \right) q^a{}_\lambda = 0. \tag{B.12}$$

In order to evaluate the Riemann form:

$$R^a{}_{b\nu\mu} = \partial_\nu \omega^a{}_{\mu b} - \partial_\mu \omega^a{}_{\nu b} + \omega^a{}_{\nu c} \omega^c{}_{\mu b} - \omega^a{}_{\mu c} \omega^c{}_{\nu b} \tag{B.13}$$

we need:

$$\omega^a{}_{\nu c} = q^a{}_\mu q^\lambda{}_c \Gamma^\mu{}_{\nu\lambda} - q^\lambda{}_c \partial_\nu q^a{}_\lambda \tag{B.14}$$

$$\omega^a{}_{\mu b} = q^c{}_\nu q^\lambda{}_b \Gamma^\nu{}_{\mu\lambda} - q^\lambda{}_b \partial_\mu q^c{}_\lambda \tag{B.15}$$

$$\omega^a{}_{\mu c} = q^a{}_\nu q^\lambda{}_c \Gamma^\nu{}_{\mu\lambda} - q^\lambda{}_c \partial_\mu q^a{}_\lambda \tag{B.16}$$

$$\omega^c{}_{\nu b} = q^c{}_\mu q^\lambda{}_b \Gamma^\mu{}_{\nu\lambda} - q^\lambda{}_b \partial_\nu q^c{}_\lambda. \tag{B.17}$$

It is then possible to evaluate products such as:

$$\omega^a{}_{\nu c} \omega^c{}_{\mu b} = \left(q^a{}_\mu q^\lambda{}_c \Gamma^\mu{}_{\nu\lambda} - q^\lambda{}_c \partial_\nu q^a{}_\lambda \right) \left(q^c{}_\nu q^\lambda{}_b \Gamma^\nu{}_{\mu\lambda} - q^\lambda{}_b \partial_\mu q^c{}_\lambda \right). \tag{B.18}$$

The Riemann tensor can then be evaluated using:

$$R^\sigma{}_{\lambda\nu\mu} = q^\sigma{}_a q^b{}_\lambda R^a{}_{b\nu\mu}. \tag{B.19}$$

In order to evaluate Eq (B.19) first rearrange dummy indices in Eq. (B.18) as follows:

$$
\begin{aligned}
& q^{\lambda}{}_{c} q^{a}{}_{\mu} q^{\lambda}{}_{b} q^{c}{}_{\nu} \Gamma^{\mu}{}_{\nu\lambda} \Gamma^{\nu}{}_{\mu\lambda} \\
& \qquad \downarrow (\mu \rightarrow \sigma) \\
& q^{\lambda}{}_{c} q^{a}{}_{\sigma} q^{\lambda}{}_{b} q^{c}{}_{\nu} \Gamma^{\sigma}{}_{\nu\lambda} \Gamma^{\nu}{}_{\mu\lambda} \\
& \qquad \downarrow (\lambda \rightarrow \rho, \nu \rightarrow \rho) \\
& q^{\rho}{}_{c} q^{a}{}_{\sigma} q^{\lambda}{}_{b} q^{c}{}_{\rho} \Gamma^{\sigma}{}_{\nu\rho} \Gamma^{\rho}{}_{\mu\lambda} = q^{a}{}_{\sigma} q^{\lambda}{}_{b} \Gamma^{\sigma}{}_{\nu\rho} \Gamma^{\rho}{}_{\mu\lambda}
\end{aligned}
\tag{B.20}
$$

Secondly cancel the term $q^{\lambda}{}_{b} \Gamma^{\sigma}{}_{\nu\rho} \partial_{\nu} q^{a}{}_{\sigma}$ in Eq. (B.11) with the term $-(q^{\lambda}{}_{c} \partial_{\nu} q^{a}{}_{\lambda})$ $(q^{\lambda}{}_{b} q^{c}{}_{\nu} \Gamma^{\nu}{}_{\mu\lambda})$ In Eq. (B.18) by rearranging dummy indices as follows:

$$
\begin{aligned}
& -q^{\lambda}{}_{c} q^{\lambda}{}_{b} q^{c}{}_{\nu} \Gamma^{\nu}{}_{\mu\lambda} \partial_{\nu} q^{a}{}_{\lambda} \\
& \qquad \downarrow (\lambda \rightarrow \sigma) \\
& -q^{\sigma}{}_{c} q^{\lambda}{}_{b} q^{c}{}_{\nu} \Gamma^{\nu}{}_{\mu\lambda} \partial_{\nu} q^{a}{}_{\sigma} \\
& \qquad \downarrow (\nu \rightarrow \sigma) \\
& -q^{\sigma}{}_{c} q^{\lambda}{}_{b} q^{c}{}_{\sigma} \Gamma^{\sigma}{}_{\mu\lambda} \partial_{\nu} q^{a}{}_{\sigma} = -q^{\lambda}{}_{b} \Gamma^{\sigma}{}_{\mu\lambda} \partial_{\nu} q^{a}{}_{\sigma} .
\end{aligned}
\tag{B.21}
$$

Finally cancel the term $-q^{b}{}_{\lambda} \omega^{a}{}_{\nu b} \partial_{\mu} q^{\lambda}{}_{b}$ in Eq. (B.11) with the term $q^{\lambda}{}_{c} q^{\lambda}{}_{b}$ $(\partial_{\nu} q^{a}{}_{\lambda})(\partial_{\mu} q^{c}{}_{\lambda}) - q^{a}{}_{\mu} q^{\lambda}{}_{c} q^{\lambda}{}_{b} \Gamma^{\mu}{}_{\nu\lambda} \partial_{\mu} q^{c}{}_{\lambda}$ in Eq. (B.18). To do this rewrite the Eq (B.18) term as $q^{\lambda}{}_{c} q^{\lambda}{}_{b} \partial_{\mu} q^{c}{}_{\lambda} (\partial_{\nu} q^{a}{}_{\lambda} - q^{a}{}_{\mu} \Gamma^{\mu}{}_{\nu\lambda})$ and use the tetrad postulate:

$$
\partial_{\nu} q^{a}{}_{\lambda} = q^{a}{}_{\mu} \Gamma^{\mu}{}_{\nu\lambda} - q^{b}{}_{\lambda} \omega^{a}{}_{\nu b} \tag{B.22}
$$

to obtain:

$$
-q^{\lambda}{}_{c} q^{\lambda}{}_{b} q^{b}{}_{\lambda} \omega^{a}{}_{\nu b} \partial_{\mu} q^{c}{}_{\lambda} = -q^{c}{}_{\lambda} \omega^{a}{}_{\nu b} \partial_{\mu} q^{c}{}_{\lambda} . \tag{B.23}
$$

We therefore obtain:

$$
-q^{b}{}_{\lambda} \omega^{a}{}_{\nu b} \partial_{\mu} q^{\lambda}{}_{b} - \left(-q^{\lambda}{}_{c} \omega^{a}{}_{\nu b} \partial_{\mu} q^{c}{}_{\lambda}\right) = -\omega^{a}{}_{\nu b} \left(q^{c}{}_{\lambda} \partial_{\mu} q^{\lambda}{}_{c} + q^{\lambda}{}_{c} \partial_{\mu} q^{c}{}_{\lambda}\right) . \tag{B.24}
$$

In order to show that this is zero use:

$$
q^{\lambda}{}_{c} q^{c}{}_{\lambda} = 1 \tag{B.25}
$$

and differentiate:

$$
\partial_{\mu} (q^{\lambda}{}_{c} q^{c}{}_{\lambda}) = 0. \tag{B.26}
$$

Finally use the Leibnitz Theorem to obtain:

$$
q^{\lambda}{}_{c} \partial_{\mu} q^{c}{}_{\lambda} + q^{c}{}_{\lambda} \partial_{\mu} q^{\lambda}{}_{c} = 0. \tag{B.27}
$$

The remaining terms give the Riemann tensor for any gamma connection:

$$
R^{\lambda}{}_{\sigma\nu\mu} = \partial_{\nu} \Gamma^{\sigma}{}_{\mu\lambda} - \partial_{\mu} \Gamma^{\sigma}{}_{\nu\lambda} + \Gamma^{\sigma}{}_{\nu\rho} \Gamma^{\rho}{}_{\mu\lambda} - \Gamma^{\sigma}{}_{\mu\rho} \Gamma^{\rho}{}_{\nu\lambda} \tag{B.28}
$$

quod erat demonstrandum.

C

The First Bianchi Identity

The first Bianchi identity of differential geometry is a balance of spin and curvature

$$D \wedge T^a := R^a{}_b \wedge q^b \tag{C.1}$$

and becomes the homogeneous field equation of the Evans unified field theory:

$$D \wedge F^a := R^a{}_b \wedge A^b \tag{C.2}$$

using the Evans Ansatz:

$$A^a = A^{(0)} q^a \tag{C.3}$$

So it is important to thoroughly understand the structure and meaning of the first Bianchi identity as in this Appendix. In order to proceed we need the following general definitions [2] of the exterior derivative and wedge product for any differential form:

$$(d \wedge A)_{\mu_1 \cdots \mu_{p+1}} = (p+1)\, \partial_{[\mu_1} A_{\mu_2 \cdots \mu_{p+1}]} \tag{C.4}$$

$$(A \wedge B)_{\mu_1 \cdots \mu_{p+q}} = \frac{(p+q)!}{p!q!}\, (p+1)\, A_{[\mu_1 \cdots \mu_p} B_{\mu_{p+1} \cdots \mu_{p+q}]}. \tag{C.5}$$

Eq (C.4) defines the exterior derivative of a p-form and Eq. (C.5) defines the wedge product of a p-form and a q-form. We also use the fact that the spin connection is a one-form [2]. The exterior covariant derivative of a one-form $X^a{}_\mu$, for example, then follows as:

$$(D \wedge X)^a{}_{\mu\nu} = (d \wedge X)^a{}_{\mu\nu} + (\omega \wedge X)^a{}_{\mu\nu} \tag{C.6}$$

where:

$$(d \wedge X)^a{}_{\mu\nu} = \partial_\mu X^a{}_\nu - \partial_\nu X^a{}_\mu \tag{C.7}$$

$$(\omega \wedge X)^a{}_{\mu\nu} = \omega^a{}_{\mu b} X^b{}_\nu - \omega^a{}_{\nu b} X^b{}_\mu. \tag{C.8}$$

Eqs. (C.7) and (C.8) follow using:

$$p = 1, q = 1, \mu_1 = \mu, \mu_2 = \nu \tag{C.9}$$

and

$$(d \wedge A)_{\mu_1 \mu_2} = (d \wedge A)_{\mu\nu} = 2\partial_{[\mu} A_{\nu]} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{C.10}$$

$$(A \wedge B)_{\mu_1 \cdots \mu_{p+q}} = (A \wedge B)_{\mu\nu} = \frac{2!}{1!1!} A_{[\mu} B_{\nu]} = A_\mu B_\nu - A_\nu B_\mu \tag{C.11}$$

Now extend this method to the exterior covariant derivative of a two-form, using:

$$\begin{aligned}(d \wedge A)_{\mu_1 \mu_2 \mu_3} &= 3\partial_{[\mu_1} A_{\mu_2 \mu_3]} \\ &= \partial_\mu A_{\nu\rho} + \partial_\nu A_{\rho\mu} + \partial_\rho A_{\mu\nu}\end{aligned} \tag{C.12}$$

and

$$\begin{aligned}(A \wedge B)_{\mu_1 \mu_2 \mu_3} &= \frac{3!}{2!1!} A_{[\mu_1} B_{\mu_2 \mu_3]} = 3A_{[\mu} B_{\nu\rho]} \\ &= A_\mu B_{\nu\rho} + A_\nu B_{\rho\mu} + A_\rho B_{\mu\nu}\end{aligned} \tag{C.13}$$

Therefore the exterior covariant derivative of the torsion or spin form used in the first Bianchi identity is:

$$(D \wedge T)^a{}_{\mu\nu\rho} = (d \wedge T)^a{}_{\mu\nu\rho} + (\omega \wedge T)^a{}_{\mu\nu\rho} \tag{C.14}$$

where:

$$(d \wedge T)^a{}_{\mu\nu\rho} = \partial_\mu T^a{}_{\nu\rho} + \partial_\nu T^a{}_{\rho\mu} + \partial_\rho T^a{}_{\mu\nu} \tag{C.15}$$

$$(\omega \wedge T)^a{}_{\mu\nu\rho} = \omega^a{}_{\mu b} T^b{}_{\nu\rho} + \omega^a{}_{\nu b} T^b{}_{\rho\mu} + \omega^a{}_{\rho b} T^b{}_{\mu\nu} \tag{C.16}$$

and where:

$$T^a{}_{\mu\nu} = \left(\Gamma^\lambda{}_{\mu\nu} - \Gamma^\lambda{}_{\nu\mu} \right) q^a{}_\lambda \tag{C.17}$$

Similarly:

$$\begin{aligned}R^a{}_b \wedge q^b &= R^a{}_{b\mu\nu} q^b{}_\rho + R^a{}_{b\nu\rho} q^b{}_\mu + R^a{}_{b\rho\mu} q^b{}_\nu \\ &= R^a{}_{\mu\nu\rho} + R^a{}_{\nu\rho\mu} + R^a{}_{\rho\mu\nu} \\ &= \left(R^\sigma{}_{\mu\nu\rho} + R^\sigma{}_{\nu\rho\mu} + R^\sigma{}_{\rho\mu\nu} \right) q^a{}_\sigma .\end{aligned} \tag{C.18}$$

So the first Bianchi identity becomes:

$$\partial_\mu T^a{}_{\nu\rho} + \omega^a{}_{\mu b} T^b{}_{\nu\rho} + \cdots = R^\sigma{}_{\mu\nu\rho} q^a{}_\sigma + \cdots \quad . \tag{C.19}$$

Using Eq. (C.17), Eq. (C.19) becomes:

$$\begin{aligned}\partial_\mu \left(\left(\Gamma^\lambda{}_{\nu\rho} - \Gamma^\lambda{}_{\rho\nu} \right) q^a{}_\lambda \right) + \omega^\lambda{}_{\mu b} \left(\Gamma^\lambda{}_{\nu\rho} - \Gamma^\lambda{}_{\rho\nu} \right) q^b{}_\lambda + \cdots \\ = R^\lambda{}_{\mu\nu\rho} q^a{}_\lambda + \cdots\end{aligned} \tag{C.20}$$

Using the Leibnitz Theorem Eq. (C.20) becomes:

$$\left(\partial_\mu \Gamma^\lambda{}_{\nu\rho} - \partial_\mu \Gamma^\lambda{}_{\rho\nu}\right) q^a{}_\lambda + \left(\partial_\mu q^a{}_\lambda + \omega^a{}_{\mu b} q^b{}_\lambda\right)\left(\Gamma^\lambda{}_{\nu\rho} - \Gamma^\lambda{}_{\rho\nu}\right) + \cdots = R^\lambda{}_{\mu\nu\rho} q^a{}_\lambda + \cdots \tag{C.21}$$

Now use the tetrad postulate:

$$\partial_\mu q^a{}_\rho + \omega^a{}_{\mu b} q^b{}_\sigma = \Gamma^\lambda{}_{\mu\sigma} q^a{}_\lambda \tag{C.22}$$

in Eq. (C.21) to obtain:

$$\begin{aligned}
&\partial_\mu \Gamma^\lambda{}_{\nu\rho} - \partial_\nu \Gamma^\lambda{}_{\mu\rho} + \Gamma^\lambda{}_{\mu\sigma} \Gamma^\sigma{}_{\nu\rho} - \Gamma^\lambda{}_{\nu\sigma} \Gamma^\sigma{}_{\mu\rho} \\
&+ \partial_\nu \Gamma^\lambda{}_{\rho\mu} - \partial_\rho \Gamma^\lambda{}_{\nu\mu} + \Gamma^\lambda{}_{\nu\sigma} \Gamma^\sigma{}_{\rho\mu} - \Gamma^\lambda{}_{\rho\sigma} \Gamma^\sigma{}_{\nu\mu} \\
&+ \partial_\rho \Gamma^\lambda{}_{\mu\nu} - \partial_\mu \Gamma^\lambda{}_{\rho\nu} + \Gamma^\lambda{}_{\rho\sigma} \Gamma^\sigma{}_{\mu\nu} - \Gamma^\lambda{}_{\mu\sigma} \Gamma^\sigma{}_{\rho\nu} \\
&:= R^\lambda{}_{\rho\mu\nu} + R^\lambda{}_{\mu\nu\rho} + R^\lambda{}_{\nu\rho\mu}.
\end{aligned} \tag{C.23}$$

The Riemann tensor for any connection (Appendix two) is:

$$R^\lambda{}_{\rho\mu\nu} = \partial_\mu \Gamma^\lambda{}_{\nu\rho} - \partial_\nu \Gamma^\lambda{}_{\mu\rho} + \Gamma^\lambda{}_{\mu\sigma} \Gamma^\sigma{}_{\nu\rho} - \Gamma^\lambda{}_{\nu\sigma} \Gamma^\sigma{}_{\mu\rho}, \tag{C.24}$$

and so Eq. (C.23) is an identity made up of the cyclic sum of three Riemann tensors on either side. The familiar Bianchi identity of the famous Einstein gravitational theory is the SPECIAL CASE when the cyclic sum vanishes:

$$R^\lambda{}_{\rho\mu\nu} + R^\lambda{}_{\mu\nu\rho} + R^\lambda{}_{\nu\rho\mu} = 0. \tag{C.25}$$

Eq. (17.25) is true if and only if the gamma connection is the symmetric Christoffel symbol:

$$\Gamma^\lambda{}_{\mu\nu} = \Gamma^\lambda{}_{\nu\mu}. \tag{C.26}$$

It is not at all clear using tensor notation (Eq. (17.23)) that the first Bianchi identity is a balance of spin and curvature. In order to see this we need the differential form notation of Eq. (C.1) and this is of key importance for the development of the Evans unified field theory.

D

The Second Bianchi Identity

The second Bianchi identity is:

$$\begin{aligned} D \wedge R^a{}_b &= d \wedge R^a{}_b + \omega^a{}_c \wedge R^c{}_b + \omega^c{}_b \wedge R^a{}_c \\ &:= 0. \end{aligned} \tag{D.1}$$

using the results of Appendix Three we may write out Eq. (D.1) in tensor notation:

$$D_\rho R^a{}_{b\mu\nu} + D_\mu R^a{}_{b\nu\rho} + D_\nu R^a{}_{b\rho\nu} := 0 \tag{D.2}$$

where:

$$\begin{aligned} D_\rho R^a{}_{b\mu\nu} &= \partial_\rho R^a{}_{b\mu\nu} + \omega^a{}_{\rho c} R^c{}_{b\mu\nu} + \omega^c{}_{\rho b} R^a{}_{c\mu\nu} \\ &\quad et \;\; cyclicum. \end{aligned} \tag{D.3}$$

Now use:

$$R^a{}_{b\mu\nu} = q^\sigma{}_b R^a{}_{\sigma\mu\nu}. \tag{D.4}$$

The Leibnitz Theorem and tetrad postulate give the result:

$$D_\rho R^a{}_{b\mu\nu} = D_\rho \left(q^\sigma{}_b R^a{}_{\sigma\mu\nu} \right) = q^\sigma{}_b D_\rho R^a{}_{\sigma\mu\nu} \tag{D.5}$$

which implies:

$$D_\rho R^a{}_{\sigma\mu\nu} + D_\mu R^a{}_{\sigma\nu\rho} + D_\nu R^a{}_{\sigma\rho\mu} := 0 \tag{D.6}$$

Now use:

$$R^a{}_{\sigma\mu\nu} = q^a{}_\kappa R^\kappa{}_{\sigma\mu\nu} \tag{D.7}$$

The Leibnitz Theorem and tetrad postulate are used again to find:

$$D_\rho R^\kappa{}_{\sigma\mu\nu} + D_\mu R^\kappa{}_{\sigma\nu\rho} + D_\nu R^\kappa{}_{\sigma\rho\mu} := 0 \tag{D.8}$$

which is the second Bianchi identity in tensor notation for any gamma connection, quod erat demonstrandum.

The second Bianchi identity is true for ANY gamma connection because it is equivalent to:

$$[D_\rho, [D_\mu, D_\nu]] + [D_\mu, [D_\nu, D_\rho]] + [D_\nu, [D_\rho, D_\mu]] := 0. \qquad \text{(D.9)}$$

and Eq. (D.9) can be summarized symbolically as a round trip with covariant derivatives around a cube: The second Bianchi identity is the geometrical

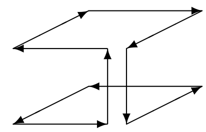

Fig. D.1. Covariant Derivatives Around a Cube

foundation for the conservation law of the Evans unified field theory:

$$D \wedge T^a{}_b := 0. \qquad \text{(D.10)}$$

Eq. (D.10) is the required generalization of the Noether Theorem for the unified field theory. The second Bianchi identity is also the foundation for the generalization of the Einstein field equation in the Evans unified field theory:

$$R^a{}_b = kT^a{}_b\,. \qquad \text{(D.11)}$$

References

1. A. Einstein, *The Meaning of Relativity* (Princeton Univ. Press, 1921 and subsequent editions).
2. S. P. Carroll, *Lecture Notes in General Relativity* (Univ California Santa Barbara, graduate course) arXiv:gr-gq/9712019 v1 3 Dec 1997.
3. G. Stephenson, *Mathematical Methods for Science Students* (Longmans, London, 1968).
4. M. W. Evans, *Generally Covariant Unified Field Theory: the Geometrization of Physics*, (in press, 2005).
5. M. W. Evans, *Found. Phys. Lett.*, **16**, 367 (2003).
6. M. W. Evans, *Found. Phys. Lett.*, **16**, 507 (2003).
7. M. W. Evans, *Found. Phys. Lett.*, **17**, 25 (2004).
8. M. W. Evans, *Found. Phys. Lett.*, **17**, 149 (2004).
9. M. W. Evans, *Found. Phys. Lett.*, **17**, 267 (2004).
10. M. W. Evans, *Found. Phys. Lett.*, **17**, 301 (2004).
11. M. W. Evans, *Found. Phys. Lett.*, **17**, 393 (2004).
12. M. W. Evans, The Evans Lemma of Differential Geometry, *Found. Phys. Lett.*, **17**, 433 (2004)
13. M. W. Evans, Derivation of the Evans Wave Equation from the Lagrangian and Action, Origin of the Planck Constant in General Relativity, *Found. Phys. Lett.*, **17**, 535 (2004)
14. M. W. Evans and the AIAS Author Group, Development of the Evans Wave Equation in the Weak Field Limit, *Found. Phys. Lett.*, **17**, 497 (2004)
15. M. W. Evans, New Concepts from the Evans Unified Field Theory: Part One, The Evolution of Curvature, Oscillatory Universe without Singularity, Causal Quantum Mechanics, and General Force and Field Equations, *Found. Phys. Lett.*, in press (preprint on www.aias.us).
16. M. W. Evans, ibid., Part two: Derivation of the Heisenberg Equation and Replacement of the Heisenberg Uncertainty Principle, *Found. Phys. Lett.*, in press (preprint on www.aias.us).
17. M. W. Evans, Derivation of the Lorentz Boost from the Evans Wave Equation, *Found. Phys. Lett.*, **17**, 663 (2004)
18. M. W. Evans, The Electromagnetic Sector of the Evans Unified Field Theory, *Found. Phys. Lett.*, in press.

19. M. W. Evans (ed.), *Modern Non-Linear Optics*, special topical issues of I. Prigogine an S. A. Rice, *Advances in Chemical Physics* (Wiley Interscience, New York, 1992 to 2001, first and second eds.), vol. 85 and 119; M. W. Evans, J.-P. Vigier et al., *The Enigmatic Photon* (Kluwer, Dordrecht, 1994 to 2002 hardback and softback), in five volumes; M. W. Evans and L. B. Crowell, *Classical and Quantum Electrodynamics and the* $\boldsymbol{B}^{(3)}$ *Field*, (World Scientific, Singapore, 2001); M. W. Evans and A. A. Hasanein, The Photomagneton in Quantum field Theory, (World Scientific, Singapore, 1994).
20. L. Felker, *The Evans Equations of Unified Field Theory* preprint on www.aias.us.
21. M. W. Evans, STAIF 2004 papers on www.aias.us.
22. M. W. Evans, Physica B, **182**, 227, 237 (1992).
23. M. Tatarikis et al., *Measurement of the Inverse Faraday Effect in High Intensity Laser produced Plasmas*, CFDL Annual report, 1998 / 1999, Rutherford Appleton Laboratory.
24. M. W. Evans, G. J. Evans, W. T. Coffey and P. Grigolini, *Molecular Dynamics and the Theory of Broad Band Spectroscopy* (Wiley-Interscience, New York, 1980).
25. P. Pinter, private communication.