Numerical Solutions for the Resonance Equations of ECE Theory

Horst Eckardt

Alpha Foundation's Institute for Advanced Study (A.I.A.S),

www.aias.us, www.atomicprecision.com

HorstEck@aol.com

Abstract

By formulating the equations of ECE field theory as expressions of the potential field, Evans has shown that resonance absorption from ECE spacetime and counter gravitation is possible. There is an urgent need to inspect these mechanisms further by numerically solving the ECE equations. This paper is a first step towards such solutions. The resonance equations are rewritten in a form suitable for numerical treatment without losing too much essential information. Resonance frequencies should be obtainable. A simple solution for the boundary problem is presented, and a numerical scheme for an implementation by computer code is derived.

1 Introduction

A covariant unified field theory (ECE, Einstein – Cartan – Evans theory) has been developed by Evans since 2003 [1]. Gravitation and electromagnetism are shown to originate from space time geometry and are described by a unifed field. The field equations are most clearly formulated by Cartan geometry. Alternatively a tensor representation is appropriate which is justified by the fact that the theory is based on Einstein's general relativity. Finally a vector form can be chosen to clarify the connection to electrodynamics and technical applications. All three forms are mathematically equivalent. In this paper we choose the vector representation since this form is closest related to numerical standard methods being available today. Cartan and tensor formulations may be best suited for symbolic manipulations by computer algebra systems.

The ECE equations are within a scalar valued factor the equations of Cartan geometry. As in the standard model of Maxwell-Heaviside electrodynamics, the equations can be formulated by using either the field tensor or a potential field. The field tensor is within a factor equal to Cartan torsion, and the potential field is within the same factor identical to the tetrad, the basis vectors of the tangent space of ECE space time.

From experiments it is known that there can be spin connections - and therefore potentials - where no fields exist (for example in the Aharonov Bohm effect [4]). Hence the potential field is more fundamental than the field tensor. In particular this becomes apparent when resonance effects are considered. Recently Evans has shown that resonant solutions for the ECE equations exist by which it is possible to gather electric energy from spacetime [2] and even get counter gravitation effects [3]. In both cases a resonance behaviour appears for the potential field. One of the equations has been shown to change to a well known equation for a resonant driven oscillator in classical dynamics if certain approximations are applied. So it is ensured by theory that resonance effects in ECE spacetime exist.

For comparison with experiments and in order to design new applications the equations have to be solved numerically. Even in this case, certain approximations have to be introduced, albeit being less restrictive than for analytic solutions. In particular, an approximate expression for the spin connectons has to be found. In a full-blown solution these connections must also be handled as variables like the tetrads or the potential fields, respectively. But this would require solving nonlinear equations for about 100 variables simultaneously, which is an ambitious task even for today's supercomputers.

The solution method proposed in this paper is the discretization of the equations on a regular 3D grid. Three space dimensions are required since the resonance equations contain vector products which cannot be reduced to less dimensions. This is an effect of Cartan geometry where the description of torsion requires at least three dimensions. In this paper we restrict our considerations to a cubic grid. Cylindric or spherical symmetry may also be important for corresponding application cases.

A further point concerning the solution of differential equations is the choice of appropriate boundary conditions. The derivatives are of second order in space coordinates, so we have to define boundary values of the potential field and possibly its first derivative. Both must fulfil the differential equations on the boundary. Finding such values can be difficult as is known from the solution of Einstein's equations [5]. In this paper we restrict consideration to configurations in free space where masses and charges are located far enough from the boundaries so that approximate solutions as for the Poisson equation can be chosen on the borders. Source terms not being present in ECE theory will be added to make the inner structure of the definition volume easier to compute by numerical methods.

In the next section we bring the ECE resonance equations into a form being suitable for numerical treatment. In the third section approximations for the spin connections are discussed. The boundary conditions are handled in section four, and examples for the concrete iteration scheme are given in the fifth section.

2 The equations to be solved

In order to come to a numerical treatment of the resonance effects we will develop the resonance equations into a suitable form. We do this for the spacetime current resonances first.

2.1 Resonance currents

The resonance equations of ref. [2] in the original form (eqs. 63, 66, 95, 99) are:

$$\underline{\nabla}\cdot\left(\underline{\omega}^{a}{}_{b}\times\underline{A}^{b}\right)=-\mu_0\,\tilde{j}^{0a} \qquad (1)$$

$$\frac{\partial}{\partial t}\left(\underline{\omega}^{a}{}_{b}\times\underline{A}^{b}\right)-c\,\underline{\nabla}\times\left(\underline{\omega}^{a}{}_{b}A^{0b}-\omega^{0a}{}_{b}\underline{A}^{b}\right)=\mu_0\,\underline{\tilde{j}}^{a} \qquad (2)$$

$$\underline{\nabla}\cdot\underline{\nabla}A^{0a}+\frac{1}{c}\underline{\nabla}\frac{\partial\underline{A}^{a}}{\partial t}+\underline{\nabla}\cdot\left(\omega^{0a}{}_{b}\underline{A}^{b}\right)-\underline{\nabla}\cdot\left(\underline{\omega}^{a}{}_{b}A^{0b}\right)=-\mu_0\,\tilde{j}^{0a} \qquad (3)$$

$$\frac{1}{c^2}\frac{\partial^2\underline{A}^{a}}{\partial t^2}+\frac{1}{c}\left(\underline{\nabla}A^{0a}-A^{0b}\underline{\omega}^{a}{}_{b}+\omega^{0a}{}_{b}\underline{A}^{b}\right)$$

$$+\underline{\nabla}\times\left(\underline{\nabla}\times\underline{A}^{a}\right)-\underline{\nabla}\times\left(\underline{\omega}^{a}{}_{b}\times\underline{A}^{b}\right)=\frac{\mu_0}{c}\,\underline{\tilde{j}}^{a} \qquad (4)$$

Here A^a is the potential field, $\omega^a{}_b$ is the spin connection, j^a is the homogeneous and J^a the inhomogeneous current. The range of the polarization indices is a,b = 0,1,2,3. The variables are explained in detail in [2].

First let's check if the equations are well defined. This requires us to count the number of unknowns and to compare it with the number of equations. We consider the spin connections as predefined functions here to retain the (formally) linear structure of the equations. The currents j^a and J^a are considered to be the "driving forces" and predefined also. The potential field A^a is unknown. As explained in [2] it can be decomposed into a scalar part A^{0a} and a vector part $\underline{A}^a$. This gives 4+12=16 components. Compared to this, equations (1)-(4) represent 32 single equations. This mismatch has to be resolved.

The idea is to handle the equations for the homogeneous and inhomogeneous current separately. This leads to two resonant structures, one for each type of current. Equations (1) and (2) are then decoupled from equations (3) and (4), and we have arrived at two well defined sets of equations.

Next we consider the time dependence of the equations. From the theory of damped oscillators [6] it is known that time-dependent behaviour plays only a role during a transient phase after the external force has been applied. We could try to neglect the time derivatives completely, but this would change the nature of the equations since the "force term" would be missing then. Furthermore, the driving force is oscillatory and therefore time dependent [6]. In addition, the presence of the spin connections in equations (1)-(4) is connected to the existence of an oscillation since the frequency is part of the chosen form of $\omega^a{}_b$ as discussed in section 3.

So we make a less restrictive approximation. We assume the time dependence of A^a, j^a and J^a to be of the harmonic oscillatory form

$$\exp(-i\omega t).$$

(Notice that ω here stands for the frequency scalar value, not for the spin connection.) For example the potential field takes the form

$$A^a{}_\mu = A^a{}_\mu(x, y, z, \omega) \exp(-i\omega t).$$

Such an assumption is justified for electromagnetic waves in vacuo since an arbitrary solution can be build by Fourier superposition of the solutions for fixed frequencies [7]. However, the spin connections are also time dependent in general. In section 3 we make an assumption for the form of them showing that each element of $\omega^a{}_b$ is represented by a linear combination of elements of A^a. As a consequence all spin connections show the same time dependence as the potential field A^a. Inserting these factors in equations (1) to (4) leads to

$$\underline{\nabla} \cdot \left(\underline{\omega}^a{}_b \times \underline{A}^b \right) e^{-2i\omega t} = -\mu_0 \tilde{\underline{j}}^{0a} e^{-i\omega t} \qquad (5)$$

$$\frac{\partial}{\partial t}\left(e^{-2i\omega t}\right)\left(\underline{\omega}^a{}_b \times \underline{A}^b\right) - c\,\underline{\nabla} \times \left(\underline{\omega}^a{}_b A^{0b} - \omega^{0a}{}_b \underline{A}^b\right) e^{-2i\omega t} = \mu_0 \tilde{\underline{j}}^a e^{-i\omega t} \qquad (6)$$

$$\frac{1}{c}\frac{\partial}{\partial t}\left(e^{-i\omega t}\right)\underline{\nabla} \cdot \underline{A}^a + \underline{\nabla} \cdot \underline{\nabla} A^{0a} e^{-i\omega t} + \underline{\nabla} \cdot \left(\omega^{0a}{}_b \underline{A}^b\right) e^{-2i\omega t} - \underline{\nabla}\left(\underline{\omega}^a{}_b A^{0b}\right) e^{-2i\omega t} = -\mu_0 \tilde{j}^{0a} e^{-i\omega t} \qquad (7)$$

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\left(e^{-i\omega t}\right)\underline{A}^a + \frac{1}{c}\left(\underline{\nabla} A^{0a} e^{-i\omega t} - A^{0b}\,\underline{\omega}^a{}_b e^{-2i\omega t} + \omega^{0a}{}_b \underline{A}^b e^{-2i\omega t}\right) + \underline{\nabla} \times \left(\underline{\nabla} \times \underline{A}^a\right) e^{-i\omega t} - \underline{\nabla} \times \left(\underline{\omega}^a{}_b \times \underline{A}^b\right) e^{-2i\omega t} = \frac{\mu_0}{c} \tilde{\underline{j}}^a e^{-i\omega t} \qquad (8)$$

Due to the nonlinearities induced by the spin connections, not all exponential time factors cancel out. On the other hand, a stationary solution is highly desirable for computational simplicity. Obviously the remaining time factors lead to an oscillatory behaviour of the terms with spin connections. This is the effect of coupling electromagnetism with gravitation. We restrict ourselves to calculating the maximum effect. The impact of the spin connections is maximal when the real part of the phase factor is unity, we can therefore choose t=0. With this additional restriction the equations become time-independent, taking the form

$$\underline{\nabla}\cdot\left(\underline{\omega}^{a}{}_{b}\times\underline{A}^{b}\right)=-\mu_0\,\tilde{j}^{0a} \tag{9}$$

$$-\frac{2i\omega}{c}\,\underline{\omega}^{a}{}_{b}\times\underline{A}^{b}-\underline{\nabla}\times\left(\underline{\omega}^{a}{}_{b}\,A^{0b}-\omega^{0a}{}_{b}\,\underline{A}^{b}\right)=\frac{\mu_0}{c}\,\tilde{\underline{j}}^{a} \tag{10}$$

$$-\frac{i\omega}{c}\,\underline{\nabla}\cdot\underline{A}^{a}+\underline{\nabla}\cdot\underline{\nabla}A^{0a}+\underline{\nabla}\cdot\left(\omega^{0a}{}_{b}\,\underline{A}^{b}\right)-\underline{\nabla}\cdot\left(\underline{\omega}^{a}{}_{b}\,A^{0b}\right)=-\mu_0\,\tilde{j}^{0a} \tag{11}$$

$$\begin{aligned}&-\frac{\omega^2}{c^2}\,\underline{A}^{a}+\frac{1}{c}\left(\underline{\nabla}A^{0a}-A^{0b}\,\underline{\omega}^{a}{}_{b}+\omega^{0a}{}_{b}\,\underline{A}^{b}\right)\\&\qquad-\nabla^2\underline{A}^{a}+\underline{\nabla}\left(\underline{\nabla}\cdot\underline{A}^{a}\right)-\underline{\nabla}\times\left(\underline{\omega}^{a}{}_{b}\times\underline{A}^{b}\right)=\frac{\mu_0}{c}\,\tilde{\underline{j}}^{a}\end{aligned} \tag{12}$$

where we have applied the vector identity

$$\underline{\nabla}\times\left(\underline{\nabla}\times\underline{A}^{a}\right)=-\nabla^2\underline{A}^{a}+\underline{\nabla}\left(\underline{\nabla}\cdot\underline{A}^{a}\right) \tag{13}$$

in the last equation. This is a form suitable for discretization on a grid for the space coordinates. The solution method and iteration scheme are discussed later in section 5.

The electric and magnetic field can be calculated from the vector part of A by the equations

$$\underline{E}^{a}=-\frac{\partial\underline{A}^{a}}{\partial t}-c\,\underline{\nabla}A^{0a}-c\,\omega^{0a}{}_{b}\,\underline{A}^{b}+c\,\underline{\omega}^{a}{}_{b}\,A^{0b} \tag{14}$$

$$\underline{B}^{a}=\underline{\nabla}\times\underline{A}^{a}-\underline{\omega}^{a}{}_{b}\times\underline{A}^{b} \tag{15}$$

where the polarization index is a=1,2,3.

2.2 Resonant counter gravitation

We repeat the above steps for the equations of counter gravitation which have been derived in [3]. In the exact expressions for the Coulomb law and Ampère-Maxwell law, the spin connections occur in contravariant form and additionally in their covariant version $\omega^{a}{}_{b}$'. These can be obtained from the contravariant versions by applying the metric $g_{\mu\nu}$:

$$\omega_{\mu}{}^{a}{}_{b}{}'=g_{\mu\nu}\,\omega^{\nu a}{}_{b} \tag{16}$$

and the metric can be derived from the tetrad or the A field respectively by

$$g_{\mu\nu} = g^{\mu\nu} = \eta_{ab}\, q^{\mu a}\, q^{\nu b} = \frac{1}{(A^{(0)})^2}\,\eta_{ab}\, A^{\mu a} A^{\nu b} \qquad (17)$$

However this would lead to highly nonlinear and complicated equations, hence we introduce the limit of weak interaction between the electromagnetic and gravitational fields as was done also in [3]. As a result, the terms with ω' vanish. In contrast to the further approximations made in [3] we do not neglect the time dependence. The Coulomb law (equation (41) of [3]) then leads to

$$\frac{1}{c}\underline{\nabla}\frac{\partial \underline{A}^a}{\partial t} + \underline{\nabla}\cdot\underline{\nabla} A^{0a} + \underline{\nabla}\left(\omega^{0a}{}_b \underline{A}^b - \underline{\omega}^a{}_b A^{0b}\right) = -\underline{A}^{b\prime}\cdot\underline{R}^a{}_b := \mu_0 J^{0a\prime} \qquad (18)$$

where $\omega^a{}_b$ is the contravariant spin connection dual to the tetrad as before. We have subsumed the term $A^{b\prime}$ at the right-hand side into the resonance current J^{0a}. This may be justified as long as only the resonance behaviour is of interest.

The Ampère-Maxwell law was given in [3] in a static form for the magnetic field only. We derive a less restrictive form. From eq. (32) of [3] we obtain by inserting above equations (14) and (15) and neglecting the terms with $\omega^a{}_b$' :

$$\frac{1}{c^2}\frac{\partial^2 \underline{A}^a}{\partial t^2} + \frac{1}{c}\underline{\nabla}\frac{\partial \underline{A}^a}{\partial t} + \frac{1}{c}\frac{\partial}{\partial t}\left(\omega^{0a}{}_b \underline{A}^b\right) - \frac{1}{c}\frac{\partial}{\partial t}\left(\underline{\omega}^a{}_b A^{0b}\right) + \underline{\nabla}\times\left(\underline{\nabla}\times\underline{A}^a\right) - \underline{\nabla}\times\left(\underline{\omega}^a{}_b \times \underline{A}^b\right) = \mu_0 \underline{J}^{a\prime} \qquad (19)$$

Introducing the harmonic time dependence

exp(-iωt)

of all quantities (including $\omega^a{}_b$) and using the vector identity (13) we finally obtain from (18), (19) the equations

$$-i\frac{\omega}{c}\underline{\nabla}\cdot\underline{A}^a + \nabla^2 A^{0a} + \underline{\nabla}\left(\omega^{0a}{}_b \underline{A}^b - \underline{\omega}^a{}_b A^{0b}\right) = \mu_0 J^{0a\prime} \qquad (20a)$$

$$-\frac{\omega^2}{c^2}\underline{A}^a - \frac{i\omega}{c}\left(\underline{\nabla}\cdot\underline{A}^a + 2\omega^{0a}{}_b \underline{A}^b - 2\underline{\omega}^a{}_b A^{0b}\right) - \nabla^2 \underline{A}^a + \underline{\nabla}\left(\underline{\nabla}\cdot\underline{A}^a\right) - \underline{\nabla}\times\left(\underline{\omega}^a{}_b \times \underline{A}^b\right) = \mu_0 \underline{J}^{a\prime} \qquad (20b)$$

3 Approximation for the spin connections

As already mentioned in section 2, the spin connection for the free electromagnetic field is dual to the tetrad. This holds approximately for weak interaction between electromagnetism and gravity. We use this approximation here. We have

$$\omega^{\mu a}{}_{b} = -\frac{k}{2}\varepsilon^{a}{}_{bc}\, q^{\mu c} = -\frac{k}{2A^{(0)}}\eta^{ad}\varepsilon_{dbc}\, A^{\mu c} \qquad (21)$$

with

$$k = \frac{\omega}{c}, \quad q^{\mu c} = \frac{1}{A^{(0)}} A^{\mu c}, \qquad (22)$$

$$\varepsilon^{a}{}_{bc} = \eta^{ad}\varepsilon_{dbc}, \quad \eta^{ad} = \mathrm{diag}(-1,1,1,1)$$

$$\varepsilon_{abc} = \begin{cases} 1 & \text{for even permutation} \\ -1 & \text{for odd permutation} \\ 0 & \text{elsewhere} \end{cases}$$

where ε_{ijk} is the Levi-Civita tensor and η^{ab} the Minkowski metric. In contrast to the usual definition of ε_{ijk} the indices run from 0 to 3, the value set is larger than the number of indices. Permutations of {0,1,2}, {0,2,3}, {0,1,3}, and {1,2,3} have to be considered. In addition it is required to specify a numerical value for $A^{(0)}$.

The approximation of the free electromagnetic field may be too restrictive in certain cases. For example Evans has shown [2] that in this case

$$\underline{\omega}^{a}{}_{b} \times \underline{A}^{b} = 0 \qquad (23)$$

holds. Therefore equation (9) gets singular and the equation system is not well defined. A simple remedy is to add small statistical values to $\omega^{a}{}_{b}$ in cases where $\omega^{a}{}_{b} \neq 0$. This is not a solution based on physics, but up to now nothing is known about the dependence of the spin connections on the interaction between electromagnetism and gravitation.

4 Source terms and boundary conditions

In the previous section we have derived stationary equations for the resonance behaviour. To get numerical solutions of the equations we have to provide boundary conditions. Initial conditions are not required because we have no explicit time dependence. In such a situation the physical system under consideration has to be described completely by specifying the potential field on the outer and inner boundaries, and possibly its derivatives. This is a viable way for free space computations, but the resonance effects we are looking for will occur by influencing certain materials, so free space conditions are not sufficient. We have to find a way to describe these effects of matter within the definition volume without specifying the potential field at inner boundaries because this information is not known.

Since ECE theory is a pure field theory, there are no particles of mass or charge a priori, these come out as a divergence of the fields. To obtain a practical solution we assume explicit source terms as in standard theory. The ECE field equations are in barebones notation:

$$d^{1}F = \mu_0 j \qquad (24)$$

$$d^{1}\tilde{F} = \mu_0 J \qquad (25)$$

In standard theory the first equation reads

$$d^1 F = 0$$

as there is no homogeneous current j. So we can add a source current term only to the second equation. This is justified because the phenomenological source terms are contained in the original J. We split J into the resonant part J_R due to interaction of electromagnetism and gravitation and the source current part J_S.

$$d^1 F = \mu_0 j. \qquad (26)$$

$$d^1 \tilde{F} = \mu_0 (J_R + J_S) \qquad (27)$$

Consequently, the J^a terms in equations (11), (12) have to be replaced by

$$\tilde{j}^{0a} \rightarrow \tilde{j}^{0a}_R + \tilde{j}^{0a}_S \qquad (28)$$

$$\underline{\tilde{j}}^{a} \rightarrow \underline{\tilde{j}}^{a}_R + \tilde{j}^{a}_S \qquad (29)$$

The analogous holds for the equations of counter gravitation (20a,b). $\underline{J}^{\sim a}$ corresponds to a conventional current density vector measured in units of A/m^2. $J^{\sim 0a}$ describes a static charge density distribution ρ^a given by

$$\tilde{j}^{0a} = c\rho^a$$

and has the same units. ρ^a is a scalar, so the only polarization index of relevance is a=0. The current $\underline{J}^{\sim a}$ is a vector with three polarization directions a=1, 2, 3.

The question remains how to proceed with equations (9) and (10) which correspond to the resonant Gauss law and Faraday induction law. Both laws hold for free space and do not contain source terms in the limit of standard theory. We propose to solve the equations (11), (12) first (resonant Coulomb law and Ampère-Maxwell law) and insert the solution for the potential field A into eqs. (9) and (10). The latter equations should deliver then a homogenious current. The frequency dependence of the resonance may be different from that of the inhomogeneous current.

Next we discuss the boundary conditions. As already stated, we restrict ourselves to configurations where the boundaries of the volume, where the equations are to be solved, lie in free space. Source charges and currents are far enough away from them so that we can expect an approximate free space behaviour there. For the resonance equations this means that the effects of spin connections are negligible and the potential field is dominated by the Poisson terms in the equations.

All the relevant equations (11), (12), (20a,b) contain a leading second order derivative term. Far from the sources they behave like a Poisson equation. For a physically meaningful solution of the Poisson equation it is required that a condition of the boundary values is fulfilled. Interpreting the A field as a streaming field, it is required that the flow through the borders is equal to the sum of sources and sinks in the definition volume [8]. Otherwise the the process can not be stationary. This is described by the integral conditions

$$\int_V \tilde{j}^{0a}\,dV - \int_S \frac{\partial A^{0a}}{\partial n}\,dS = 0 \qquad (30)$$

$$\int_V \underline{\nabla}\cdot\underline{A}^{a}\,dV - \int_S \underline{A}^{a}\cdot\underline{n}\,dS = 0 \qquad (31)$$

V is the definition volume, S the corresponding surface, and $\underline{n}$ the normal vector in outer direction on the surface. Eq. (30) corresponds to a Neumann type boundary condition while eq. (31) represents a Dirichlet type boundary condition. This is similar to the Navier-Stokes equations in fluid dynamics where A^{0a} corresponds to the scalar pressure and $\underline{A}^{a}$ to the velocity field [8].

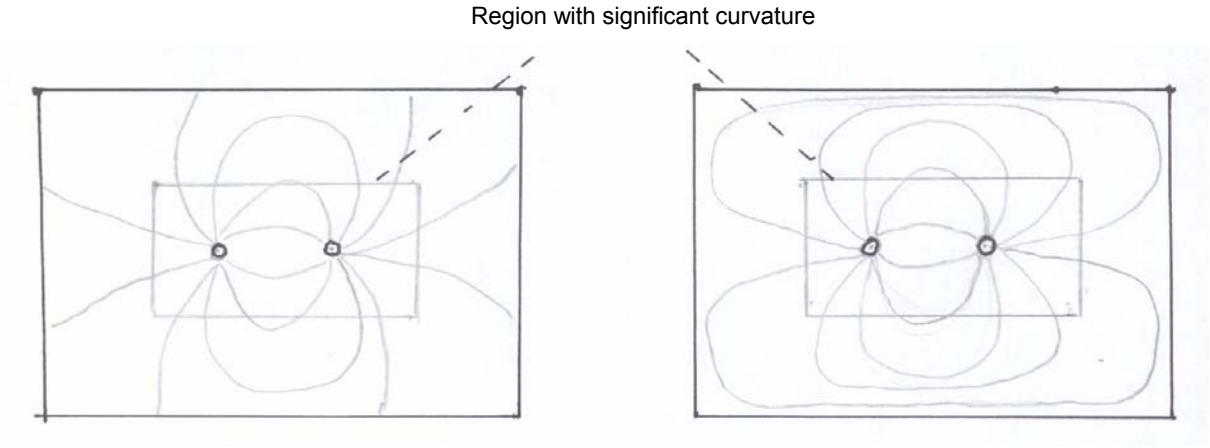

Fig. 1a. Electric field of a dipole (exact)

Fig. 1b. Electric field of a dipole with zero boundary conditions

In the case of resonant ECE equations we assume that the source term densities do not extend to the borders of the definition volume, see Fig. 1a,b. We select electrically neutral configurations and can set

$$A^{0a} = 0$$

and

$$\underline{A}^{a} = 0$$

on all boundaries. Then eqs. (30), (31) are fulfilled trivially. The type of error is visible by comparing Fig. 1a (exact solution) with Fig. 1b (approximate solution). The difference should not be relevant in the interior of the definition volume where interaction effects of torsion and curvature will play a role.

We finish this section by stating that condition (31) does not mean that the divergence of $\underline{A}^{a}$ vanishes in the definition volume. Only the average value of the divergence is zero.

5 Solution scheme on a grid

In this article we consider only rectangular grids and definition volumes. Furthermore we use the finite difference method for solving the equations. This means that all functions have to be computed at the grid points, and differential operators are to be evaluated by algebraic

expressions of some neighboring points. Let's enumerate the grid points in x, y, z direction by indices i, j, k and assume a homogeneous grid spacing

$$\Delta x = \Delta y = \Delta z = h.$$

The partial derivative in x direction of a function f(x, y, z) discretized at grid points x_i, y_j, z_k is approximated by

$$\frac{\partial f_{ijk}}{\partial x} = \frac{1}{2h}\left(f_{i+1,j,k} - f_{i-1,j,k}\right) + O(h^2) \qquad (32)$$

and the Laplace operator by

$$\nabla^2 f = \frac{1}{h^2}\left(f_{i+1,j,k} + f_{i,j+1,k} + f_{i,j,k+1} + f_{i-1,j,k} + f_{i,j-1,k} + f_{i,j,k-1} - 6 f_{ijk}\right) \qquad (33)$$

From these all other operators can be constructed (see Appendix A).

To solve the resonance equations we use the successive overrelaxation (SOR) scheme which is well known from the literature [9] for partial differential equations without time dependence. We describe this method shortly for the Poisson equation

$$\nabla^2 f = \rho \qquad (34)$$

with a fixed function ρ (x, y, z). Discretization according to eq. (33) gives

$$f_{ijk}^{(n+1)} = \frac{1}{6}\left(f_{i+1,j,k}^{(n)} + f_{i,j+1,k}^{(n)} + f_{i,j,k+1}^{(n)} + f_{i-1,j,k}^{(n)} + f_{i,j-1,k}^{(n)} + f_{i,j,k-1}^{(n)} - h^2 \rho_{ijk}\right) \qquad (35)$$

Writing it this form, a value for f at a grid point (i, j, k) can be computed from the surrounding grid points appearing at the right hand side. In this way the values for iteration n+1 are computed from the preceding step n. The iterations are denoted as upper indices in parentheses. To obtain faster convergence (in some cases also to obtain convergence at all) the newly computed values $f^{(n)}$ are weighted with the former values by a mixing factor w by

$$f^{(n+1)} := (1-w) f^{(n)} + w f^{(n+1)} \qquad (36)$$

where w is to be chosen between 1 and 2 (therefore called "overrelaxation"). Convergence can be accelerated further by using the new values of ieration n+1 also at the right-hand side as soon as they have been computed in the current iteration sweep (Gauss-Seidel iteration).

Now the resonance equations (11, 12, 20a,b), which all contain a Laplace operator, can be formulated as an iteration scheme as described for the Poisson equation. For example, eq. (11) reads

$$A^{0a\,(n+1)}_{ijk} = \frac{1}{6}\Big(A^{0a\,(n)}_{i+1,j,k} + A^{0a\,(n)}_{i,j+1,k} + A^{0a\,(n)}_{i,j,k+1} + A^{0a\,(n)}_{i-1,j,k} + A^{0a\,(n)}_{i,j-1,k} + A^{0a\,(n)}_{i,j,k-1} + h^2\Big(-\frac{i\omega}{c}\Big[\underline{\nabla}\cdot\underline{A}^{a\,(n)}\Big]_{ijk} + \Big[\underline{\nabla}\cdot\big(\omega^{0a}{}_{b}\,\underline{A}^{b\,(n)}\big)\Big]_{ijk} - \Big[\underline{\nabla}\cdot\big(\underline{\omega}^{a}{}_{b}\,A^{0b\,(n)}\big)\Big]_{ijk} + \mu_0\big(\tilde{j}^{0a}_{R\,ijk} + \tilde{j}^{0a}_{S\,ijk}\big)\Big)\Big) \qquad (37)$$

The discrete forms of the differential operators have to be inserted according to Appendix A. At the right hand side only potential fields of previous iteration n occur, thus maintaining the method introduced in eq. (35).

The nonlinearities arising due to the spin connections have still to be addressed. The spin connections $\omega^a{}_b$ are computed from the A fields as described by eq. (21). According to a method called Picard iteration [10], we compute $\omega^a{}_b$ from $A^{(n)}$, obtaining effectively an iterated $\omega^a{}_b{}^{(n)}$. Thus we maintain the iteration scheme of eq. (35) and extend it to non-linear equations.

At this point all has been said about the solution of the stationary resonance equations. Finally we give an outlook how the explicit time dependence in the original version of the equations can be covered. Time discretization schemes are handled differently from space discretization since an iterative solution of the time dependence is not possible. Time development has to be obtained in one step.

Only the first time derivative should occur in the equations. Let's consider eq. (4) for the resonance current. We can get rid of the second time derivative of $\underline{A}^a$ by defining a supplementary vector field

$$\underline{C}^a = \frac{\partial \underline{A}^a}{\partial t} \qquad (38)$$

Inserting this in eqs. (3) and (4) leads to

$$\nabla^2 A^{0a} + \frac{1}{c}\left(\underline{\nabla}\cdot\underline{C}^a + \underline{\nabla}\cdot\left(\omega^{0a}{}_b\,\underline{A}^b\right) - \underline{\nabla}\cdot\left(\underline{\omega}^a{}_b A^{0b}\right)\right) = -\mu_0 \tilde{j}^{0a} \qquad (39)$$

$$\frac{1}{c^2}\frac{\partial \underline{C}^a}{\partial t} + \frac{1}{c}\underbrace{\left(\underline{\nabla} A^{0a} - \underline{\omega}^a{}_b A^{0b} + \omega^{0a}{}_b \underline{A}^b\right)}_{\underline{f}} \qquad (40)$$

$$\underbrace{-\nabla^2 \underline{A}^a + \underline{\nabla}\left(\underline{\nabla}\cdot\underline{A}^a\right) - \underline{\nabla}\times\left(\underline{\omega}^a{}_b \times \underline{A}^b\right)}_{\underline{g}} = \frac{\mu_0}{c}\underline{\tilde{j}}^a$$

The simplest way of numerically solving these equations is by forward discretization in time:

$$\frac{\partial \underline{C}^a}{\partial t} = \frac{\underline{C}^{a(n+1)} - \underline{C}^{a(n)}}{\Delta t} - \mathcal{O}(\Delta t) \qquad (41)$$

The index n now describes the time step. In contrast to the symmetric scheme of eq. (32) this scheme is less precise, the error is of linear order instead of quadratic order in Δt. Inserting (41) in eq. (40) leads to the time integration scheme (with abbreviation functions $\underline{f}$ and $\underline{g}$ defined there)

$$\underline{C}^{a(n+1)} = \underline{C}^{a(n)} + c^2 \Delta t\left(-\frac{1}{c}\underline{f}^{(n)} - \underline{g}^{(n)} + \frac{\mu_0}{c}\underline{\tilde{j}}^a\right) \qquad (42)$$

The time integration for $\underline{A}^a$ then follows from (38) by

$$\frac{\partial \underline{A}^a}{\partial t} = \frac{\underline{A}^{a(n+1)} - \underline{A}^{a(n)}}{\Delta t} = \underline{C}^{a(n)}$$

or

$$\underline{A}^{a(n+1)} = \Delta t\, \underline{C}^{a(n)} + \underline{A}^{a(n)} \qquad (43)$$

The time derivative of the scalar component A^{0a} does not appear in eqs. (3) and (4). The time dependence is implicit and needs to be solved by an iterative scheme similar to eq. (37). All variables except A^{0a} have to be chosen for the new time step n+1. These are obtained from eqs. (42) and (43) before, and $\omega^a{}_b$ has to be computed from them also. We will not go into further detail here.

This was only a short demonstration on how the time dependence of the resonance equations can be handled. More detailed considerations are required to obtain a numerically stable scheme. For example the right-hand side of eq. (37) should be defined by a mixture of the iteration steps n and n+1 (Crank-Nicolson scheme [11]).

The boundary conditions have to be supplemented by an initial condition of the field variables all over the definition volume. This is not as easy as only to define values on the boundaries because the initial values have already to fulfil the equations which are to be solved. Several methods for resolving this problem have been mentioned in the literature for the Einstein equations of general relativity [5]. These methods should be considered for applicability.

6 Conclusion

It has been shown how the resonance equations of ECE theory for energy absorption and counter gravitation can be solved numerically. By assuming a harmonic behaviour of time dependence a stationary form could be derived. The equations can be solved by iterative methods. We have confined ourselves to the simplest form of geometry - a cubic grid. Alternatively, spherical or cylinder symmetry (e.g. for circulating currents in coils) could be applied. The solution method proposed is a straightforward overrelaxation scheme. The efficiency can be enhanced further by applying multigrid methods [12]. For a more complicated geometry, finite element methods should be deployed.

The equations for resonance absorption represent two independent sets of equations for the potential field. This is a consequence of the fact that the two structure equations of Cartan geometry are dual to one another. For getting solutions for the electric and magnetic field both sets of equations are required, but for the potential field one of them is sufficient, as far as the spin connections are not considered as independent variables. As a consequence, the homogeneous and inhomogeneous current are not independent in this approach.

In the stationary case we have added source terms to the resonance equations for practical purposes. This was done in analogy to Maxwell theory so that these terms only show up where the inhomogeneous resonant current occurs. The homogeneous current has no counterpart in Maxwell theory and can not be augmented by source terms. It is not restricted to free space scenarios and can occur in matter as well.

Boundary conditions could be formulated in a simple manner which is adequate for configurations embedded in empty space. Other configurations, for example unit cells of solids, have to be handled in a different way.

The time dependent behaviour of the potential field can be studied by using numerical time integration schemes. A rough idea for this was outlined. Generally, methods having been developed for the Navier-Stokes equations can be applied, where also nonlinearities are to be dealt with.

The resonance effects can alternatively be formulated as a dielectric theory [3]. Numerical schemes for this have to be worked out further.

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9 Appendix A

List of differential operators in Cartesian coordinates

Be f a scalar field $f(\underline{x}, t) : R^3xR^1 \rightarrow R^1$; $\underline{u}$ a vector field $\underline{u}(\underline{x}, t) : R^3xR^1 \text{ -> } R^3$. Then it is defined gradient:

$$\underline{\nabla} f = (\partial f/\partial x_1, \partial f/\partial x_2, \partial f/\partial x_3)$$

gradient of a vector field:

$$\underline{\nabla} (\underline{u}) = (\partial u_1/\partial x_1, \partial u_2/\partial x_2, \partial u_3/\partial x_3)$$

divergence :

$$\underline{\nabla} \cdot \underline{u} = \partial u_1/\partial x_1 + \partial u_2/\partial x_2 + \partial u_3/\partial x_3$$

curl:

$$\underline{\nabla} \text{ x } \underline{u} = (\partial u_3/\partial x_2 - \partial u_2/\partial x_3, \partial u_1/\partial x_3 - \partial u_3/\partial x_1, \partial u_2/\partial x_1 - \partial u_1/\partial x_2)$$

Laplace operator:

$$\nabla^2 f = \partial^2 f/\partial x_1^2 + \partial^2 f/\partial x_2^2 + \partial^2 f/\partial x_3^2$$

Laplace operator of a vector field:

$$\nabla^2 \underline{u} = (\partial^2 u_1/\partial x_1^2 + \partial^2 u_1/\partial x_2^2 + \partial^2 u_1/\partial x_3^2,$$
$$\partial^2 u_2/\partial x_1^2 + \partial^2 u_2/\partial x_2^2 + \partial^2 u_2/\partial x_3^2,$$
$$\partial^2 u_3/\partial x_1^2 + \partial^2 u_3/\partial x_2^2 + \partial^2 u_3/\partial x_3^2)$$