5(2): Radial Current Densities for Dust in a Closed a
Open Universe, FLRW Metric.

The radial current density from the FLRW metric is:

$$J_r = \frac{A^{(0)}}{\mu_0}\left(\frac{1 - k r^2}{a^2}\right)\left(\frac{2}{a^2}\left(k + \dot{a}^2\right) + \frac{\ddot{a}}{a}\right) -$$

Closed Universe

For dust in a closed universe:

$$k = 1, \quad p = 0 \qquad -(2)$$

So
$$J_r = \frac{A^{(0)}}{\mu_0}\left(\frac{1 - r^2}{a^2}\right)\left(\frac{2}{a^2} + 2\left(\frac{\dot{a}}{a}\right)^2 + \frac{\ddot{a}}{a}\right) - (3)$$

where:
$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{16\pi G}{3}\rho - \frac{2}{a^2}, \quad \frac{\ddot{a}}{a} = -\frac{4\pi}{3}G\rho$$

i.e.
$$\boxed{J_r = \frac{A^{(0)}}{\mu_0}\left(\frac{1 - r^2}{a^2}\right) \cdot 4\pi G\rho} \qquad -(4)$$

where
$$a^2 = \frac{C^2}{4}\left(1 - \cos\phi\right)^2, \qquad -(5)$$
$$t = \frac{C}{2}\left(\phi - \sin\phi\right). \qquad -(6)$$

Open Universe

For dust in an open universe:

$$k = -1, \quad p = 0 \qquad -(7)$$

$$J_r = \frac{A^{(0)}}{\mu_0}\left(\frac{1 + r^2}{a^2}\right)\left(2\left(\frac{\dot{a}}{a}\right)^2 - \frac{2}{a^2} + \frac{\ddot{a}}{a}\right)$$

$$= A^{(0)} \left(1 + r^2 \ldots\right)\left(16\pi G \ldots\right.$$

2) Thus:

$$\boxed{J_r = \frac{A^{(0)}}{\mu_0}\left(\frac{1+r^2}{a^2}\right) \cdot 4\pi G\rho} \quad -(8)$$

where:

$$a^2 = \frac{c^2}{2}\left(\cosh\phi - 1\right)^2 \quad -(9)$$

$$t = \frac{c}{2}\left(\sinh\phi - \phi\right) \quad -(10)$$

Results for FLRW Metric Applied to Dust

Charge Density

$$\boxed{\rho_e = 4\pi\epsilon_0 \phi G\rho} \quad -(11)$$

Radial Current Density

1) Flat Universe

$$\boxed{J_r = \frac{A^{(0)}}{\mu_0}\left(\frac{4\pi G\rho}{a^2}\right)} \quad -(12)$$

2) Closed Universe

$$\boxed{J_r = \frac{A^{(0)}}{\mu_0}\left(4\pi G\rho\left(\frac{1-r^2}{a^2}\right)\right)} \quad -(13)$$

3) Open Universe

$$\boxed{J_r = \frac{A^{(0)}}{\mu_0}\left(4\pi G\rho\left(\frac{1+r^2}{a^2}\right)\right)} \quad -(14)$$

Note in SI Units

$$A^{(0)} = J s C^{-1} m^{-1}, \quad \mu_0 = J s^2 C^{-2} m^{-1} \quad -(15)$$

$$\text{So } \frac{A^{(0)}}{\mu_0} = C s^{-1} = \text{Amps} \quad -(16)$$

3) $G = m\,kgm^{-1}, \quad \rho = kgm\,m^{-3}, \quad -(17)$

So:

$$J_1 = Cs^{-1}m^{-2} = Amp\,m^{-2} \quad -(18)$$

This is the correct SI unit of current density, provided that the following factors are unitless:

$$\frac{1}{a^2}, \quad \frac{1-r^2}{a^2}, \quad \frac{1+r^2}{a^2} \quad -(19)$$

These factors come from the usual way in which the FLRW metric is defined:

$$ds^2 = -c^2dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right) \quad -(20)$$

It is seen that the factor:

$$f = \frac{a^2}{1-kr^2} \quad -(21)$$

is unitless in S.I. units. Since 1 is unitless, kr^2 is also unitless and a^2 is unitless.

Therefore the units in eqns. (11) to (14) are self-consistent in S.I.

PS In Eq. (12):

$$a^2 = \left(\frac{9C}{4}\right)^{2/3} t^{4/3} \quad -(22)$$