Paper 93(4): Checking Part 4

General Expression for Curl

If a general $\omega_0 = \omega_0(t, x, y, z)$

Then:

$$\underline{\nabla} \times (\omega_0 \underline{\dot{A}}) = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \omega_0 A_x & \omega_0 A_y & \omega_0 A_z \end{vmatrix}$$

The $\underline{k}$ component is

$$\left(\frac{\partial}{\partial x}(\omega_0 A_y) - \frac{\partial}{\partial y}(\omega_0 A_x) \right) \underline{k}$$

$$= \left(A_y \frac{\partial \omega_0}{\partial x} + \omega_0 \frac{\partial A_y}{\partial x} - A_x \frac{\partial \omega_0}{\partial y} - \omega_0 \frac{\partial A_x}{\partial y} \right) \underline{k}$$

In cylindrical polar coordinates:

$$\underline{\nabla} \times (\omega_0 \underline{\dot{A}}) = \left(\frac{1}{r} \frac{\partial(\omega_0 \dot{A}_z)}{\partial \phi} - \frac{\partial(\omega_0 \dot{A}_\phi)}{\partial z} \right) \underline{e}_r + \left(\frac{\partial(\omega_0 \dot{A}_r)}{\partial z} - \frac{\partial(\omega_0 \dot{A}_z)}{\partial r} \right) \underline{e}_\phi + \frac{1}{r} \left(\frac{\partial}{\partial r}(r \omega_0 \dot{A}_\phi) - \frac{\partial(\omega_0 \dot{A}_r)}{\partial \phi} \right) \underline{e}_z$$