Checking Paper 94(3)

1) I think that eq. (12) should be:

$$\frac{\dot{\underline{E}}}{c} = -\frac{\ddot{\underline{A}}}{c} - \ddot{\omega}\,\underline{A} - \omega^{\circ}\,\ddot{\underline{A}}$$

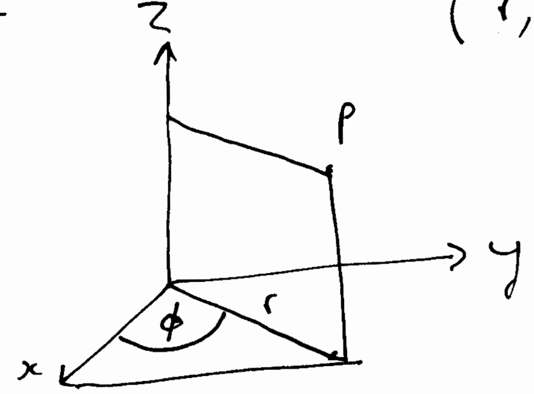
minor error

2) Cylindrical Polar Coordinates (r, ϕ, z)

$x = r\cos\phi$

$y = r\sin\phi$

$z = z$



$$\underline{\nabla}\cdot\underline{F} = \frac{1}{r}\frac{\partial(rF_r)}{\partial r} + \frac{1}{r}\frac{\partial F_\phi}{\partial\phi} + \frac{\partial F_z}{\partial z}$$

$$\underline{\nabla}\times\underline{F} = \left(\frac{1}{r}\frac{\partial F_z}{\partial\phi} - \frac{\partial F_\phi}{\partial z}\right)\underline{e}_r + \left(\frac{\partial F_r}{\partial z} - \frac{\partial F_z}{\partial r}\right)\underline{e}_\phi + \frac{1}{r}\left(\frac{\partial(rF_\phi)}{\partial r} - \frac{\partial F_r}{\partial\phi}\right)\underline{e}_z$$

("Vector Analysis Problem Solver", p. 1071)

So if: $\underline{A} = A_r\,\underline{e}_r + A_\phi\,\underline{e}_\phi$

$$\underline{\nabla}\times\underline{A} = \frac{1}{r}\frac{\partial(rA_\phi)}{\partial r}\,\underline{e}_z$$

checked

We have: $\underline{\omega} = \omega_r\,\underline{e}_r + \omega_\phi\,\underline{e}_\phi$

$\underline{A} = A_r\,\underline{e}_r + A_\phi\,\underline{e}_\phi$

2) On VAPS p. 1027:

$$\underline{e}_r = \underline{i}\cos\phi + \underline{j}\sin\phi$$
$$\underline{e}_\phi = -\underline{i}\sin\phi + \underline{j}\cos\phi$$
$$\underline{e}_z = \underline{k}$$

So:
$$\underline{\omega} = \omega_r(\underline{i}\cos\phi + \underline{j}\sin\phi) + \omega_\phi(-\underline{i}\sin\phi + \underline{j}\cos\phi)$$
$$\underline{A} = A_r(\underline{i}\cos\phi + \underline{j}\sin\phi) + A_\phi(-\underline{i}\sin\phi + \underline{j}\cos\phi)$$

$$\underline{\omega}\times\underline{A} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ (\omega_r\cos\phi - \omega_\phi\sin\phi) & (\omega_r\sin\phi + \omega_\phi\cos\phi) & 0 \\ (A_r\cos\phi - A_\phi\sin\phi) & (A_r\sin\phi + A_\phi\cos\phi) & 0 \end{vmatrix}$$

$$= \Big((\omega_r\cos\phi - \omega_\phi\sin\phi)(A_r\sin\phi + A_\phi\cos\phi) - (\omega_r\sin\phi + \omega_\phi\cos\phi)(A_r\cos\phi - A_\phi\sin\phi)\Big)\underline{k}$$

$$= \Big(\omega_r A_\phi\cos^2\phi - \omega_\phi A_r\sin^2\phi + \cancel{\omega_r A_r\cos\phi\sin\phi} - \cancel{\omega_\phi A_\phi\sin\phi\cos\phi} - \cancel{\omega_r A_r\sin\phi\cos\phi} - \omega_\phi A_r\cos^2\phi + \omega_r A_\phi\sin^2\phi + \cancel{\omega_\phi A_\phi\sin\phi\cos\phi}\Big)\underline{k}$$

$$= \Big(\omega_r A_\phi(\cos^2\phi + \sin^2\phi) - \omega_\phi A_r(\cos^2\phi + \sin^2\phi)\Big)\underline{k}$$

3)

$$= (\omega_r A_\phi - \omega_\phi A_r)\, \underline{k}$$

$$= (\omega_r A_\phi - \omega_\phi A_r)\, \underline{e}_z$$

Eq. (25) should be:

$$\underline{\omega} \times \underline{A} = \begin{pmatrix} 0 \\ 0 \\ \omega_r A_\phi - \omega_\phi A_r \end{pmatrix}$$