The Reissner Nordström Metric

This is obtained by solving the Einstein field equation with a particular form of the electromagnetic stress energy tensor:

$$ds^2 = -\left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right) dt^2 + \left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega^2 \quad — (1)$$

in reduced units. So:

$$g_{00} = -\left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right), \; g_{11} = \left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right)^{-1}$$

$$g_{22} = r^2, \quad g_{33} = r^2 \sin^2\theta. \quad — (2)$$

This was discovered independently by Reissner and Nordström, 1916 – 1918, and was an early attempt at field unification.

The Charged Kerr or Newman, Penrose et al. Metric

In 1963, Kerr discovered another stationary solution of the EH field equation. In 1965 this was extended by Newman et al. to the metric of a rotating charged mass. This metric is a generalization of the Reissner Nordström metric and is as follows:

2)

$$ds^2 = -\left(\frac{\Delta - a^2\sin^2\theta}{\Sigma}\right)dt^2 + \frac{\Sigma}{\Delta}\,dr^2 + \Sigma\, d\theta^2$$

$$-\frac{2a\sin^2\theta\,(r^2+a^2-\Delta)}{\Sigma}\,dt\,d\phi$$

$$+\frac{1}{\Sigma}\left((r^2+a^2)^2 - \Delta a^2\sin^2\theta\right)\sin^2\theta\, d\phi^2 \qquad -(3)$$

where:

$$\Sigma = r^2 + a^2\cos^2\theta,$$
$$\Delta = r^2 + a^2 + e^2 - 2Mr.$$

Special Cases.

1) When $e = 0$, the vacuum Kerr solution is obtained.

2) When $a = 0$, the Reissner Nordström solution is required.

3) When $a = e = 0$ the Schwarzschild solution is required.

No other stationary solutions exist, but dynamical solutions such as Robertson Walker exist.

Before coding, g_{00}, g_{11}, g_{22} and g_{33} are needed from eq. (3).