85(4): Anomalous Magnetic g factor of the Electron

In ECE theory (paper 18 or chapter 20 of volume 1) this is given by:

$$\boxed{g_{ECE} = 2 + \frac{\alpha}{\pi} + \frac{\alpha^2}{8\pi^2}} \quad -(1)$$

Using: $\alpha(\text{exptl.}) = 0.007297(3)$ — (2)

gives: $\boxed{g(ECE) = 2.0023235}$ — (3)

This compares with:

$$\boxed{g(\text{exptl}) = 2.002319304\,8\ldots\ldots} \quad -(4)$$

i.e. $g/2(\text{exptl}) = 1.001\,159\,652\,180\,85\,(76)$ — (5)

as of Feb. 2007 by a method based on a single electron in a Penning trap. The original calculation by Schwinger in the forties gave:

$$\boxed{g(\text{Schwinger}) = 2.00232} \quad -(6)$$

using the early methods of QED.

Therefore the ECE method differs from the 2007 experimental value of g only by:

$$\boxed{g(ECE) - g(\text{exptl}) = 0.0000042} \quad -(7)$$

This means that ECE reproduces the anomalous g factor to one part in a million.

2)

The α (experimental) of eq. (2) is rounded off to experimental uncertainty in $\hbar$:

$$\Delta \hbar = \pm 1.7 \times 10^{-7} \text{ Js.} \quad -(8)$$

Here, in S.I. units:

$$\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c} \quad -(9)$$

so α is known to:

$$\boxed{\Delta \alpha (\text{expt}) = \pm 1.7 \times 10^{-7}}$$

in relative standard uncertainty. In calculating α the speed of light is:

$$c = 2.997925 \times 10^{8} \text{ ms}^{-1} \quad -(10)$$

$$e = -1.60219 \times 10^{-19} \text{ C} \quad -(11)$$

$$4\pi\epsilon_0 = 1.112650 \times 10^{-10} \text{ J}^{-1}\text{C}^2\text{m}^{-1} \quad -(12)$$

$$\hbar = (1.05459 \pm 0.00000017) \times 10^{-34} \text{ Js} \quad -(13)$$

which gives:

$$1/\alpha = (137.0360 \pm 0.00017). \quad -(14)$$

to the same relative standard uncertainty.

Therefore QED cannot possibly claim to give a theoretical estimate many orders of magnitude more precise than the experimental relative standard uncertainty.

The Wolfram site states that α is calculated from QED using:

$$g = 2\left(1 + \frac{\alpha}{2\pi} - 0.328\left(\frac{\alpha}{\pi}\right)^2 + 1.181\left(\frac{\alpha}{\pi}\right)^3 - 1.510\left(\frac{\alpha}{\pi}\right)^4 + \dots + 4.393 \times 10^{-12}\right) \quad -(15)$$

(www.scienceworld.wolfram). It is stated that the value of α is actually derived by combining measurements of g with "accurate expansion using Feynman diagrams" (sic.) This involves the computation of thousands of Feynman diagrams.

It is seen from eqs. (15) and (1) that the accuracy of ECE and QED is the same to first order in α. Since α is known only to a relative standard uncertainty of $\pm 1.7 \times 10^{-7}$, g cannot be estimated theoretically to a higher precision.

ECE Method (Paper 18, i.e. vol. 1, chapter 20)

This is based on the minimal prescription:

$$eA^{(vac)} = \hbar \kappa, \quad -(16)$$

where $A^{(vac)}$ is the zero point level of the quantized electromagnetic field. The fine structure constant is then identified as:

$$\frac{\alpha}{4\pi} := \frac{eA^{(vac)}}{\hbar \kappa} \quad -(17)$$

4)

Thus:

$$A^{(vac)} = \left(\frac{e}{16\pi^2 \epsilon_0 \hbar c^2} \right) \hbar\omega \quad \text{— (18)}$$

where $\hbar\omega/2$ is the vacuum energy level of the quantized electromagnetic field:

$$E_n = \left(n + \frac{1}{2} \right) \hbar\omega \quad \text{— (19)}$$

regarded as a harmonic oscillator as usual. Thus

$$\boxed{A^{(vac)} := \frac{\alpha}{4\pi e c} E_n^{(vac)}} \quad \text{— (20)}$$

where $E_n^{(vac)}$ is the vacuum energy. In QED the latter gives rise to virtual photons and electron positron pairs.

Now extend eq (20) to energy - momentum:

$$e A_\mu^{(vac)} := \frac{\alpha}{4\pi} p_\mu^{(vac)} . \quad \text{— (21)}$$

It is seen that this is a kind of minimal prescription. Eq (21) is used in the Dirac equation of the electron:

$$\left(\gamma^\mu p_\mu \left(1 + \frac{\alpha}{4\pi} \right) - mc \right) \psi = 0 \quad \text{— (22)}$$

i. e.

$$\left(\gamma^\mu p_\mu \left(1 + \alpha' \right) - mc \right) \psi = 0 \quad \text{— (23)}$$

5)

where:

$$\alpha' := \frac{e^2}{16\pi^2 \epsilon_0 \hbar c} \quad -(24)$$

Eq. (23) is equivalent to:

$$\gamma^\mu \rightarrow (1+\alpha')\gamma^\mu \quad -(25)$$

which is an increase in the Dirac matrix. In QE

$$\gamma^\mu \rightarrow \gamma^\mu + \Lambda^{\mu(2)} \quad -(26)$$

where $\Lambda^{\mu(2)}$ is a convergent vertex.

In ECE, α' is a spacetime property in general relativity, in QED it is still special relativity.

The factor 2 of Dirac theory of the electron comes from:

$$2\eta^{\mu\nu} = \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu \quad -(27)$$

where $\eta^{\mu\nu}$ is diag $(-1, 1, 1, 1)$, the Minkowski metric. So:

$$\boxed{g\,(\text{Dirac}) = 2} \quad -(28)$$

In ECE theory:

$$\gamma^\mu \rightarrow (1+\alpha')\gamma^\mu \quad -(29)$$

$$\gamma^\nu \rightarrow (1+\alpha')\gamma^\nu \quad -(30)$$

So :

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu \rightarrow 2(1+\alpha')^2 (\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu) \quad - (31)$$

Thus :

$$\boxed{g(\text{ECE}) = 2(1+\alpha')^2 = 2\left(1+\frac{\alpha}{4\pi}\right)^2} \quad - (32)$$

which is eq. (1).

Notes

It is seen that this method depends on the definition of the vacuum ~~vector~~ potential magnitude in eq. (17), and comes directly from the minimal prescription. It over-estimates the latest experimental value of g (Feb. 2007) by only one part in a million. Therefore the hypothesis (21) is accurate to this degree of precision. It states essentially that the average contribution of $eA_\mu^{(vac)}$ to p_μ of the electron is given through $\alpha / 4\pi$. The factor 4π can be thought of as introduced by SI units, and coming from an averaging process.

7)

Fitting to Experimental Result

As the Wolfram site indicates, there is no way of deriving α fundamentally. It is defined by the combination of constants in eq. (9). It could equally well be defined in ECE as:

$$\frac{\alpha^2}{8\pi^2} + \frac{\alpha}{\pi} + 2 = 2.002319304 8 \quad -(33)$$

i.e.

$$\frac{\alpha}{\pi}\left(\frac{\alpha}{8\pi} + 1\right) = 0.0023193048 \quad -(34)$$

giving:

$$\alpha \sim 0.0023193048\,\pi$$

$$\boxed{\alpha = 0.0072863} \quad -(35)$$

This means:

$$\boxed{\frac{e^2}{4\pi\epsilon_0 \hbar c} = 0.0072863} \quad -(36)$$

This means that the α (exptl.) of eq. (2), found from eqs. (10) to (13), must be adjusted by

$$\Delta\alpha = 0.0072863 - 0.007297$$
$$\sim -0.000004 \quad -(37)$$

as in eq. (7)

8).

This means that the ECE result (1) would be exact if this adjustment were made. It could be made if the value of ϵ_0 were adjusted by international treaty. At present it is exact:

$$\epsilon_0 = 8.854\,187\,817 \times 10^{-12} \ J^{-1} C^2 m^{-1}.$$

The speed of light c is also taken as exact:

$$c = 2.99792458 \times 10^{8} \ m s^{-1}$$

while e is measured:

$$e = 1.6021753 \times 10^{-19} \ C$$
$$\pm 0.0000014 \times 10^{-19}$$

If either ϵ_0 or c were adjusted, then the ECE result would be exact and much simpler than QED.

Lamb Shift

In this case $A^{(vac)}$ of eq (17) has to be used with the Dirac equation of the H atom, and has to affect $2S_{1/2}$ and $2P_{1/2}$ energy levels to a different extent: 1060 MHz.