Motion of One Electron in a Radiated $\underline{B}^{(3)}$ Field and a Static Magnetic Field

1) Motion in a Static Magnetic Field

Angular frequency:

$$\Omega = \frac{eB}{m} . \qquad -(1)$$

Angular momentum:

$$J = erA \qquad -(2)$$

Kinetic energy:

$$T = e^2A^2/(2m) \qquad -(3)$$

2) Motion in the $\underline{B}^{(3)}$ Radiated Field.

a) Direct Integration of Einstein Equation

Angular frequency:

$$\Omega = \frac{\left(\gamma m v + eA^{(0)}\right)c^2}{\left(r_0 + \frac{eA^{(0)}}{\gamma m \omega}\right)\left(mc^2(1+\gamma) + e\phi\right)} \qquad -(4)$$

Angular Momentum: $\qquad -(5)$

$$\underline{J} = \gamma \underline{r_0} \times \underline{p} + e\underline{r_0} \times \underline{A} + e\int \underline{A}\, dt \times \underline{v} + \frac{e^2}{m\gamma}\int \underline{A}\, dt \times \underline{A}$$

$$J = \left(r_0 + \frac{eA^{(0)}}{\gamma m \omega}\right)\left(\gamma m v + eA^{(0)}\right) \qquad -(5a)$$

a) <u>Kinetic Energy</u>:

$$T = \frac{\left(\gamma m v + e A^{(0)}\right)^2 c^2}{m c^2 (1+\gamma) + e\phi} \quad -(6)$$

b) <u>Hamilton - Jacobi Method</u>

Angular frequency:

$$\Omega = \left(\omega^2 + \frac{e^2 B^{(0)2}}{m^2}\right)^{1/2} \quad -(7)$$

Angular Momentum:

$$\underline{J}^{(3)} = \frac{e^2 c^2}{\omega^2} \left(\frac{B^{(0)}}{\left(m^2\omega^2 + e^2 B^{(0)2}\right)^{1/2}} \right) \underline{B}^{(3)} \quad -(8)$$

Kinetic Energy:

$$T = \frac{1}{2}\Omega J$$

$$= \frac{e^2 c^2}{2 m \omega^2} B^{(0)2} \quad -(9)$$

<u>Notes</u>

Ω = angular frequency of the electron

ω = angular frequency of the electromagnetic field

$$\gamma = \left(1 - \frac{u^2}{c^2}\right)^{-1/2}$$

v = velocity of electron, r = position of electron

$$\underline{r} = \underline{r}_0 + \frac{e}{m\gamma}\int \underline{A}\, dt$$

3)

Notes on Static Magnetic Field

The angular momentum is:

$$\underline{J} = \underline{r} \times \underline{p} = e\,\underline{r} \times \underline{A} \quad -(10)$$

from the minimal prescription. In the standard model:

$$\underline{B} = \underline{\nabla} \times \underline{A}, \quad \underline{A} = \frac{1}{2}\underline{B} \times \underline{r} \quad -(11)$$

so:

$$J = erA, \quad A = Br. \quad -(12)$$

The kinetic energy is:

$$T = \frac{e^2 A^2}{2m} \quad -(13)$$

so the angular velocity is:

$$\Omega = \frac{2T}{J} = \frac{eA}{rm} = \frac{eB}{m}. \quad -(14)$$

The $\underline{B}^{(3)}$ Field from Equation (5)

This is given by:

$$\underline{J}^{(3)} = \frac{e^2}{m\gamma} \int \underline{A}^{(1)} dt \times \underline{A}^{(2)} \quad -(15)$$

where:

$$\underline{A}^{(1)} = \frac{A^{(0)}}{\sqrt{2}} (\underline{i} - i\underline{j}) e^{i(\omega t - \kappa z)} \quad -(16)$$

$$\underline{A}^{(2)} = \frac{A^{(0)}}{\sqrt{2}} (\underline{i} + i\underline{j}) e^{-i(\omega t - \kappa z)} \quad -(17)$$

so:

$$\underline{J}^{(3)} = \frac{e^2 A^{(0)2}}{m\gamma\omega} \underline{e}^{(3)} \quad -(18)$$

We now use:

4)

$$A^{(0)} = \frac{c}{\omega} B^{(0)} \quad -(19)$$

and:
$$\underline{B}^{(3)} = B^{(0)} \underline{e}^{(3)} \quad -(20)$$

to find:
$$\boxed{\underline{J}^{(3)} = \frac{e^2 c^2}{\gamma m \omega^3} B^{(0)} \underline{B}^{(3)}} \quad -(21)$$

In the non-relativistic limit:

$$\gamma \to 1, \quad m\omega >> eB^{(0)} \quad -(22)$$

and eqn. (21) becomes eqn (8) from the HJ method

For a static magnetic field:

$$\boxed{J = er^2 B = e\Phi} \quad -(22)$$

where
$$\Phi = r^2 B = \text{magnetic flux.}$$

Therefore the quantum of flux is:

$$\Phi = \frac{\hbar}{e} \quad (\text{i.e. } J \to \hbar). \quad -(23)$$

From eq. (21):

$$\boxed{J = \frac{e^2 c^2}{\gamma m \omega^3} B^{(0)2} = \frac{\mu_0 e^2 c}{\gamma m \omega} \left(\frac{I}{\omega^2}\right)}$$

5)

Here:

$$\underline{I} = \frac{c}{\mu_0} B^{(0)2} \quad - (24)$$

is the electromagnetic field's power density (watts per square metre).

From a comparison of eqns. (22) and (23) it is seen that $\underline{B}^{(3)}$ and $\underline{B}$ are completely different.

Induced Magnetic Dipole Moment

1) Static Magnetic Field

This is calculated from the gyromagnetic ratio

$$\underline{m} = \frac{e}{2m} \underline{J} \quad - (25)$$

$$= \left(\frac{e^2 r^2}{2m} \right) \underline{B} = \frac{e^2}{2m} \Phi \underline{k} \quad - (26)$$

So:

$$\boxed{\chi = \frac{e^2 r^2}{2m}} \quad - (27)$$

is the susceptibility.

2) The $\underline{B}^{(3)}$ Field

$$\underline{m}^{(3)} = \left(\frac{e^3 c^2}{2\gamma m^2 \omega^3} \right) B^{(0)} \underline{B}^{(3)} \quad - (28)$$

6)

Here:

$$\boxed{\beta = \frac{e^3 c^2}{2\gamma m^2 \omega^3}} \quad \text{— (29)}$$

is the magnetic hypermagnetizability.

Conclusions

1) A static magnetic field interacts with one electron through the static magnetic susceptibility (27) to induce a magnetic dipole moment.

2) The $\underline{B}^{(3)}$ field interacts with one electron through the frequency dependent hypermagnetizability (29) to induce a magnetic dipole moment.