1)

8i(7): Most General Relation between ω and Ω.

We have:

$$\Omega = \frac{(\gamma m v + eA)c^2}{\left(r + \frac{eA}{\gamma m \omega}\right)\left(mc^2(1+\gamma) + e\phi\right)} \quad -(1)$$

So:

$$\Omega\left(r + \frac{eA}{\gamma m \omega}\right) = x \quad -(2)$$

where:

$$x = \frac{(\gamma m v + eA)c^2}{mc^2(1+\gamma) + e\phi} \quad -(3)$$

So:

$$\boxed{\omega = \frac{eA\Omega}{\gamma m(x - \Omega r)}} \quad -(4)$$

In the limit:

$$v = 0 \quad -(5)$$

(for an initially stationary electron):

$$x = \frac{eAc^2}{mc^2(1+\gamma) + e\phi} \quad -(6)$$

If we now consider:

$$mc^2(1+\gamma) \gg e\phi \quad -(7)$$

and

$$\gamma \rightarrow 1 \quad -(8)$$

2)

$$x \rightarrow \frac{eA}{2m} \quad -(9)$$

So:

$$\omega \rightarrow \frac{eA\Omega}{m\left(\frac{eA}{2m} - \Omega r\right)} \quad -(10)$$

Finally, if the electron is initially at:

$$r = 0 \quad -(11)$$

then

$$\boxed{\omega \rightarrow 2\Omega} \quad -(12)$$