

81(5): The Faraday Effect, Rotation of Plane of Polarization by a Static Magnetic Field of Electromagnetic Radiation Interacting with One Electron.

In note 81(4) it was shown that the relation between the angular frequency of the e/m radiation,  $\omega$ , and the angular frequency of the electron,  $\Omega$  is:

$$\omega = \frac{1}{\gamma} \left( 1 + \gamma + \frac{e\phi}{mc^2} \right) \Omega \quad - (1)$$

where: 
$$\gamma = \left( 1 - \frac{u^2}{c^2} \right)^{-1/2} \quad - (2)$$

Here  $\phi$  is the scalar potential and  $u$  is the velocity of one frame with respect to another in a Lorentz transformation. Eq. (1) has assumed that the radiation is circularly polarized:

$$\underline{A} = A^{(0)} \left( \underline{i} \cos \omega t + \underline{j} \sin \omega t \right) \quad - (3)$$

in general.

The Faraday effect is the rotation of the plane of e/m radiation with a static magnetic field. This is equivalent to changing circular to elliptical polarisation. The phase of the electromagnetic field is:

$$\phi = \omega t - \kappa z$$

$$= \omega \left( t - \frac{z}{c} \right) \quad - (4)$$

Using eq. (1):

$$\phi = \frac{1}{\gamma} \left( 1 + \gamma + \frac{e\phi}{mc^2} \right) \left( t - \frac{z}{c} \right) \Omega \quad - (5)$$

The effect of the extra magnetic field is to change the angular frequency  $\Omega$  of the electron, so it changes  $\phi$ . As shown in previous notes this leads to a change from circular to elliptical polarization.

### Details of the Effect of a Static Magnetic Field on the Motion of an Electron.

As per previous notes the extra angular momentum imparted to the electron by the static magnetic field is:

$$\Omega = \frac{e}{2m} B_{\text{static}} \quad - (6)$$

so the magnetic field changes  $\Omega$  given to:

$$\phi' = \frac{1}{\gamma} \left( 1 + \gamma + \frac{e\phi}{mc^2} \right) \left( t - \frac{z}{c} \right) \left( 1 + \frac{e}{2m} B_{\text{static}} \right)$$

which is the electron Faraday effect.