1)

Notes 81(1): Inverse Faraday Effect and Faraday Effect from the $B^{(3)}$ Spin Field.

These notes define a new method of directly solving the Einstein equation by expressing the relativistic kinetic energy as:

$$T = E - E_0 = \frac{p^2 c^2}{E + E_0} \quad -(1)$$

where the relativistic energy is:

$$E = \gamma m c^2, \quad -(2)$$

the rest energy is:

$$E_0 = m c^2, \quad -(3)$$

the relativistic momentum is:

$$\underline{p} = \gamma m \underline{v}, \quad -(4)$$

and:

$$\gamma = \left(1 - v^2/c^2\right)^{-1/2}. \quad -(5)$$

One electron is considered to interact with an electromagnetic field through the minimal prescription:

$$p^\mu \rightarrow p^\mu + e A^\mu. \quad -(6)$$

where

$$p^\mu = \left(\frac{E}{c}, \underline{p}\right), \quad -(7)$$

$$A^\mu = \left(\frac{\phi}{c}, \underline{A}\right). \quad -(8)$$

Thus:

$$E = \gamma m c^2 \rightarrow \gamma m c^2 + e\phi, \quad -(9)$$

$$p = \gamma m \underline{v} \rightarrow \gamma m \underline{v} + e\underline{A}. \quad -(10)$$

So the relativistic kinetic energy describing the interaction is:

$$\boxed{T = \frac{\gamma^2 m^2 v^2 c^2 + e^2 A^2 c^2}{\gamma m c^2 + e\phi + m c^2}} \quad -(11)$$

2) The interaction part is:

$$T_{int} = \frac{e^2 A^2 c^2}{\gamma mc^2 + mc^2 + e\phi} \quad -(12)$$

where $$|A| = A^{(0)} = \frac{cB^{(0)}}{\omega}. \quad -(13)$$

Thus: $$T_{int} = \frac{e^2 B^{(0)2} c^4}{\omega^2 (mc^2(1+\gamma) + e\phi)}. \quad -(14)$$

Non-Relativistic Limit

This is the limit:

$$mc^2(1+\gamma) >> e\phi, \quad -(15)$$

$$\gamma \rightarrow 1, \quad -(16)$$

so: $$T_{int} \rightarrow \left(\frac{e^2 c^2}{2m\omega^2}\right) B^{(0)2}. \quad -(17)$$

In this limit:

$$T_{int} = \frac{1}{2}\omega J \quad -(18)$$

where J is the magnitude of the orbital angular momentum of the electron in the e/m field. So

$$\boxed{J = \frac{e^2 c^2}{m\omega^3} B^{(0)2}} \quad -(19)$$

as from the Hamilton-Jacobi method.

3) The non-relativistic limit is the condition for the Prague experiments, to detect the Larmor radius and the RFR effect. The induced magnetic dipole moment from eq. (19) is:

$$\underline{m}^{(3)} = \left(\frac{e^3 c^2}{2 m^2 \omega^3} \right) B^{(0)} \underline{B}^{(3)} . \quad -(20)$$

The RFR effect is given directly from eq. (17) as:

$$\hbar \omega_{res} = \frac{e^2 c^2 B^{(0)2}}{2 m \omega^2} \left(1 - (-1) \right) \quad -(21)$$

using the SU(2) basis. This gives:

$$\boxed{f_{res} = \left(\frac{e^2 \mu_0 c}{2 \pi \hbar m} \right) \frac{I}{\omega^2}} . \quad -(22)$$

So the RFR effect is very fundamental, it can be derived directly from the minimal prescription. Relativistic corrections to RFR are given by eq. (14). In this non-relativistic limit the Larmor radius may be worked out from:

$$J = pr = \frac{e^2 c^2}{m \omega^3} B^{(0)2} , \quad -(23)$$

$$p = eA = \frac{ec}{\omega} B^{(0)} \quad -(24)$$

so:

$$\boxed{r_L = \frac{p}{m\omega} = \frac{ecB^{(0)}}{m\omega^2} = \frac{ec}{m\omega^2} \left(\frac{\mu_0 I}{c} \right)^{1/2}} \quad -(25)$$

using $I = \frac{c B^{(0)2}}{\mu_0}$. $-(26)$

CROSS-CHECKED

1) Notes 81(2), Third Cross Check of Self-Consistency

The relativistic momentum is defined by:

$$\underline{p} = m\frac{d\underline{r}}{d\tau} = m\frac{d\underline{r}}{dt}\frac{dt}{d\tau} \quad -(1) \qquad \underline{r} = \frac{1}{\gamma}\int \underline{p}\,dt \quad -(1a)$$

where:

$$\frac{dt}{d\tau} = \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(2)$$

so

$$\underline{r} = \frac{1}{m}\int \underline{p}\,d\tau = \frac{1}{\gamma m}\int \underline{p}\,dt \quad -(3) \qquad = \frac{1}{\gamma}\int \underline{v}\,dt$$

relativistic

- If we now consider the interaction momentum:

$$\underline{p} = e\underline{A} \quad -(4)$$

then

$$\underline{r} = \frac{e}{m}\frac{1}{\gamma}\int \underline{A}\,dt \quad -(5)$$

Now assume the plane wave:

$$\underline{A} = A^{(0)}\left(\underline{i}\cos\phi + \underline{j}\sin\phi\right) \quad -(6)$$

then:

$$p_x = eA^{(0)}\cos\phi, \quad p_y = eA^{(0)}\sin\phi$$

$$r_x = \frac{eA^{(0)}}{\gamma\omega}\sin\phi, \quad r_y = -\frac{eA^{(0)}}{\gamma\omega}\cos\phi$$

So:

$$p = \left(p_x^2 + p_y^2\right)^{1/2} = eA^{(0)} \quad -(7)$$

$$\boxed{r = \left(r_x^2 + r_y^2\right)^{1/2} = \frac{1}{m\omega\gamma}eA^{(0)}} \quad -(8)$$

$$J = rp = \frac{e^2A^{(0)2}}{\gamma m\omega} = \frac{e^2c^2B^{(0)2}}{m\omega^3\gamma} \quad -(9)$$

LARMOR RADIUS

DOUBLE CROSS CHECKED IN THREE WAYS

2)

More generally:

$$\underline{p} = \gamma m \underline{v} + e\underline{A} \quad -(10)$$

$$\underline{r} = \frac{1}{m}\left(\int m\underline{v}\,dt + \frac{1}{\gamma}\int e\underline{A}\,dt\right) \quad -(11)$$

$$\underline{r} = \int \underline{v}\,dt + \frac{e}{m\gamma}\int \underline{A}\,dt \quad -(12)$$

and

$$\underline{J} = \underline{r} \times \underline{p}. \quad -(13)$$

The relativistic velocity is:

$$\underline{v} = \underline{v} + \frac{e}{m\gamma}\underline{A} \quad -(14)$$

and the angular velocity of the electron in the e/m field is:

$$\Omega = \left|\frac{\underline{v}}{|\underline{r}|}\right|, \quad -(15)$$

while its total kinetic energy is:

$$T = \frac{\gamma^2 m^2 v^2 c^2 + e^2 c^2 A^2}{mc^2(1+\gamma) + e\phi}. \quad -(16)$$

<u>Hyper-Relativistic Limit</u>

This is the limit:

$$e\phi \gg mc^2(1+\gamma) \quad -(17)$$

so

$$T_{int} = ec^2\frac{A^2}{\phi} \quad -(18)$$

If we assume that:

$$|A| = A^{(0)}, \quad |\phi| = cA^{(0)} \quad -(19)$$

3) Then:

$$T_{int} = ecA^{(0)} \quad -(20)$$

using the de Broglie wave-particle duality

$$eA^{(0)} = \hbar\kappa \quad -(21)$$

then:

$$T_{int} = \hbar c\kappa = \hbar\omega \quad -(22)$$

which is the quantum of energy of a photon.

- If we define:

$$g = \frac{e}{\hbar} = \frac{\kappa}{A^{(0)}} \quad -(23)$$

then:

$$\boxed{\underline{B}^{(3)*} = -ig\,\underline{A}^{(1)} \times \underline{A}^{(2)}} \quad -(24)$$

In this limit, the definition of g defines the ECE spin field in free space. The $eA^{(0)}$ interaction momentum is equal to the photon momentum $\hbar\kappa$. A photon of e/m radiation has transferred all its momentum and energy to the electron. This is therefore an elastic collision between photon and electron. In the opposite (inelastic) limit:

$$\underline{m}^{(3)} = \frac{e}{2m}\underline{J}^{(3)} = -\frac{ie^3}{2m^2\omega}\underline{A}^{(1)} \times A^{(2)}$$

$$\boxed{\underline{B}^{(3)*} = -i\left(\frac{\mu_0}{V}\frac{e^3}{2m^2\omega}\right)\underline{A}^{(1)} \times \underline{A}^{(2)}} \quad -(25)$$

4) Calculation of Angular Momentum and Energy

Assuming that:

$$\underline{v} = v^{(0)} \left(\underline{i} \cos\phi + \underline{j} \sin\phi \right) \quad -(26)$$

the relativistic angular momentum can be calculated as follows from eq. (13). We have:

$$r_x = \frac{1}{\omega} \left(v_x + \frac{eA^{(0)}}{m\gamma} \right) \sin\phi \quad -(27)$$

$$r_y = -\frac{1}{\omega} \left(v_y + \frac{eA^{(0)}}{m\gamma} \right) \cos\phi \quad -(28)$$

$$p_x = \left(\gamma m v_x + eA^{(0)} \right) \cos\phi \quad -(29)$$

$$p_y = \left(\gamma m v_y + eA^{(0)} \right) \sin\phi \quad -(30)$$

$$J_z = r_x p_y - r_y p_x, \quad -(31)$$

$$J_z = \frac{m v^{(0)2}}{\omega} + \frac{e v^{(0)} A^{(0)}}{\gamma \omega} + \frac{e^2 A^{(0)2}}{\gamma m \omega} \quad -(32)$$

So the rotational kinetic energy is:

$$T = \frac{1}{2} \left(m v^{(0)2} + \frac{e v^{(0)}}{\gamma} A^{(0)} + \frac{e^2 A^{(0)2}}{\gamma m} \right) \quad -(33)$$

$$= \frac{1}{2} \omega J_z$$

Comparison with Hamilton - Jacobi Equation

In the non-relativistic limit the same second order interaction term is recovered:

$$T = \frac{1}{2} \frac{e^2 A^{(0)2}}{m} = \frac{e^2 c^2}{2 m \omega^2} B^{(0)2} \quad -(34)$$

CHECKED IN FOUR WAYS

The first order term is:

$$T_1 = \frac{1}{2} e v^{(0)} \left(1 - \frac{u^2}{c^2}\right)^{1/2} A^{(0)} \quad -(35)$$

In the limit:

$$v^{(0)} \to c, \quad u \to 0 \quad -(36)$$

$$T_1 \to \frac{1}{2} e c A^{(0)} = \frac{e c^2}{2 \omega} B^{(0)} \quad -(37)$$

which is the ultra-relativistic limit of the Hamilton-Jacobi method. In this limit the velocity of the orbiting electron approaches the speed of light, at which no frame can move faster, so $u \to 0$. So this method gives the same results as the Hamilton-Jacobi method. The latter therefore applies to an electron orbiting according to eq. (26).

Finally the angular velocity of the electron in the e/m field is given by:

5)

$$\Omega = \frac{d\theta}{dt} = \frac{v}{r_L} \quad -(38)$$

where:

$$r_L = \left(r_X^2 + r_Y^2 \right)^{1/2} \quad -(39)$$

$$v = \left(v_X^2 + v_Y^2 \right)^{1/2} \quad -(40)$$

The Larmor radius is:

$$r_L = \left(\left(\frac{v^{(0)}}{\omega} \right)^2 + \left(\frac{eA^{(0)}}{\gamma m \omega} \right)^2 \right)^{1/2} \quad -(41)$$

and the net orbital velocity is:

$$v = \left(\left(\gamma v^{(0)} \right)^2 + \left(\frac{eA^{(0)}}{m} \right)^2 \right)^{1/2} \quad -(42)$$

In the non-relativistic limit:

$$\gamma \rightarrow 1 \quad -(43)$$

and:

$$\Omega \rightarrow \left(\frac{\left(v^{(0)} \right)^2 + \left(\frac{eA^{(0)}}{m} \right)^2}{\left(\left(\frac{v^{(0)}}{\omega} \right)^2 + \left(\frac{eA^{(0)}}{m\omega} \right)^2 \right)} \right)^{1/2} \quad -(43)$$

$$= \omega$$

which is the result of paper 78.