79(3): The Faraday Effect and the Inverse Faraday Effect for One Electron.

The Faraday effect is the rotation of the plane of polarization of light by a magnetic field applied to any material. The inverse Faraday effect is the magnetization of any material by a circularly polarized electromagnetic field of any frequency. In ECE physics both effects are due to the spin torsion of spacetime. It is shown in this note that both effects are described by the same equation for one electron. This equation is:

$$\underline{J}^{(3)} = \frac{e^2 c^3 B^{(0)} \underline{B}^{(3)}}{\gamma \omega^2} \quad -(1)$$

from paper 78. In the limit:

$$\omega \ll \frac{e}{m} B^{(0)} \quad -(2)$$

eq. (1) becomes:

$$\boxed{\underline{J}^{(3)} = \frac{e c^2}{\omega^2} \underline{B}^{(3)}} \quad -(3)$$

2) In the inverse Faraday effect eq. (3) gives the magnetization due to the $\underline{B}^{(3)}$ spin field. In the Faraday effect $\underline{B}^{(3)}$ is the applied static magnetic field. The quantity:

$$\chi_0 = \frac{ec^2}{\omega^2} \quad -(4)$$

is the magnetizability of one electron. It is more generally a molecular property tensor, and is the same for both effects. This is why the inverse Faraday effect is so called. For other materials χ is more complicated but the law is the same.

In the Faraday effect, the plane polarized light's electric field is:

$$\underline{E}^{(1)} = \underline{E}_R^{(1)} + \underline{E}_L^{(1)}$$

$$= \frac{2}{\sqrt{2}}\, \underline{i} \exp\left(i(\omega t - \kappa z)\right) \quad -(5)$$

The electromagnetic phase is:

3)
$$\phi = \omega t - \kappa Z = S/\hbar = J/\hbar \quad -(6)$$

where S is action, which has the same units as angular momentum. So the extra angular momentum of eq. (3) results in:

$$J \rightarrow J + J^{(3)} \quad -(7)$$

and
$$\phi \rightarrow \phi + \phi^{(3)} = \phi_1. \quad -(8)$$

If the electric field is initially the plane wave:

$$\underline{E} = \frac{E^{(0)}}{\sqrt{2}} \left(\underline{i} \cos\phi + \underline{j} \sin\phi \right) \quad -(9)$$

it becomes:

$$\underline{E}_1 = \frac{E^{(0)}}{\sqrt{2}} \left(\underline{i} \cos\phi_1 + \underline{j} \sin\phi_1 \right) \quad -(10)$$

$$= \frac{E^{(0)}}{\sqrt{2}} \left(a\underline{i} \cos\phi + b\underline{j} \sin\phi \right) \quad -(11)$$

where $\cos\phi_1 = a\cos\phi$, $\sin\phi_1 = b\sin\phi$ $\quad -(12)$

e.g. if $\phi = 45^\circ$, $\phi_1 = 60^\circ$ $\quad -(13)$

4) Then:

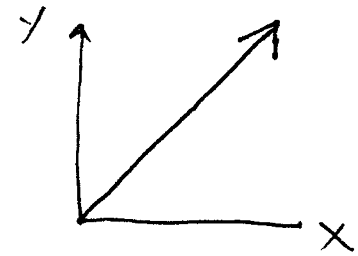
$$a = 1.414, \quad b = 0.816. \qquad (14)$$

The wave (9) is circularly polarized but the wave (11) is elliptically polarized. The plane of polarization of $\underline{E}$ if $\phi = 45°$ is, from eq. (9):

$$\underline{E} = \frac{E^{(0)}}{\sqrt{2}} \left(0.707 \left(\underline{i} + \underline{j} \right) \right) \qquad (15)$$

$$= \frac{0.707\, E^{(0)}}{\sqrt{2}} \left(\underline{i} + \underline{j} \right)$$

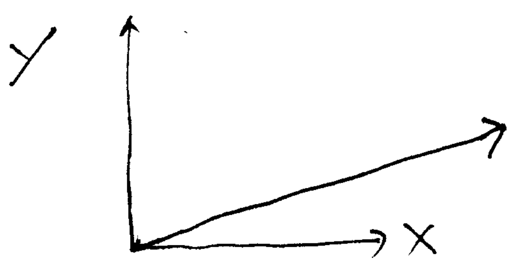
which is 45° to $\underline{i}$ and $\underline{j}$:



If we take eq. (11) for $\phi = 45°$:

$$\underline{E} = \frac{0.707\, E^{(0)}}{\sqrt{2}} \left(a\,\underline{i} + b\,\underline{j} \right) \qquad (16)$$

$$= \frac{0.707\, E^{(0)}}{\sqrt{2}} \left(1.414\,\underline{i} + 0.816\,\underline{j} \right)$$



5)

This means that the angular momentum of eq. (3) has changed the plane or direction of polarization of the light clockwise towards the X axis. This is the Faraday effect.

Conclusion

Both effects are described for one electron by eq. (3). In the IFE, $\underline{B}^{(3)}$ is the spin field of ECE physics. In the FE it is a static magnetic field. Both effects must by definition be described by the same magnetizability χ, so it follows that the $\underline{B}^{(3)}$ field exists in electromagnetic radiation. In ECE physics both $\underline{B}^{(3)}$ and a static magnetic field are due to the spin torsion. They are examples of:

$$F = d \wedge A + \omega \wedge A \quad -(17)$$