78(4): Effect of Gravitation on RFR: Field Equations

In general the field equations must be set up in SU(2) representation space to describe the interaction of a fermion with an electromagnetic field. The effect of gravitation must then be incorporated through a non zero homogeneous current. The RFR term is obtained straightforwardly from the definition of the magnetic field in ECE ~~theory~~:

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b \quad -(1)$$

When there is no gravitation:

$$\underline{B}^{(1)*} = \underline{\nabla} \times \underline{A}^{(1)*} - ig\, \underline{A}^{(3)} \times \underline{A}^{(1)} \quad -(2)$$

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The $\underline{B}^{(3)}$ component is:

$$\underline{B}^{(3)*} = -ig\, \underline{A}^{(1)} \times \underline{A}^{(2)} \quad -(3)$$

In the SU(2) basis:

$$\underline{\sigma} \cdot \underline{B}^{(3)*} = -ig\, \underline{\sigma} \cdot \underline{A}^{(1)} \times \underline{A}^{(2)} \quad -(4)$$

where $\underline{\sigma}$ is the Pauli ~~matrix~~ vector. The basis set in SU(2) is made up of the three Pauli matrices. The right hand side in eq. (4) is proportional to the RFR term.

Finally it can be shown that if the gravitational field affects the electromagnetic field, there must be an effect on RFR (e.g. paper 69). This is because eq. (1) does not reduce to eq. (2) when there is gravitation present. So the RFR term is changed to:

$$\underline{\sigma} \cdot \underline{B}^a = \underline{\sigma} \cdot \left(\underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b \right) \quad - (5)$$

In this case the spin connection is defined by both curving and spinning. In eq. (2) it is defined only by spinning.

<u>Tetrad Definition in the SU(2) and SU(3) Bases.</u>

The SU(2) basis is defined by the commutation of Pauli matrices:

$$\left[\frac{\sigma_x}{2}, \frac{\sigma_y}{2} \right] = i \epsilon_{xyz} \frac{\sigma_z}{2} \quad - (6)$$

The SU(3) basis is defined by:

$$\left[\frac{\lambda_a}{2}, \frac{\lambda_b}{2} \right] = i f_{abc} \frac{\lambda_c}{2} \quad - (7)$$

where c sums from 1 to 8, and where:

$$f_{123} = 1,$$

$$f_{147} = -f_{156} = f_{246} = f_{257} = f_{345} = -f_{367} = \frac{1}{2}$$

$$f_{458} = f_{678} = \sqrt{3}/2$$

3) In SU(3) there are eight gauge potentials. These are defined (Ryder, 2nd. ed., p. 124) by:

$$A_\mu = A_\mu^a \frac{\lambda_a}{2} = \frac{1}{2}\begin{bmatrix} A_\mu^3 + \frac{1}{\sqrt{3}} A_\mu^8 & A_\mu^1 - i A_\mu^2 & A_\mu^4 - i A_\mu^5 \\ A_\mu^1 + i A_\mu^2 & -A_\mu^3 + \frac{1}{\sqrt{3}} A_\mu^8 & A_\mu^6 - i A_\mu^7 \\ A_\mu^4 + i A_\mu^5 & A_\mu^6 + i A_\mu^7 & -\frac{2}{\sqrt{3}} A_\mu^8 \end{bmatrix} \quad -(8)$$

The field is then defined by:

$$G_{\mu\nu}^1 = \partial_\mu A_\nu^1 - \partial_\nu A_\mu^1 + g f_{1bc} A_\mu^b A_\nu^c$$

$$= \partial_\mu A_\nu^1 - \partial_\nu A_\mu^1 + g \Big(A_\mu^2 A_\nu^3 - A_\mu^3 A_\nu^2 + \frac{1}{2}\left(A_\mu^4 A_\nu^7 - A_\mu^7 A_\nu^4\right) - \frac{1}{2}\left(A_\mu^5 A_\nu^6 - A_\mu^6 A_\nu^5\right)\Big) \quad -(9)$$

It is seen that eqns. (8) and (9) are examples of Cartan geometry with a particular type of spin connection, producing ~~three~~ commutators in eq. (9). Thus eq. (9) is an example of the general definition:

4)

$$G^a_{\mu\nu} = d \wedge A^a_\mu + \omega^a_{\mu b} \wedge A^b_\nu \quad -(10)$$

where the spin connection involves the SU(3) structure factor f_{ibc}. Eq. (8) is an example of the textbook definition:

$$A_\mu = A^a_\mu \frac{\lambda_a}{2} \quad -(11)$$

where:

$$A^a_\mu = A^{(0)} q^a_\mu . \quad -(12)$$

Similarly, in the SU(2) basis:

$$A_\mu = A^a_\mu \frac{\sigma_a}{2} \quad -(13)$$

and:

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g \epsilon_{abc} A^b_\mu A^c_\nu . \quad -(14)$$

The vector field in eqs (11) and (13) is A_μ, the components are A^a_μ and the basis elements of SU(3) and SU(2) are $\lambda_a / 2$ and $\sigma_a / 2$ respectively. In ECE this is <u>general relativity</u> and not gauge theory.