INVARIANCE OF THE TETRAD POSTULATE AS A FUNDAMENTAL PRINCIPLE OF UNIFIED FIELD THEORY

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ABSTRACT

The tetrad postulate is proven to be an invariant of the general coordinate transformation in Cartan Riemann geometry. In Einstein Cartan Evans (ECE) unified field theory this inference implies that the ECE Lemma and wave equation are covariant in such a way that only the tetrad eigenfunction changes upon general coordinate transformation. The phase of the eigenfunction therefore changes by the addition of a dimensionless factor that is independent of distance and time. This factor is the origin of non-local effects in general relativity. There is no contradiction in concept between the local theory of general relativity and the non-local nature of this factor, because the latter is obtained by the fundamental principle of relativity, coordinate covariance. The invariance of the tetrad postulate is adopted as a fundamental principle ro replace the gauge principle in ECE theory. A discussion is given of several experimental effects which can be described by this non-local factor, non-local in the sense that it does not depend on time and distance.

Keywords: Invariance of the tetrad postulate, ECE unified field theory, non-locality, coordinate transformation, general relativity.

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1. INTRODUCTION

It is well known that the tetrad postulate {1} is the link between Cartan geometry, inferred in 1922, and Riemann geometry, inferred in the early nineteenth century. Without the tetrad postulate Cartan cannot be translated into Riemann as required for self consistency {2-10}. The tetrad postulate is proven rigorously in refs. (1) to (10) and is a consequence of the fundamental definition of the second rank mixed index tensor known as the tetrad:

$$V^a = q^a_\mu V^\mu. \quad -(1)$$

Here V^a is a vector in Minkowski space-time and V^μ a vector in the base manifold. The tetrad is q^a_μ, and has two indices, a and μ. It is therefore a second rank tensor whose covariant derivative is defined {1-10} to be:

$$D_\mu q^a_\nu := \partial_\mu q^a_\nu + \omega^a_{\mu b} q^b_\nu - \Gamma^\lambda_{\mu\nu} q^a_\lambda \quad -(2)$$

where $\Gamma^\lambda_{\mu\nu}$ is the gamma connection of Riemann geometry and $\omega^a_{\mu b}$ the spin connection of Cartan geometry. The well known Christoffel connection is a special case of the gamma connection when the lower two indices are symmetric:

$$\Gamma^\lambda_{\mu\nu} = \Gamma^\lambda_{\nu\mu} \quad -(3)$$

and when the torsion tensor is zero:

$$T^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu} = 0. \quad -(4)$$

The "tetrad postulate" (1-10} is a direct consequence of the fact that a vector field is independent of the coordinate system in which it is expressed. "Tetrad postulate" is therefore something of a misnomer because it is in fact a fundamental property:

$$D_\mu q^a_{\ \nu} = 0 \quad -(5)$$

independent of all connections {1-10}.

In Section 2 it is shown that Eq. (5) is invariant under the general coordinate transformation, in other words the tetrad postulate is true in any frame of reference and is a fundamental property of general relativity. This property is adopted in Section 2 as a fundamental principle of Einstein Cartan Evans (ECE) field theory {2-10} and replaces the gauge principle by Ockham's Razor. In consequence of this invariance principle the ECE Lemma and wave equation are shown in Section 2 to be covariant under the general coordinate transformation as required by general relativity. Furthermore the Lemma and wave equation are covariant in such a way that only the tetrad eigenfunction changes under the general coordinate transformation. The eigenoperator and eigenvalues do not change.

In Section 3 this property of the Lemma and wave equation is interpreted in terms of a dimensionless factor added to the phase of the eigenfunction under general coordinate transformation. For example, if the original phase is considered to be:

$$\begin{aligned}\phi &= \kappa^\mu x_\mu \\ &= \omega t - \kappa Z\end{aligned} \quad -(6)$$

the phase after general coordinate transformation is

$$\phi' = \kappa^\mu x_\mu + \alpha \quad -(7)$$

where α is independent of x^μ, i.e. independent of time and distance. Here κ^μ is the wave-number four vector:

$$\kappa^\mu = \left(\frac{\omega}{c}, \underline{\kappa} \right) \quad -(8)$$

and x^{μ} the coordinate four-vector:

$$x_{\mu} = (ct, -\underset{\sim}{r}) \quad -(9)$$

where ω is angular frequency, t is time, Z is a coordinate for propagation along the Z axis and κ the wave-number magnitude. Therefore α is "non-local" in the sense that it is independent of x^{μ}. There is no contradiction with the local theory of relativity, because α is generated by a coordinate transformation - one of the fundamental principles of general relativity itself. Non-locality is defined therefore as the distance and time independence of α, a feature introduced through coordinate transformation of the ECE wave equation, the fundamental wave equation of physics.

In Section 4 it is shown that α is responsible for several experimental effects and a discussion is given of the shortcomings of gauge theory. The invariance of the tetrad postulate is adopted in preference to the gauge principle in generally covariant unified field theory.

2. FRAME INVARIANCE OF THE TETRAD POSTULATE

The general coordinate transformation in Riemann geometry for a tensor of any rank is {1-10}:

$$T^{\mu_1' \ldots \mu_k'}_{\nu_1' \ldots \nu_l'} = \left(\frac{\partial x^{\mu_1'}}{\partial x^{\mu_1}} \ldots \frac{\partial x^{\mu_k'}}{\partial x^{\mu_k}} \right) \left(\frac{\partial x^{\nu_1}}{\partial x^{\nu_1'}} \ldots \frac{\partial x^{\nu_l}}{\partial x^{\nu_l'}} \right) T^{\mu_1 \ldots \mu_k}_{\nu_1 \ldots \nu_l} \quad -(10)$$

and the most basic property of general relativity is that the tensor transform as in Eq. (10). Otherwise it is not a tensor. For this reason the gamma connection is not a tensor {1} as is well known. A tensor in one frame of reference must be a tensor in any other frame of reference, moving arbitrarily with respect to the original frame. The Christoffel connection, for example, transforms {1} as:

$$\Gamma^{\nu'}_{\mu'\lambda'} = \frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial x^{\lambda}}{\partial x^{\lambda'}} \frac{\partial x^{\nu'}}{\partial x^{\nu}} \Gamma^{\mu}_{\nu\lambda} - \frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial x^{\lambda}}{\partial x^{\lambda'}} \frac{\partial^2 x^{\nu'}}{\partial x^{\mu} \partial x^{\lambda}} \quad - (11)$$

and because of the second term on the right hand side, does not transform as a tensor.

The spin connection on the other hand {1} transforms as a vector under the general coordinate transformation:

$$\omega^{a}_{\mu' b} = \left(\frac{\partial x^{\mu}}{\partial x^{\mu'}} \right) \omega^{a}_{\mu b} \quad - (12)$$

and the tetrad transforms as:

$$q^{a}_{\mu'} = \left(\frac{\partial x^{\mu}}{\partial x^{\mu'}} \right) q^{a}_{\mu} . \quad - (13)$$

In these transformations the a and b indices are kept constant, but the μ index is transformed. For pure rotational motion {2-10}:

$$\omega^{a}_{\mu b} = - \frac{\kappa}{2} \epsilon^{a}{}_{bc} q^{c}_{\mu} \quad - (14)$$

so, self-consistently:

$$\omega^{a}_{\mu' b} = - \frac{\kappa}{2} \epsilon^{a}{}_{bc} q^{c}_{\mu'} . \quad - (15)$$

This property means that Eq. (14) is generally covariant as required by general relativity. Eq. (14) is therefore a true tensorial equation and is a key equation in the derivation {2-10} of the ECE spin field. This equation can be constructed only in Cartan geometry, otherwise, as we have seen, the connection does not transform as a tensor. General covariance in the spin connection and tetrad must be defined as in Eqs. (12) and (13) for self-consistently.

If the Lorentz transformation {1} of special relativity is applied:

$$\omega^{a'}_{\mu b'} = \Lambda^{a}_{a'} \Lambda^{b}_{b'} \omega^{a}_{\mu b} - \Lambda^{c}_{b'} \partial_\mu \Lambda^{a'}_{c} \quad -(16)$$

but:

$$q^{a'}_{\mu} = \Lambda^{a'}_{a} q^{a}_{\mu} \quad -(17)$$

and so Eq. (14) is not covariant under the Lorentz transformation. This finding is consistent with the fact that Eq. (14) is not an equation of translation, it is an equation of rotation. The Lorentz transform applies only to one frame translating at a constant velocity with respect to another (the limit of special relativity). This is the case for the Maxwell Heaviside (MH) field theory - a theory only of special relativity. Applying the Lorentz transform to MH theory is internally inconsistent therefore because MH theory deals with rotational motion as is well known. ECE theory is therefore the correct way to describe electrodynamics because ECE is correctly and generally covariant, not just Lorentz covariant. We have just illustrated this with respect to Eq. (14). Every equation of Cartan geometry is generally covariant, so every equation of ECE theory is generally covariant.

The frame invariance of the tetrad postulate is the property:

$$D_\mu q^{a}_{\nu} = \left(D_\mu q^{a}_{\nu}\right)' = 0 \quad -(18)$$

under the general coordinate transformation. This is proven as follows, but firstly some basic definitions are given. The partial derivative transforms generally as {1}:

$$\partial_{\mu'} = \frac{\partial x^{\mu}}{\partial x^{\mu'}} \partial_\mu . \quad -(19)$$

A vector in the base manifold transforms generally as:

$$V^{\mu'} = \left(\frac{\partial x^{\mu'}}{\partial x^{\mu}}\right) V^{\mu} \quad -(20)$$

The Lorentz transform is the special case:

$$x^{\mu'} = \Lambda^{\mu'}{}_{\mu} x^{\mu}. \quad -(21)$$

The symmetric metric transforms generally as a second rank tensor:

$$g_{\mu'\nu'} = \left(\frac{\partial x^{\mu}}{\partial x^{\mu'}}\right)\left(\frac{\partial x^{\nu}}{\partial x^{\nu'}}\right) g_{\mu\nu}. \quad -(22)$$

The covariant derivative of Riemann geometry is defined as

$$D_{\mu} V^{\nu} = \partial_{\mu} V^{\nu} + \Gamma^{\nu}_{\mu\lambda} V^{\lambda} \quad -(23)$$

and is DEFINED as a tensor, so that:

$$D_{\mu'} V^{\nu'} := \left(\frac{\partial x^{\mu}}{\partial x^{\mu'}}\right)\left(\frac{\partial x^{\nu'}}{\partial x^{\nu}}\right) D_{\mu} V^{\nu}. \quad -(24)$$

This definition means that the Christoffel connection cannot be a tensor as we have argued already. The general mixed index tensor transforms as:

$$T^{a'\mu'}{}_{b'\nu'} = \Lambda^{a'}{}_{a} \frac{\partial x^{\mu'}}{\partial x^{\mu}} \Lambda_{b'}{}^{b} \frac{\partial x^{\nu}}{\partial x^{\nu'}} T^{a\mu}{}_{b\nu} \quad -(25)$$

if both the Latin and Greek indices are transformed. However, the general coordinate transformation of the spin connection and tetrad are DEFINED {1} by Eqs. (12) and (13) as we have argued. Therefore the tetrad postulate transforms generally as:

$$D_{\nu'} q^a_{\mu'} = \left(\frac{\partial x^\nu}{\partial x^{\nu'}}\right)\left(\frac{\partial x^\mu}{\partial x^{\mu'}}\right) D_\nu q^a_\mu \quad -(26)$$

$$= 0$$

because:

$$D_\nu q^a_\mu = 0 \quad -(27)$$

Q.E.D.

Therefore the tetrad postulate is an invariant of Cartan Riemann geometry under the general coordinate transformation. We adopt this property as a fundamental principle to replace the gauge principle.

A direct consequence of this principle is that the ECE Lemma transforms as:

$$\Box q^a_\mu = R q^a_\mu \quad -(28)$$

$$\downarrow$$

$$\Box' q^a_{\mu'} = R' q^a_{\nu'} \quad -(29)$$

meaning that the Lemma is generally covariant as required. The transformation in Eq. (29) simplifies as follows. It is known that R is a scalar curvature {2-10}, so that:

$$R = R'. \quad -(30)$$

It is defined by:

$$R = -kT \quad -(31)$$

where k is Einstein's constant and where T is another scalar quantity, the index reduced canonical energy momentum density defined by:

$$T = g^{\mu\nu} T_{\mu\nu} \quad -(32)$$

where $T_{\mu\nu}$ is the canonical energy-momentum tensor. Furthermore the d'Alembertian is also frame invariant:

$$\square = \partial^{\mu}\partial_{\mu} = \square' = \partial^{\mu'}\partial_{\mu'} \quad -(33)$$

so the Lemma transforms generally as follows:

$$\square q^{a}_{\mu} = R q^{a}_{\mu} \rightarrow \square q^{a}_{\mu'} = R q^{a}_{\mu'} . \quad -(34)$$

The wave equation transforms generally as follows:

$$\left[(\square + kT) q^{a}_{\mu} = 0 \right] \rightarrow \underbrace{(\square + kT) q^{a}_{\mu'}}_{= 0} . \quad -(35)$$

In electrodynamics the potential field is defined through the ECE ansatz as:

$$A^{a}_{\mu} = A^{(0)} q^{a}_{\mu} \quad -(36)$$

where $cA^{(0)}$ has the units of volts. Here c is the speed of light, a constant of relativity.

Therefore the fundamental wave equation of electrodynamics is:

$$(\square + kT) A^{a}_{\mu} = 0 \quad -(37)$$

and under general transformation becomes:

$$(\square + kT) A^{a}_{\mu'} = 0 . \quad -(38)$$

When the electromagnetic field becomes independent of all other fields (notably, but not only, the gravitational field) the following limit is defined by the ECE principle of equivalence, a generalization of the Einstein principle of equivalence for the gravitational field:

$$kT = \left(\frac{mc}{\hbar}\right)^2 \quad -(39)$$

Here m is the mass of the photon, and $\hbar$ is the quantum of action or angular momentum. In this limit Eq. (37) is a generalization of the Proca equation {2-10}:

$$\left(\square + \left(\frac{mc}{\hbar}\right)^2\right) A^a_\mu = 0. \quad -(40)$$

For each index a (a polarization) Eq. (40) is the Proca equation. There is no difficulty in deriving the Proca equation from the fundamental principle (18) given the ECE Ansatz (36). In contrast the Proca equation cannot be derived from the gauge principle {11} as is well known. However, photon mass is essential to the Einstein Hilbert (EH) theory of the deflection of light by gravity, a theory known to be precise to one part on a hundred thousand after experiments by NASA Cassini in the solar system. Therefore the gauge principle should be abandoned and replaced by the invariance principle (18).

For fermionic fields {2-10} the general coordinate transform is:

$$\underbrace{(\square + kT)\psi}_{=0} \rightarrow \underbrace{(\square + kT)\psi'}_{=0} \quad -(41)$$

and for gravitational fields {2-10} it is:

$$(\square + kT) q^a_\mu = 0 \rightarrow (\square + kT) q^a_{\mu'} = 0. \quad -(42)$$

The field equations {2-10} of ECE theory are also generally covariant. Adopting a short-hand notation in which indices are left out for clarity, the electromagnetic field transforms generally as:

$$F = D \wedge A = d \wedge A + \omega \wedge A \quad -(43)$$

$$\downarrow$$

$$F' = (D \wedge A)' = (d \wedge A)' + (\omega \wedge A)' \quad -(44)$$

The homogeneous field equation transforms as:

$$d \wedge F = \mu_0 j \rightarrow (d \wedge F = \mu_0 j)' . \quad -(45)$$

The inhomogeneous field equation transforms as:

$$d \wedge \tilde{F} = \mu_0 J \rightarrow (d \wedge \tilde{F} = \mu_0 J)' . \quad -(46)$$

In these equations the homogeneous current j transforms as:

$$j = \frac{A^{(0)}}{\mu_0} (R \wedge q - \omega \wedge T) \rightarrow j' \quad -(47)$$

where R is the curvature form and T is the torsion form. Here μ_0 is the vacuum permeability, and:

$$j' = \frac{A^{(0)}}{\mu_0} (R \wedge q - \omega \wedge T)' . \quad -(48)$$

The inhomogeneous current J is the Hodge dual of j and also transforms covariantly. In electrodynamics the basic invariance principle (18) becomes:

$$D_\mu A^a_\sim \rightarrow (D_\mu A^a_\sim)' = 0 . \quad -(49)$$

In contrast {1} gauge theory is superfluous by Ockham's Razor, because it introduces an internal vector space that is an abstract concept, not a geometrical concept. The internal space is not related to the base manifold geometrically in gauge theory. The internal vector space of gauge theory is defined {1} as "..an independent addition to the manifold." The fibre bundle of gauge theory is the mathematical union of base manifold and the internal vector space. The gauge transform is the transform of a field defined in the fiber bundle. The gauge principle is that physical quantities are invariant under a gauge transform in the fiber bundle. Gauge theories are invariant under a gauge transform. Without having to go into any further detail of gauge theory it is seen that it is based on an additional postulate not present in Cartan Riemann geometry. This postulate is not needed by Ockham's Razor in ECE theory, where everything is defined geometrically and where the tangent bundle {1-10} replaces the fiber bundle. As we have seen, ECE theory successfully generalizes the Proca equation, and also reduces {2-10} to the EH theory in the limit of zero torsion. Gauge theory cannot produce the Proca equation, and so ECE is preferred to gauge theory for this reason as well as by Ockham's Razor.

3. NON-LOCALITY FROM THE INVARIANCE PRINCIPLE.