71(4). Example of Scalar Invariants in General Relativity

An example of an invariant is:

$$g_{\mu\nu} g^{\mu\nu} = 4 \quad -(1)$$

as given by Einstein in "The meaning of Relativity". The relation (1) is always true for any metric and inverse metric. These transform according to:

$$g_{\mu'\nu'} = \left(\frac{\partial x^{\mu}}{\partial x^{\mu'}}\right)\left(\frac{\partial x^{\nu}}{\partial x^{\nu'}}\right) g_{\mu\nu} \quad -(2)$$

$$g^{\mu'\nu'} = \left(\frac{\partial x_{\mu}}{\partial x_{\mu'}}\right)\left(\frac{\partial x_{\nu}}{\partial x_{\nu'}}\right) g^{\mu\nu} \quad -(3)$$

but:

$$g_{\mu'\nu'} g^{\mu'\nu'} = 4. \quad -(4)$$

This is because the number 4 does not change between frames.

Another example is

$$q^{a}_{\mu} q^{\mu}_{a} = 1. \quad -(5)$$

The tetrads transform as:

$$q^{a'}_{\mu'} = \Lambda^{a'}_{a} \frac{\partial x^{\mu}}{\partial x^{\mu'}} q^{a}_{\mu} \quad -(6)$$

$$q^{\mu'}_{a'} = \Lambda^{a}_{a'} \frac{\partial x^{\mu'}}{\partial x^{\mu}} q^{\mu}_{a} \quad -(7)$$

 but:

$$q^{a'}_{\mu'} \, q^{\mu'}_{a'} = 1. \quad -(8)$$

The d'Alembertian operator is:

$$\Box := \partial^{\mu}\partial_{\mu} = \partial^{\mu'}\partial_{\mu'} \quad -(9)$$

and is also frame invariant.

The scalar curvature:

$$R = q^{\lambda}_{a}\,\partial^{\mu}\left(\Gamma^{\nu}_{\mu\lambda}\,q^{a}_{\nu} - \omega^{a}_{\mu b}\,q^{b}_{\lambda}\right) -(10)$$

is similarly frame invariant, so the ECE Lemma transforms as:

$$\boxed{\begin{array}{c} \Box q^{a}_{\mu} = R q^{a}_{\mu} \\ \downarrow \\ \Box q^{a'}_{\mu'} = R q^{a'}_{\mu'} \end{array}} \quad \begin{array}{c} -(11) \\ \\ -(12) \end{array}$$

Therefore only the eigenfunctions q^{a}_{μ} change under general coordinate transformation. The eigenoperator $\Box$ and eigenvalues R remain unchanged. This means that if: $\quad -(13)$

$$q^{a}_{\mu} = q^{a}_{\mu}(0)\exp\left(i(\omega t - \kappa Z)\right)$$

for example,

3) then:

$$q^{a'}_{\mu'} = q^{a'}_{\mu'}(0) \exp\left(i(\omega t - \kappa z + \alpha)\right) \quad - (14)$$

where α is not a function of x^μ.

This has the important consequence that seemingly non-local effects can arise from general relativity, because α is not a function of x^μ. This does not violate causality or objectivity because it is the addition of a number α in the phase of the tetrad upon general coordinate transform. So entangled states and Aharonov Bohm effects may be developed in a causal, objective theory of general relativity. They are seen as non-local effects of ~~f~~ a frame or phase transformation.

Another way to see that R is a scalar invariant is to start from the EH field equation:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = k T_{\mu\nu} \quad - (15)$$

4) in the usual notation. Now contract indices in the way indicated by Einstein:

$$R = g^{\mu\nu} R_{\mu\nu} \quad -(16)$$

$$T = g^{\mu\nu} T_{\mu\nu} \quad -(17)$$

so:

$$g^{\mu\nu} R_{\mu\nu} - \frac{1}{2} R g^{\mu\nu} g_{\mu\nu} = k g^{\mu\nu} T_{\mu\nu} \quad -(18)$$

Now use eq (1) to find:

$$R = - k T. \quad -(19)$$

It is seen that upon general coordinate transform

$$R' = - k T', \quad -(20)$$

and:

$$R = R'. \quad -(22)$$

$$T = T'. \quad -(23)$$

The scalar curvature R in eq (10) is the generalization of R in eq. (16) to Cartan geometry. The invariance of R in both cases is due to summation over repeated indices.

Therefore the ECE wave equation transforms as follows:

5)

$$(\Box + kT)\, q^a_\mu = 0 \quad -(24)$$

$$\downarrow$$

$$(\Box + kT)\, q^{a\,\prime}_\mu = 0 \quad -(25)$$

$$\boxed{\text{Conclusion}}$$

Conclusion

Non-local effects such as quantum entanglement and the Aharonov Bohm effects are due to general coordinate transformation of the ECE wave equation.

It is to be noted that the Heisenberg Uncertainty Principle has not been used. This is because it is not causal and not objective, and violates the principle of general relativity.

Sagnac Effect (and Ring Laser Gyro).

In this case the number α appears from the rotation of the frame ~~on~~ upon which the Sagnac interferometer is built. The appearance of α is seen experimentally by interferometry.

6) Aharonov Bohm Effects

The eigenfunction is A^a_μ, the electromagnetic potential. In general this is, in any reference frame:

$$A^a_\mu = A^a_\mu(0) \exp\left(i\left(\omega t - \kappa z + \alpha\right)\right) \quad -(26)$$

where α is not a function of x^μ. Therefore A^a_μ has a non-local part, which causes the Aharonov Bohm effects in regions where the electromagnetic field is not present. The complete potential is a product of eigen-functions:

$$A^a_\mu = A^a_\mu(0) \underbrace{e^{i(\omega t - \kappa z)}}_{\text{local}} \underbrace{e^{i\alpha}}_{\text{non local}} \quad -(27)$$

The local part is:

$$A^a_\mu = A^a_\mu(0) e^{i(\omega t - \kappa z)} \quad -(28)$$

and this is the part confined to region where the field $F^a_{\mu\nu}$ is non-zero. In regions where $F^a_{\mu\nu}$ is zero ~~and~~ the potential is given by eq. (27), and has a non-local component.

7) The Aharonov Bohm effects arise from a rotational coordinate transformation from one region to another. In the first region the e/m field is present. In the second region it is not present.

Furthermore, the phase factor $e^{i\alpha}$ arises from the spinning of spacetime. So the complete effect is:

$$A^a_\mu \rightarrow A^a_\mu e^{i\alpha} \quad -(29)$$

on the local potential A^a_μ. So α may be regarded as a rotation generator. We also know that the spinning of spacetime results in (standard notation):

$$F^a = d \wedge A^a + \omega^a_b \wedge A^b \quad -(30)$$

i.e.

$$\left.\begin{aligned} F &= d \wedge A \quad \left(\begin{matrix}\text{Maxwell}\\ \text{Heaviside}\end{matrix}\right) \\ &\downarrow \\ F^a &= d \wedge A^a + \omega^a_b \wedge A^b \end{aligned}\right\} \quad -(31) \quad \text{(ECE)}$$

So the AB effects are due to $\omega^a_b \wedge A^b$ as is volumes one to four.

Therefore in summary the spinning of spacetime causes:

8)

$$A(\text{local}) \rightarrow A(\text{local})\, e^{i\alpha} \quad -(32)$$

$$\left.\begin{aligned} F(\text{local}) &= (d \wedge A)(\text{local}) \\ &\downarrow \\ F^a &= (d \wedge A^a)^{(\text{local})} + \omega^a{}_b \wedge A^b \end{aligned}\right] (\text{non-local}) \quad -(33)$$

where ω is the spin connection.

Conclusion

The Sagnac and A-B effects are frame ~~or~~ phase transformation effects due to the spinning of spacetime.

PS on eq (33)

The e/m field is:

$$F = d \wedge A^a(\text{local}) + \omega^a{}_b \wedge A^b(\text{non-local})$$

where:

$$A^b(\text{non-local}) = A^b(\text{local})\, e^{i\alpha}$$

The visible part of a laser beam is due to the local A^a, but it causes an AB effect due to the non-local A^b.
