Notes 63(23): Equivalent Volume Analysis

In the off-resonant condition it is known that:

$$\phi = \frac{e}{4\pi\epsilon_0 r} = \frac{\rho(0)}{\epsilon_0}\left(\frac{\kappa^2 r^4 \cos^2(\kappa r)}{1+\kappa^4 r^4\cos(\kappa r)+5\kappa^3 r^3\sin(\kappa r)-4\kappa^2 r^2\cos(\ldots}\right. \quad -(1)$$

Therefore if:

$$\rho(0) = e/\bar{V} \quad -(2)$$

then

$$\bar{V} = r^3\left(\frac{r^2\kappa^2\cos^2(\kappa r)}{1+\kappa^4 r^4\cos(\kappa r)+5\kappa^3 r^3\sin(\kappa r)-4\kappa^2 r^2\cos(\kappa\ldots}\right. \quad -(3)$$

is the volume r^3 with a relativistic factor.

Considering the special case:

$$\kappa r = 2\pi n \quad -(4)$$

Then for $n = 0$:

$$V_0 = 0, \quad -(5)$$

for $n = 1$:

$$\bar{V}_1 = \left(\frac{4\pi^2}{1+16\pi^4-16\pi^2}\right) r^3 \quad -(6)$$

for $n = 2$

$$\bar{V}_2 = \left(\frac{16\pi^2}{1+256\pi^4-64\pi^2}\right) r^3 \quad -(7)$$

and as $n \to \infty$

$$V_\infty \to 0. \quad -(8)$$

In each case the inverse square law is