Easiest Route to Reduction of the Resonance Equation

Start with:
$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0 \quad -(1)$$

and use:
$$\underline{E} = -\underline{\nabla} \phi + \underline{\omega} \phi \quad -(2)$$

to obtain:
$$\nabla^2 \phi - \underline{\omega} \cdot \underline{\nabla} \phi - \underline{\nabla} \cdot \underline{\omega} \phi = -\rho / \epsilon_0 \quad -(3)$$

Let:
$$\underline{\omega} = \frac{1}{Z} \underline{k} \quad -(4)$$

to obtain:
$$\frac{\partial^2 \phi}{\partial Z^2} - \frac{1}{Z} \frac{\partial \phi}{\partial Z} + \frac{1}{Z^2} \phi = -\rho / \epsilon_0 \quad -(5)$$

Let
$$Z = Z_0 e^{i k_z x} \quad -(6)$$

so
$$\langle Z^2 \rangle = Z Z^* = Z_0^2 \quad -(7)$$

to obtain:
$$\frac{\partial^2 \phi}{\partial Z^2} + \frac{1}{Z_0^2} \phi = -\frac{\rho}{\epsilon_0} \quad -(8)$$

Define:
$$\rho := -\rho_0 \cos(\kappa Z) \quad -(9)$$

and
$$\boxed{\frac{\partial^2 \phi}{\partial Z^2} + \frac{1}{Z_0^2} \phi = \frac{\rho_0}{\epsilon_0} \cos(\kappa Z)} \quad -(10)$$

2) If we assume:

$$Z = Z_0 = \langle Z_0^2 \rangle^{1/2} \quad -(11)$$

we obtain an undamped oscillator:

$$\frac{\partial^2 \phi}{\partial Z^2} - \frac{1}{Z_0}\frac{\partial \phi}{\partial Z} + \frac{1}{Z_0^2}\phi = -\rho / \epsilon_0 \quad -(12)$$

as used in paper 61. If we use:

$$Z^2 = e^{2ik_Z x} \quad -(13)$$

then eq. (5) must be reduced with the method of paper 63, using:

$$\underline{E} = -(\underline{\nabla} + \underline{\omega})\phi \quad -(14)$$

i.e. spin connection defined with a negative sign. This gives another undamped oscillator:

$$\frac{\partial^2 \phi}{\partial Z^2} + \kappa_0^2 \phi = \frac{\rho_0}{\epsilon_0}\cos(\kappa' Z) \quad -(15)$$

In all cases we obtain resonance.
