61(3): Minimal Prescription, RFR, Schrödinger Pauli Equation.

The simplest type of minimal prescription in ECE theory is

$$\partial_\mu \rightarrow \partial_\mu - i\omega_\mu \quad -(1)$$

where the indices of the spin connection have been omitted for clarity. In electrodynamics, eqn. (1) is equivalent to

$$\partial_\mu \rightarrow \partial_\mu - i\frac{e}{\hbar}A_\mu \quad -(2)$$

or to:

$$\partial_\mu \rightarrow \partial_\mu - i\frac{\kappa}{A^{(0)}}A_\mu. \quad -(3)$$

On the classical level therefore:

$$\underline{A} = \frac{A^{(0)}}{\kappa}\underline{\omega}. \quad -(4)$$

RFR emerges from the minimal prescription used in the SU2 representation space (M. W. Evans and L. B. Crowell, "Classical and Quantum Electrodynamics and the $\underline{B}^{(3)}$ Field" (World Scientific, Singapore, 2001):

$$T = (\underline{\sigma}\cdot\underline{p})(\underline{\sigma}\cdot\underline{p})/(2m$$

$$\rightarrow \frac{1}{2m}\underline{\sigma}\cdot(\underline{p}+e\underline{A})\underline{\sigma}\cdot(\underline{p}+e\underline{A}^*)$$

$$= \frac{1}{2m}(\underline{p}\cdot\underline{p}+i\underline{\sigma}\cdot\underline{p}\times\underline{p}) + \frac{e}{2m}(\underline{p}\cdot\underline{A}^*+\underline{A}\cdot\underline{p}$$

$$+ i(\underline{\sigma}\cdot\underline{p}\times\underline{A}^*+\underline{\sigma}\cdot\underline{A}\times\underline{p}))$$

$$+ \frac{e^2}{2m}(\underline{A}\cdot\underline{A}^*+i\underline{\sigma}\cdot\underline{A}\times\underline{A}^*) \quad -(5)$$

The RFR energy is:

$$H_{RFR} = \frac{ie^2}{2m} \underline{\sigma} \cdot \underline{A} \times \underline{A}^*$$

$$= \frac{\mu_0 c e^2}{2m} \frac{I}{\omega^2} \sigma_z \quad - (6)$$

RFR occurs at:

$$\hbar \omega_{res} = \frac{\mu_0 c e^2}{2m} \frac{I}{\omega^2} (1 - (-1))$$

$$\boxed{\omega_{res} = \left( \frac{\mu_0 c e^2}{\hbar m} \right) \frac{I}{\omega^2}} \quad - (7)$$

This is eqn. (27) of notes 61(2). So it is seen that RFR is a direct consequence of the use of a SU(2) rep., and the same rep. gives the well known half integral spin and Zeeman effect, ESR, NMR and MRI. The easiest way to see this is to use the Schrödinger - Pauli equation:

$$H \phi = E \phi \quad - (8)$$

with:

$$H = i \frac{e}{2m} \underline{\sigma} \cdot ( \underline{p} \times \underline{A} + \underline{A} \times \underline{p} ) \quad - (9)$$

and

$$\underline{p} \rightarrow -i\hbar \underline{\nabla} \quad - (10)$$

Thus:

$$H \phi = \frac{ie\hbar}{2m} \underline{\sigma} \cdot \left( (\underline{\nabla} \times \underline{A}) \phi + (\underline{\nabla} \phi) \times \underline{A} \right.$$

$$= \frac{e\hbar}{2m} \left( \underline{\sigma} \cdot \underline{B} \right) \phi \quad -(11)$$

This is the famous half-integral spin of the Zeeman effect.

In ECE theory RFR and the well known half-integral spin must be understood in terms of a face-line resonance. The magnetic field in ECE is always:

$$\underline{B} = \underline{\nabla} \times \underline{A} - \underline{\omega} \times \underline{A} \quad -(12)$$

and from eq. (4):

$$\boxed{\underline{\omega} = \frac{\kappa}{A^{(0)}} \underline{A} := g \underline{A}.} \quad -(13)$$

If $\underline{A}$ is complex valued, and using the complex circular basis:

$$\boxed{\underline{B} = \underline{\nabla} \times \underline{A} - i g \underline{A} \times \underline{A}^*} \quad -(14)$$

So:

$$\underline{\sigma} \cdot \underline{B} = \underline{\sigma} \cdot \left( \underline{\nabla} \times \underline{A} - i g \underline{A} \times \underline{A}^* \right). \quad -(15)$$

Therefore the basic information in eq. (5) emerges from the spin connection (13) and the minimal prescription (1).

The definition (14) applies to any magnetic field, but if $\underline{A}$ is real valued, only the $\underline{\nabla} \times \underline{A}$ part of eq. (14) is non-zero. So for consistency with general relativity $\underline{A}$ must be complex valued.

4) The resonance equation for A is:

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{A} - \underline{\omega} \times \underline{A}) = \frac{\mu_0}{c} \underline{J} \quad -(15)$$

if we consider only magnetic effects. This has been developed in papers 52, 53 and 60. In the SU(2) rep:

$$(\underline{\nabla} \times (\underline{\nabla} \times \underline{A} - \underline{\omega} \times A)) \cdot \underline{\sigma} = \frac{\mu_0}{c} \underline{J} \cdot \underline{\sigma}. \quad -(16)$$

The RFR resonance or the ordinary ESR, NMR or MRI resonances may be identified as resonance from eq. (16), which is a driven damped oscillator equation as usual, these are space-time resonances.

Conclusions

The minimal prescription in ECE is, at the simplest level:

$$\partial_\mu \rightarrow \partial_\mu - i\omega_\mu \quad -(17)$$

and the $\omega_\mu$ denotes the existence of spinning spacetime. On the classical level:

$$A_\mu = \frac{A^{(0)}}{\kappa} \omega_\mu := \frac{1}{g} \omega_\mu \quad -(18)$$