1) 60(5): Resonant Electrodynamics in ECE Theory

The relevant equations are:

$$\underline{\nabla} \cdot \underline{E}^a = c\mu_0 \tilde{J}^{0a} \quad - (1)$$

$$\underline{\nabla} \times \underline{B}^a - \frac{1}{c^2}\frac{\partial \underline{E}^a}{\partial t} = \frac{\mu_0}{c} \underline{\tilde{J}}^a \quad - (2)$$

$$\underline{E}^a = -\frac{\partial \underline{A}^a}{\partial t} - \underline{\nabla}\phi^a - c\omega^{0a}{}_b \underline{A}^b + \underline{\omega}^a{}_b \phi^b \quad - (3)$$

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b. \quad - (4)$$

To simplify the problem, the polarization indices can be dropped. This process is equivalent to the use of a simplified spin convention. So eqs (1) - (4) become:

$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0 \quad - (5)$$

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} = \frac{\mu_0}{c} \underline{\tilde{J}} \quad - (6)$$

$$\underline{E} = -\frac{\partial \underline{A}}{\partial t} - \underline{\nabla}\phi - c\omega^0 \underline{A} + \phi \underline{\omega} \quad - (7)$$

$$\underline{B} = \underline{\nabla} \times \underline{A} - \underline{\omega} \times \underline{A}. \quad - (8)$$

$$\tilde{J}^\mu = (c\rho, \underline{\tilde{J}}) \quad - (9)$$

If the scalar potential is neglected:

$$\underline{E} = -\frac{\partial \underline{A}}{\partial t} - c\omega^0 \underline{A} \quad - (10)$$

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and we obtain:

$$\frac{1}{c^2}\frac{\partial^2 \underline{A}}{\partial t^2} + \frac{\omega^\circ}{c}\frac{\partial \underline{A}}{\partial t} + (\underline{\nabla}\cdot\underline{\omega})\underline{A} - (\underline{A}\cdot\underline{\nabla})\underline{\omega}$$

$$-\nabla^2\underline{A} + \underline{\nabla}(\underline{\nabla}\cdot\underline{A}) - \underline{\omega}(\underline{\nabla}\cdot\underline{A}) + (\underline{\omega}\cdot\underline{\nabla})\underline{A} = \frac{\mu_0}{c}\tilde{\underline{J}}$$

— (11)

Now assume a quasi plane-wave approximation

$$\underline{\nabla}\cdot\underline{A} \sim 0 \qquad \text{— (12)}$$

and split $\underline{J}$ into a time dependent and $\underline{r}$ dependent sum:

$$\tilde{\underline{J}} \sim \tilde{\underline{J}}(t) - \tilde{\underline{J}}(\underline{r}) \qquad \text{— (13)}$$

to obtain two resonance equations:

$$\boxed{\frac{1}{c^2}\frac{\partial^2 \underline{A}}{\partial t^2} + \frac{\omega^\circ}{c}\frac{\partial \underline{A}}{\partial t} + (\underline{\nabla}\cdot\underline{\omega})\underline{A} = \frac{\mu_0}{c}\tilde{\underline{J}}(t)}$$

— (14)

and:

$$\boxed{\nabla^2\underline{A} - (\underline{\omega}\cdot\underline{\nabla})\underline{A} + (\underline{A}\cdot\underline{\nabla})\underline{\omega} = \frac{\mu_0}{c}\tilde{\underline{J}}(\underline{r})}$$

— (15)

both of which have <u>analytical</u> solutions.