Notes 59(7): Resonance Oscillations in the H Atom

The background to this work is the driven damped oscillator equation:

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = A\cos\omega t \quad -(1)$$

The particular integral is:

$$x_p(t) = D\cos(\omega t - \delta) \quad -(2)$$

$$= \frac{A\cos(\omega t - \delta)}{\left((\omega_0^2 - \omega^2)^2 + 4\omega^2\beta^2\right)^{1/2}}$$

where:

$$\delta = \tan^{-1}\left(\frac{2\omega\beta}{\omega_0^2 - \omega^2}\right).$$

At resonance:

$$\omega_0 = \omega \quad -(3)$$

and so $x_p(t)$ becomes highly oscillatory. Therefore a small initial oscillation can be greatly amplified at resonance. The purpose of paper 59 is to demonstrate in a simple approximation that a resonance phenomenon may occur within the H atom to break it apart into proton and electron. This work is founded on general relativity and on the work of the Mexican group. This work indicates that there is a process in nature which can produce electric power from spacetime. The latter is represented by the spin connection. In the standard model this process does not exist, even on the qualitative level.

In the standard model the Poisson equation is:

$$\frac{\partial^2\phi}{\partial Z^2} = -\frac{\rho}{\epsilon_0} \quad -(4)$$

where ϕ is the scalar potential, ρ is the charge density and ϵ_0 is the free space permittivity. Eq. (4) does not produce resonance. The relevant units are:

$$\epsilon_0 = J^{-1} C^2 m^{-1}, \ \rho = C m^{-3}, \ \phi = J C^{-1}, \ Z = m.$$

Thus the units of ϕ are volts $= J C^{-1}$. The minus sign in eq. (4) is a convention. In S.I. units in the standard model:

$$\underline{E} = - \underline{\nabla} \phi \quad - (5)$$

and the usual solution (Jackson, 3rd. ed., eq. (1.16)) is:

$$\phi = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(x')}{|\underline{x} - \underline{x}'|} d^3 x' \quad - (6)$$

This may be written as:

$$\phi = \frac{e}{4\pi\epsilon_0 r} \quad - (7)$$

which has the correct units of volts. The potential energy used in the Schrödinger equation of the H atom is:

$$V = - e\phi = - \frac{e^2}{4\pi\epsilon_0} \frac{1}{r} . \quad - (8)$$

The negative sign in eq. (8) means that the energy gives rise to Coulomb attraction between one proton and one electron. In eq. (4) r has been aligned in Z for convenience. The Coulomb force law is:

$$\underline{F} = q \underline{E} \quad - (9)$$

and the work done is:

$$W = -\int_A^B \underline{F} \cdot d\underline{l} = q(\phi_B - \phi_A) \quad -(10)$$

These well known laws are sufficient for the great majority of applications, but are nevertheless laws of special relativity. In general relativity eq. (5) is replaced by:

$$\underline{E}^a = -\frac{\partial \underline{A}^a}{\partial t} - c\underline{\nabla} A^{0a} - c\,\omega^{0a}{}_b \underline{A}^b + c\,\underline{\omega}^a{}_b A^{0b} \quad -(11)$$

If the vector potential $\underline{A}^a$ is neglected eq. (11) reduces to:

$$\boxed{\underline{E} = -\underline{\nabla}\phi + \underline{\omega}\phi} \quad -(12)$$

in the very simplest case. The electric field is defined by ϕ and the spin connection vector $\underline{\omega}$. The Coulomb law in general relativity is:

$$\underline{\nabla} \cdot \underline{E} = \frac{\rho}{\epsilon_0} \quad -(13)$$

so from eqns. (12) and (13):

$$\nabla^2 \phi - \underline{\nabla} \cdot (\underline{\omega}\phi) = -\frac{\rho}{\epsilon_0}, \quad -(14)$$

i.e.

$$\boxed{\nabla^2 \phi - \underline{\omega} \cdot \underline{\nabla}\phi - (\underline{\nabla} \cdot \underline{\omega})\phi = -\frac{\rho}{\epsilon_0}} \quad -(15)$$

4)

It can be seen that eq (15) is similar in structure to eq. (1) except for sign changes due to convention. We may write eq. (15) for convenience as:

$$\nabla^2 \phi + \underline{\omega} \cdot \underline{\nabla} \phi + (\underline{\nabla} \cdot \underline{\omega}) \phi = \frac{\rho}{\epsilon_0} \quad - (16)$$

and restricting consideration to ~~the~~ Z axis:

$$\frac{\partial^2 \phi}{\partial Z^2} + \omega \frac{\partial \phi}{\partial Z} + \left(\frac{\partial \omega}{\partial Z} \right) \phi = \frac{\rho}{\epsilon_0} \quad - (17)$$

The final ingredient needed for resonance is to make ρ a function of $\cos(\kappa Z)$:

$$\rho = \rho^{(0)} \cos(\kappa Z) \quad - (18)$$

Here κ has the units of m^{-1}, and eq. (18) means that there is a small oscillation in the charge density ρ. This is well known to be present in the H atom (and all atoms and molecules) ~~through the~~ through the phenomena of Zitterbewegung (jitterbugging). This comes from q.e.d. (see Atkins for example), and is modelled in eq. (18). It should be clear that ω in eq. (17) denotes the spin connection not angular frequency.

5) So eq. (17) is:

$$\frac{\partial^2 \phi}{\partial Z^2} + \omega \frac{\partial \phi}{\partial Z} + \left(\frac{\partial \omega}{\partial Z}\right)\phi = \frac{\rho^{(0)}}{\epsilon_0} \cos(\kappa Z) \quad - (19)$$

and eq. (1) is:

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = A\cos(\omega t), \quad - (1)$$

so:

$$x \rightarrow \phi, \; 2\beta \rightarrow \omega, \; \omega_0^2 \rightarrow \partial\omega/\partial Z, \; A \rightarrow \rho^{(0)}/\epsilon_0,$$
$$\omega \rightarrow \kappa, \; t \rightarrow Z.$$

Therefore the particular solution of eq. (19) is:

$$\boxed{\phi_p(t) = \left(\frac{\rho^{(0)}}{\epsilon_0}\right) \frac{\cos(\kappa Z - \delta)}{\left(\left(\frac{\partial \omega}{\partial Z} - \kappa^2\right)^2 + \omega^2\kappa^2\right)^{1/2}} \quad - (20)}$$

where:

$$\delta = \tan^{-1}\left(\frac{\omega\kappa}{\frac{\partial\omega}{\partial Z} - \kappa^2}\right). \quad - (21)$$

At resonance:

$$\kappa_0^2 = \frac{\partial \omega}{\partial Z}, \quad - (22)$$

and the scalar potential $\phi_p(t)$ becomes highly oscillatory, breaking apart the atom.

6)

Finally, eq. (20) may be incorporated into the H atom using:

$$\phi_p(t) = \frac{e}{4\pi\epsilon_0 V_0} \cdot \frac{\cos(\kappa Z - \delta)}{((\kappa_0^2 - \kappa^2)^2 + \omega^2\kappa^2)^{1/2}} \quad -(23)$$

where V_0 is a volume. The potential energy to be used in the Schrödinger equation is then:

$$V = -\frac{e^2}{4\pi\epsilon_0 V_0} \frac{\cos(\kappa Z - \delta)}{((\kappa_0^2 - \kappa^2)^2 + \omega^2\kappa^2)^{1/2}} \quad -(24)$$

When:

$$\kappa = 0 \quad -(25)$$

eq. (24) reduces to:

$$V = -\frac{e^2}{4\pi\epsilon_0 r} \quad -(26)$$

if

$$r = V_0 \frac{\partial\omega}{\partial Z}. \quad -(27)$$

The Schrödinger eqn. is:

$$\hat{H}\phi = E\phi \quad -(28)$$

$$\hat{H} = -\frac{\hbar^2}{2\mu}\nabla^2 + V. \quad -(29)$$