1)

Notes 59(4): Example Solution.

Start from the general equation:

$$\underline{\nabla} \cdot \underline{\nabla} A^{a0} + \frac{1}{c}\frac{\partial}{\partial t}\left(\underline{\nabla} \cdot \underline{A}^{a}\right) + \underline{\nabla} \cdot \left(\omega^{0a}{}_{b} \underline{A}^{b}\right) - \underline{\nabla} \cdot \left(\underline{\omega}^{a}{}_{b} A^{0b}\right) = -\mu_0 \tilde{J}^{0a} \quad -(1)$$

Assume that for a quasi-electrostatic situation the vector potential terms are small. This means that

$$\nabla^2 A^{a0} - \underline{\omega}^{a}{}_{b} \cdot \underline{\nabla} A^{0b} - \left(\underline{\nabla} \cdot \underline{\omega}^{a}{}_{b}\right) A^{0b} = -\mu_0 \tilde{J}^{0a} \quad -(2)$$

Consider that $\tilde{J}^{0a}$ is oscillating:

$$\tilde{J}^{0a} = \tilde{J}^{0a}(0) \cos\left(\underline{\kappa} \cdot \underline{r}\right) \quad -(3)$$

Consider:

$$a = b = 0 \quad -(4)$$

and drop the superscripts for convenience. Finally consider only the Z axis to obtain:

$$\frac{\partial^2 A}{\partial Z^2} + \omega \frac{\partial A}{\partial Z} + \left(\frac{\partial \omega}{\partial Z}\right) A = \mu_0 \tilde{J}(0) \cos(\kappa Z) \quad -(5)$$

<u>Resonance</u> occurs at:

$$\kappa_R = \left(\left(\frac{\partial \omega}{\partial Z}\right)^2 - \frac{\omega^2}{2}\right)^{1/2} \quad -(6)$$

The particular integral is:

$$A_p(\kappa) = D \cos(\kappa Z - \delta) \quad -(7)$$

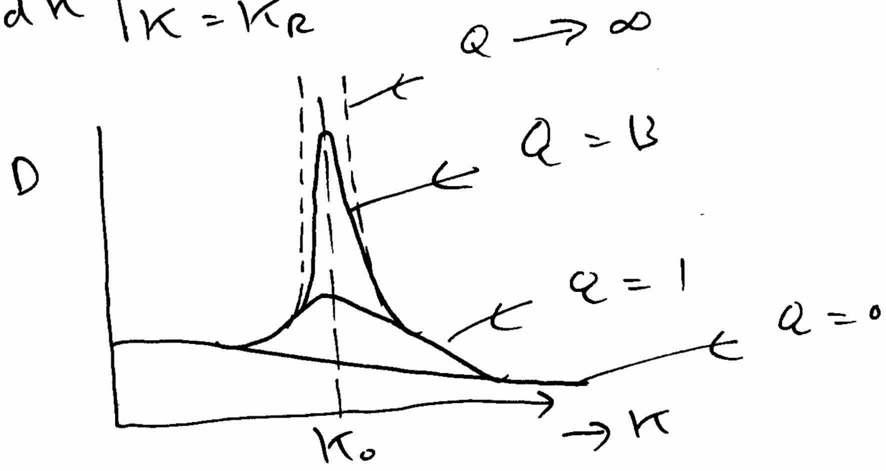
2) where:

$$\delta = \tan^{-1}\left(\frac{\omega\kappa}{\left(\frac{\partial\omega}{\partial z}\right)^2 - \kappa^2}\right) \quad - (8)$$

At resonance:

$$\left.\frac{dD}{d\kappa}\right|_{\kappa=\kappa_R} = 0 \quad - (9)$$

giving:



It can be seen that at resonance, the potential $A_p(\kappa)$ from eq. (7) is greatly amplified. This is the potential used in the Schrödinger equation. If enough energy is inputted into the H atom it breaks apart. The driving charge density $\bar{J}(0)$ is the charge density within the δ_0 H atom in the off-resonance condition.