Notes 58(6): Calculation of the Einstein/Eddington Orbit

Consider the Schwarzschild Vetrad:

$$q^1{}_1 = \left(1 - \frac{2GM}{c^2 r}\right)^{-1/2} \quad -(1)$$

where M is the mass of the sun. Now we:

$$(1-x)^{-1/2} = 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \dots \quad (2)$$

so

$$q^1{}_1 = 1 + \frac{GM}{c^2 r} \quad -(3)$$

to a good approximation. A position x is given by

$$x = x^{(0)} q^1{}_1 = x^{(0)}\left(1 + \frac{GM}{c^2 r}\right) \quad -(4)$$

so x is deflected by the curvature of spacetime. The Newtonian result is $x = x^{(0)}$. From eq. (4):

$$\frac{\partial x}{\partial r} = -\frac{x^{(0)}}{c^2}\left(\frac{GM}{r^2}\right) \quad -(5)$$

i.e.

$$\boxed{c^2 \frac{\partial q^1{}_1}{\partial r} = -\frac{GM}{r^2}.} \quad -(6)$$

The Newtonian force between photon and sun is:

$$\underline{F} = -\frac{GmM}{r^2}\underline{k} \quad -(7)$$

Therefore the total force is twice the Newtonian esult because of the extra $-GM/r^2$ from eq.). This was first observed by Eddington et alii.