Notes 58(4)

ECE Curvatures of the General Metric in Spherically Symmetric Spacetime

The general line element is:

$$ds^2 = -e^{2\alpha(t,r)}dt^2 + e^{2\beta(t,r)}dr^2 + r^2 d\Omega^2, \quad -(1)$$

so:

$$g_{\mu\nu} = \begin{bmatrix} -e^{2\alpha} & 0 & 0 & 0 \\ 0 & e^{2\beta} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\phi \end{bmatrix} . \quad -(2)$$

Therefore:

$$q^0_0 = e^{\alpha}; \; q^1_1 = e^{\beta}; \; q^2_2 = q^3_3 = 1. \quad -(3)$$

The ECE Lemmas are:

$$\Box q^0_0 = R_0 q^0_0; \; \Box q^1_1 = R_1 q^1_1 \quad -(4)$$

so:

$$\boxed{R_0 = \Box \alpha; \; R_1 = \Box \beta} \quad -(5)$$

Schwarzschild Solution

$$R_0 = \Box \left(1 - \frac{2GM}{c^2 r}\right)^{1/2} \quad -(6)$$

$$R_1 = \Box \left(1 - \frac{2GM}{c^2 r}\right)^{-1/2} \quad -(7)$$

where:

$$r = (x^2 + y^2 + z^2)^{1/2} \quad -(8)$$

$$\Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} . \quad -(9)$$

As $r \to \infty$, $R_0 \to 0$, $R_1 \to 0$ ✓