1)

Notes 57(1): Canonical Quantization

Canonical quantization of the electromagnetic field in the standard model runs into difficulties because of the assumption of a massless electromagnetic field. In special relativity this means that A_μ must have only two components, and these are usually taken as the transverse components. However, A_μ must have four components by definition, so there is a basic contradiction in the standard model. Canonical quantization is the name given to the construction of the Heisenberg equation appropriate to A_μ, (see for example L. H. Ryder, "Quantum Field Theory", CUP, 2nd ed., 1996). The standard model is based on special relativity, on the mathematical gauge theory property that if:

$$A_\mu \rightarrow A_\mu + \partial_\mu \Lambda \qquad -(1)$$

then:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad -(2)$$

is unchanged. Therefore $F_{\mu\nu}$ is "gauge invariant". The discovery of the Aharonov Bohm effects meant that this view had to be modified, because in regions where $F_{\mu\nu}$ is zero, A_μ has a physical effect. Also, in the minimal prescription:

$$p_\mu \rightarrow p_\mu + eA_\mu \qquad -(3)$$

the potential A_μ is regarded as physical.

Therefore the philosophical basis of gauge theory is questionable, the assumption that A_μ is mathematical and that $F_{\mu\nu}$ is physical is untenable, and in the standard model, gauge theory applied to the e/m, weak and strong sectors is special relativity, not general relativity as required. This leads to considerable difficulties in constructing the Heisenberg equation for A_μ.

In paper 57 it is shown that ECE theory removes these difficulties straightforwardly - a step forward in quantum field theory.

In differential form notation, eqs. (1) and (2) are:

$$A \rightarrow A + d\Lambda \quad -(4)$$

$$F = d \wedge A \quad -(5)$$

$$= d \wedge (A + d\Lambda) \quad -(6)$$

because:

$$d \wedge d\Lambda := 0. \quad -(7)$$

Eq. (7) is the Poincaré Lemma. Gauge theory in the second half of the twentieth century was based on this mathematical procedure. Eq. (4) is the simplest example of a <u>gauge transformation</u>. Eq. (6) is an example of <u>gauge invariance</u>. The principle of gauge invariance was elevated to be the most important principle in field theory. However, in relativity theory the most important principle is that all equations of physics must be covariant under the general coordinate transformation. Seen in this light, the gauge transformation principle is superfluous by Ockham's Razor, it is not needed in field theory because general relativity already contains all the needed philosophical basics.

ECE theory now shows that the generally covariant formulation of electrodynamics unifies the latter with gravitation and the weak and strong fields. The standard model is unable to do this. The relevant equations are:

$$F^a = d \wedge A^a + \omega^a{}_b \wedge A^b \quad -(8)$$

$$d \wedge F^a = \mu_0 j^a \quad -(9)$$

$$d \wedge \tilde{F}^a = \mu_0 \tilde{J}^a . \quad -(10)$$

The fundamental field is:

$$A^a = A^{(0)} q^a \quad -(11)$$

where q^a is the tetrad field and where $cA^{(0)}$ is the primordial voltage. Thus F^a is

4) constructed from A^a via the first Bianchi identity, eq. (9). The vector valued one-form A^a_μ and the vector valued two-form $F^a_{\mu\nu}$ are covariant under the general coordinate transformation according to the rules of Cartan geometry. However, under a "gauge transformation" such as:

$$A^a \rightarrow A^a + d\Lambda^a \quad -(12)$$

$$F^a \rightarrow d\wedge(A^a + d\Lambda^a) + \omega^a{}_b \wedge (A^b + d\Lambda^b) \quad -(13)$$

$$= d\wedge A^a + \omega^a{}_b \wedge A^b + \omega^a{}_b \wedge d\Lambda^b \quad -(14)$$

$$\boxed{F^a \longrightarrow F^a + \omega^a{}_b \wedge d\Lambda^b} \quad -(15)$$

So F^a is not invariant under a transformation such as (12), F^a is coordinate covariant.

There is also the additional problem that in gauge ~~theory~~, a is an abstract number superimposed on a flat or Minkowski spacetime, and again this is superfluous to the requirements of general relativity, in which a is well

5) defined by Cartan geometry as the index of the tangent space-time at P to the base manifold indexed μ. In ECE theory ~~the~~ space-time that defines the electromagnetic field is defined self-consistently by Cartan geometry alone, and in four dimensions only. Thus ECE is preferred to ___ gauge theory by the Ockham Razor of philosophy. In ECE ~~theory~~ a is already defined by geometry, in gauge ~~theory~~ it must be introduced as an extra assumption, and in gauge theory, gauge invariance must also be introduced as an extra assumption.

In ECE theory the fundamental wave equation of electrodynamics is:

$$(\square + kT) A^a_\mu = 0. \quad -(16)$$

In the limit defined by the correspondence principle of relativity:

$$kT \rightarrow \left(\frac{mc}{\hbar}\right)^2 \quad -(17)$$

6) Eq. (16) becomes the Proca Equation:

$$\left(\Box + \left(\frac{mc}{\hbar} \right)^2 \right) A_\mu^a = 0 \quad -(18)$$

where m is the non-zero photon mass.

In the standard model, the interpretation of the Proca equation is self-contradictory (see Ryder) because the Lagrangian needed to derive eq. (18) is not gauge invariant. So for this reason the Proca equation is not used for canonical quantization. The basic weak point here is again the assumption of gauge invariance. ECE removes gauge invariance and replaces it by coordinate covariance. Therefore the electromagnetic field A_μ is associated with the photon mass m. Therefore A_μ has the necessary four physical components because it is no longer a massless field. General relativity prohibits the existence of a massless field. Thus A_μ is manifestly covariant in ECE theory and can be canonically quantized.

7) As shown in paper 56, the potential field in ECE theory can be non-zero in regions where the F^a field is zero. In this case the equations defining the potential are:

$$d \wedge A^1 = g\, A^2 \wedge A^3 \quad -(19)$$

$$d \wedge A^2 = g\, A^3 \wedge A^1 \quad -(20)$$

$$d \wedge A^3 = g\, A^1 \wedge A^2 \quad -(21)$$

$$d \wedge A^0 = -\, d \wedge A^3, \quad -(22)$$

if it is assumed for the sake of simplicity of argument that the gravitational field does not influence the electromagnetic field, i.e.:

$$j^a = 0. \quad -(23)$$

It is seen that eqs. (19) to (22) all contain <u>commutators of potentials</u>. This commutator is precisely the quantity needed to construct a Heisenberg equation in the potential. This is therefore the required <u>canonical quantization</u>.

The quantity g is:

$$g = \frac{\kappa}{A^{(0)}} = \frac{e}{\hbar} \quad -(23)$$

So the quantized field is:

8)

$$A^2 \wedge A^3 = \frac{\hbar}{e} d \wedge A^1 \quad -(24)$$

et cyclicum.

In tensor notation, eqn. (24) is:

$$\boxed{[A^2_\mu, A^3_\nu] = \frac{\hbar}{e} [\partial_\mu, A^1_\nu] \quad \text{et cyclicum}} \quad -(25)$$

This is the Heisenberg type equation needed in canonical quantization, but it is important to realize that eq. (25) is an equation of <u>general relativity</u>. The original Heisenberg equation is one of <u>non-relativistic</u> quantum mechanics.

In regions where both F^a and A^a are non-zero:

$$F^1 = d \wedge A^1 - g A^2 \wedge A^3 \quad -(26)$$

et cyclicum

and

$$\boxed{[A^2_\mu, A^3_\nu] = \frac{\hbar}{e} \left([\partial_\mu, A^1_\nu] - F^1_{\mu\nu} \right) \quad \text{et cyclicum}} \quad -(27)$$

9) Summary

The commutators of potential needed for canonical quantization of the e/m field in ECE theory are derived directly from Cartan geometry and the ECE Ansatz, eq. (11). All four components of A_μ^a are physical and non-zero in general. The fundamental wave equation of e/m is eq. (16) which reduces to the Proca equation (18) using the correspondence principle. The principle of gauge invariance is discarded in favour of the older (1916) principle of coordinate covariance. Canonical quantization of A_μ^a follows straightforwardly. This is a major advance in field theory.