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WAVE MECHANICS AND ECE THEORY

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ABSTRACT

Generally covariant wave mechanics is developed from Einstein Cartan Evans (ECE) field theory. The ECE lagrangian density is identified and used in the ECE Euler Lagrange equation to identify the origin of the Planck constant as a minimized action of general relativity. It is shown that the Planck constant as used in special relativity (standard model wave mechanics) is a special case in which volume is fortuitously cancelled out. More generally the commutator equation of Heisenberg must include volume. The Cartan structure equation, Cartan torsion, and Bianchi identity are derived from the lagrangian density. The Aspect experiment is explained using ECE wave mechanics, and quantum entanglement is described using the spin connection term of ECE theory. The Bohr Heisenberg indeterminacy is discarded in favor of a causal, objective and unified wave mechanics. Phase velocity, v, in ECE wave mechanics can become much greater than c (which remains the universal constant of relativity theory) and the equations defining the condition v >> c are given.

Keywords: Einstein Cartan Evans (ECE) field theory; generally covariant wave mechanics, origin of the Planck constant; ECE lagrangian density; derivation of the Cartan structure equation, Cartan torsion and Bianchi identity; description of the Aspect experiment and quantum entanglement, greater than c phase velocity.

1. INTRODUCTION

Recently {1-38} it has been shown that the origin of generally covariant wave mechanics is the tetrad postulate of Cartan geometry {39, 40}, the fundamental requirement that a vector field be independent of the coordinate system used to describe it. General covariance in physics means that its equations are covariant under the general coordinate transformation. This means that they retain their form, a tensor in one coordinate system must be a tensor in any other coordinate system. The equations of physics are therefore objective to an observer in one reference frame moving in an arbitrary way with respect to an observer in any other reference frame. The requirement of objectivity in physics manifests itself as this fundamental principle of general relativity and without this principle there is no objective physics, nature would mean different things to different observers. Special relativity is known to be accurate to one part in twenty seven orders of magnitude and general relativity to one part in one hundred thousand for the solar system. So the principle of objectivity is tested to high precision. The other fundamental attribute of relativity theory is that the constant c be universal. This is usually interpreted to mean that no information can travel faster than c and other constants in physics are based on a fixed c in standards laboratories worldwide. The constancy of c is needed to ensure causality, to ensure that nothing happens without a cause.

Throughout the twentieth century, general relativity was thought to be incompatible with the principle of indeterminacy developed mainly by Bohr and Heisenberg. This principle states that pairs of variables such as position x and momentum p behave in such a way that if one is known exactly (for example x), the other (for example p) is unknowable. This assertion is based on a variation inferred by Heisenberg of the Schrödinger equation of non-relativistic wave mechanics. There is nothing, however, in the original Schrödinger equation which implies indeterminacy, the Schrödinger equation is based {41} on the fact that action is minimized in particles by the classical Hamilton principle of least action, and that time interval is minimized in waves by the classical Fermat principle of least time. The Heisenberg commutator equation is a restatement of the Schrödinger equation. It has been shown {1-38} that the Schrödinger equation is a well defined non-relativistic quantum limit of the Einstein Cartan Evans (ECE) wave equation of general relativity. Therefore the Schrödinger equation has been shown to be objective and causal and has been shown to be an equation of relativity theory. It follows that the Heisenberg commutator equation is also objective and causal. It cannot lead to Bohr Heisenberg indeterminacy and cannot lead to anything that is unknowable. Recently {42-45} the Bohr Heisenberg indeterminacy principle has indeed been refuted experimentally in several independent ways, all of which are repeatable and reproducible. Indeterminacy is therefore an intellectual aberration which worked itself uncritically into thousands of textbooks of the twentieth century era.

In Section 2 the lagrangian density of generally covariant unified field theory is deduced and used to derive the fundamental ECE wave equation. Therefore from the outset the concept of volume is introduced into wave mechanics because the lagrangian density has the units of energy divided by volume. It has been shown {1-38} that the experiments of Croca et al. {42}, experiments which refute indeterminacy experimentally, can be explained by ECE theory with the introduction of volume into the Heisenberg commutator equation. The ECE lagrangian density inferred in this Section is the fundamental origin of this volume. Key quantities in wave mechanics must therefore be densities, in common with the rest of general relativity. This deduction is seen at work in the fundamental ECE wave equation {1-38}:

$$(\Box + kT)\, q^a_\mu = 0 \qquad -(1)$$

Here k is Einstein's constant, T is the index reduced canonical energy momentum, a concept first introduced {46} by Einstein, and q^a_μ is the tetrad of Cartan geometry {39, 40}, the fundamental unified field of ECE theory. In the rest frame:

$$T = \frac{m}{V} \quad -(2)$$

which is mass divided by volume. The lagrangian density in this limit is:

$$\mathcal{L} = c^2 T = \frac{mc^2}{V} \quad -(3)$$

and is the rest energy divided by volume. All other wave equations of physics are limits of the ECE wave equation {1-38}, so volume is inherent in all of them. In this section it is shown that the Cartan structure equation and the Bianchi identity of Cartan geometry can be derived from the same lagrangian density. It is thus inferred that all of physics (both classical and quantum) derives from the tetrad postulate, the fundamental mathematical requirement that a complete vector field is independent of the way it is written, independent of the coordinate system used to define the vector field. This inference leads to an unprecedented degree of simplicity and fundamental understanding.

In Section 3 the fundamental origin of the Planck constant is discussed within ECE field theory using fact that action is:

$$S = \frac{1}{c} \int \mathcal{L} \, d^4x \quad -(4)$$

an integral of the lagrangian density $\mathcal{L}$ over the four-volume d^4x. Action has the units of energy multiplied by time, and these are also the units of angular momentum. Using these concepts the fundamental Planck Einstein and de Broglie equations of quantum mechanics are derived within the concepts of ECE field theory and thus of general relativity. This derivation is not possible in the standard model because there, wave mechanics is not generally covariant. The evolution of the tetrad in ECE theory is governed by:

$$q^a_\mu(x^\mu) = \exp\left(i\frac{S}{\hbar}\right) q^a_\mu(0) \quad -(5)$$

where S is the action and $\hbar$ a constant of proportionality introduced to make the exponent dimensionless as required. This is the reduced Planck constant. The fundamental origin of Eq. (5) is wave particle duality. In ECE theory there is no distinction between wave and particle, both are manifestations of ECE spacetime. The Dirac electron, for example, is defined by the limit {1-38}:

$$kT = \frac{m^2c^2}{\hbar^2} \quad -(6)$$

of the ECE wave equation (1). This is not a point particle, because from Eqs. (3) and (6) emerges the rest volume of any particle:

$$V_0 = \frac{k\hbar^2}{mc^2}. \quad -(7)$$

The wave nature of the Dirac electron is governed by the same ECE wave equation through the SU(2) representation of the tetrad {1-38}. In Section 3 it is shown that the Planck constant is a limit of Eq. (4), a limit in which the volume V fortuitously cancels. There is a lot more to the Planck constant in generally covariant unified wave mechanics than the standard model's quantum mechanics.

In Section 4 the Aspect experiment and quantum entanglement are discussed within the context of generally covariant and causal wave mechanics, and finally in Section 5 it is shown that under well defined circumstances, the phase velocity, v, of a generally covariant wave can become much larger than c, and indeed approach infinity. The phase velocity v >> c, combined with the spin connection, lead to many new inferences and possible new technologies.

2. LAGRANGIAN FORMULATION OF GENERALLY COVARIANT WAVE MECHANICS.

It is seen by inspection that the generally covariant Euler Lagrange equation:

$$\frac{\partial \mathcal{L}}{\partial q^{\sim}_{a}} = D^{\mu}\left(\frac{\partial \mathcal{L}}{\partial (D^{\mu} q^{\sim}_{a})}\right) \quad -(8)$$

with the lagrangian density:

$$\mathcal{L} = c^2 T + D_{\mu} q^{a}_{\sim} D^{\mu} q^{\sim}_{a} \quad -(9)$$

gives:

$$D^{\mu}\left(D_{\mu} q^{a}_{\sim}\right) = 0. \quad -(10)$$

This is the ECE Lemma {1-38}, which is obtained by covariant differentiation of the tetrad postulate {39, 40} of Cartan geometry:

$$D_{\mu} q^{a}_{\sim} = 0. \quad -(11)$$

Using the fundamental definition:

$$q^{a}_{\mu} q^{\mu}_{a} = 1 \quad -(12)$$

the Leibnitz Theorem is applied to give:

$$\begin{aligned} D_{\sim}\left(q^{a}_{\mu} q^{\mu}_{a}\right) &= q^{a}_{\mu}\left(D_{\sim} q^{\mu}_{a}\right) + \left(D_{\sim} q^{a}_{\mu}\right) q^{\mu}_{a} \\ &= 0. \end{aligned} \quad -(13)$$

Using Eq. (11) in Eq. (13) gives:

$$D_\nu q^\mu_a = 0. \quad -(14)$$

Therefore Eq. (9) is:

$$\mathcal{L} = c^2 T. \quad -(15)$$

In the rest frame limit this becomes Eq (3) as discussed in Section 1. Therefore the second term in Eq. (9) is needed to give the lagrangian from which the ECE Lemma and wave equation are derived by variational calculus and minimization of action. Eq. (10) can be rewritten in the form {1-38}:

$$\Box q^a_\mu = R q^a_\mu \quad -(16)$$

where R is a scalar curvature defined as follows. Using the Einstein Ansatz {46}:

$$R = -kT \quad -(17)$$

the ECE Lemma becomes the ECE wave equation (1) of Section 1. This is the fundamental and generally covariant wave equation of ECE field theory. All the major wave equations of physics can be derived from Eq. (1) {1-38} in various limits, for example the Dirac equation of special relativistic wave mechanics and the Proca equation of electrodynamics (the d'Alembert equation with photon mass). The ECE Lemma (16) follows {1-38} from the tetrad postulate:

$$D_\mu q^a_\lambda = \partial_\mu q^a_\lambda + \omega^a_{\mu b} q^b_\lambda - \Gamma^\nu_{\mu\lambda} q^a_\nu = 0. \quad -(18)$$

Here $\omega^a_{\mu b}$ is the spin connection and $\Gamma^\nu_{\mu\lambda}$ is the gamma connection {1-40}. Therefore:

$$D^\mu \left(D_\mu q^a_{\ \lambda} \right) = \partial^\mu \left(D_\mu q^a_{\ \lambda} \right) = 0 \quad -(19)$$

i.e.

$$\partial^\mu \left(\partial_\mu q^a_{\ \lambda} + \omega^a_{\ \mu b} q^b_{\ \lambda} - \Gamma^\nu_{\ \mu\lambda} q^a_{\ \nu} \right) = 0 \quad -(20)$$

or

$$\square q^a_{\ \lambda} = \partial^\mu \left(\Gamma^\nu_{\ \mu\lambda} q^a_{\ \nu} \right) - \partial^\mu \left(\omega^a_{\ \mu b} q^b_{\ \lambda} \right). \quad -(21)$$

Now define the scalar curvature:

$$R q^a_{\ \lambda} = \partial^\mu \left(\Gamma^\nu_{\ \mu\lambda} q^a_{\ \nu} - \omega^a_{\ \mu b} q^b_{\ \lambda} \right) \quad -(22)$$

and use Eq. (12) to obtain:

$$R = q^\lambda_{\ a} \partial^\mu \left(\Gamma^\nu_{\ \mu\lambda} q^a_{\ \nu} - \omega^a_{\ \mu b} q^b_{\ \lambda} \right) \quad -(23)$$

and to deduce Eq. (16), Q.E.D. Therefore Eq. (23) is the fundamental definition of scalar curvature in ECE wave mechanics:

$$R = -kT = q^\lambda_{\ a} \partial^\mu \left(\Gamma^\nu_{\ \mu\lambda} q^a_{\ \nu} - \omega^a_{\ \mu b} q^b_{\ \lambda} \right). \quad -(24)$$

This lagrangian derivation of the ECE Lemma and wave equation is fully self-consistent and is based on Hamilton's principle of least action for the particle and Fermat's principle of least time for the wave. Therefore wave and particle are terms which become obsolete: in ECE theory they are both manifestations of spacetime. Wave and particle are simultaneously observable as indicated by recent experiments {42-45}. In the now obsolete Bohr Heisenberg indeterminacy the wave and particle are never simultaneously observable.

The lagrangian formulation of ECE theory is also the lagrangian formalism of Cartan geometry itself. A powerful simplicity of understanding is achieved through the tetrad postulate, which is the fundamental mathematical requirement that a complete vector field V be independent of the system of coordinates used to define it. The key difference between Cartan and Riemann geometry resides in the basis elements used to define the tangent spacetime at a point P in the base manifold {1-40}. In Riemann geometry the basis elements always form the set of partial derivatives. In Cartan geometry the basis elements are more generally defined and labelled by a. The tetrad q^a_μ is the rank two mixed index tensor defined by:

$$V^a = q^a_\mu V^\mu \qquad (25)$$

where the vector elements V^a are defined in the tangent spacetime (Minkowski spacetime) and the vector elements V^μ are defined in the base manifold (ECE spacetime). The tetrad postulate (18) follows from the fact that the complete vector field V in the tangent spacetime must be the same with basis set labelled a and the Riemann basis set of partial derivatives. Eq. (18) is the rule for the covariant differentiation of a mixed index rank two tensor {39}, i.e. D_μ acting on the rank two tensor $q^a_\sim$). Therefore the tetrad is always a rank two tensor with a matrix structure. With these fundamentals clearly defined it is seen that the lagrangian density of ECE theory and Cartan geometry, Eq. (9), contains a product of two tetrad postulates and the ECE Lemma and wave equation are obtained by covariant differentiation of the tetrad postulate. So everything stems from the fact that a complete vector field V is independent of the vector components and basis element used to describe it. For example a vector field V in three dimensional Euclidean geometry may be represented by the cartesian unit vectors, $\underline{i}$, $\underline{j}$ and $\underline{k}$ (the basis elements), and by the cartesian vector components V_x, V_y and V_z:

$$\underline{V} = V_x \underline{i} + V_y \underline{j} + V_z \underline{k} . \quad -(26)$$

The same vector field $\underline{V}$ in spherical polar coordinates will have different components and different basis elements but is the same vector field. If we extend this reasoning to Cartan geometry the tetrad postulate (18) is the inevitable result {1-40}. All of physics stems from this property of vector field V via the ECE field theory. So physics is fundamental geometry.

The well known Cartan torsion is also a direct consequence of the tetrad postulate. This was first demonstrated in vol. 2 of ref. {1} and the proof is as follows. Consider two tetrad postulates:

$$\partial_\mu q^a_\lambda + \omega^a_{\mu b} q^b_\lambda = \Gamma^\nu_{\mu\lambda} q^a_\nu , \quad -(27)$$

$$\partial_\lambda q^a_\mu + \omega^a_{\lambda b} q^b_\mu = \Gamma^\nu_{\lambda\mu} q^a_\nu . \quad -(28)$$

All that has been done is to change the index labeling, so we have written out the tetrad postulate twice. Subtract Eq. (28) from Eq. (27) to obtain the Cartan torsion :

$$T^a_{\mu\lambda} = T^\nu_{\mu\lambda} q^a_\nu = \left(\Gamma^\nu_{\mu\lambda} - \Gamma^\nu_{\lambda\mu}\right) q^a_\nu$$

$$= \partial_\mu q^a_\lambda - \partial_\lambda q^a_\mu + \omega^a_{\mu b} q^b_\lambda - \omega^a_{\lambda b} q^b_\mu . \quad -(29)$$

In differential form notation Eq. (29) is the first Cartan structure equation {39}:

$$T^a_{\mu\nu} = \left(d \wedge q^a\right)_{\mu\nu} + \omega^a_{\mu b} \wedge q^b_\nu . \quad -(30)$$

In the standard notation of Cartan geometry the Greek indices of the base manifold are omitted, because they are always the same on both sides of any equation of Cartan or differential geometry, so Eq. (30) is written conventionally {39} as:

$$T^a = d \wedge q^a + \omega^a{}_b \wedge q^b := D \wedge q^a. \quad -(31)$$

The electromagnetic tensor is then defined directly from the Cartan torsion through the ECE Ansatz {1-38}:

$$F^a = A^{(0)} T^a \quad -(32)$$

where $A^{(0)}$ is a fundamental and universal voltage within a factor of c.

So we have obtained the electromagnetic tensor and the lagrangian density using the same tetrad postulate. The lagrangian density (9) is the same for both the wave and field equations of ECE theory. In the standard model { 47 } the lagrangian formulation is both incomplete and considerably more complicated. The complexity of the standard model is lack of understanding, the converse of the complete and simple ECE theory given here.

The field equations of ECE theory {1-38} are obtained from the first Bianchi identity of Cartan geometry:

$$D \wedge T^a = d \wedge T^a + \omega^a{}_b \wedge T^b = R^a{}_b \wedge q^b \quad -(33)$$

which states that the covariant derivative of the Cartan torsion is identically equal to a cyclic sum of Riemann tensor elements {39, 40}. The first Bianchi identity (33) again follows from the tetrad postulate, as demonstrated in full detail in the appendices of chapter 17 of ref. {1}. Using the Ansatz (32) and the equivalent ansatz:

$$A^a_\mu = A^{(0)} q^a_\mu \quad -(34)$$

in Eq. (33) produces the homogeneous field equation of ECE theory:

$$d \wedge F^a = \mu_0 j^a = A^{(0)} \left(R^a{}_b \wedge q^b - \omega^a{}_b \wedge T^b \right). \quad -(35)$$

The Hodge dual {1-40} of Eq. (35) is the inhomogeneous field equation of ECE theory:

$$d \wedge \tilde{F}^a = \mu_0 J^a = A^{(0)} \left(\tilde{R}^a{}_b \wedge q^b - \omega^a{}_b \wedge \tilde{T}^b \right). \quad -(36)$$

So we have linked all the fundamental equations of ECE theory with a lagrangian formalism based on the minimization of action (Hamilton principle) and time interval (Fermat principle).

Having achieved this unification of basic concepts it is now possible to develop the fundamental equations of the incomplete standard model quantum mechanics into principles of the completed theory sought for by Einstein and Cartan: generally covariant wave mechanics. The minimization of action and time interval are concepts which are central to wave mechanics and wave particle dualism. It is now possible to develop wave particle dualism into the concept of indistinguishability of wave and particle because wave mechanics has been recognized as a property of space-time. The ECE Lemma asserts that scalar curvature itself is quantized, space-time itself is quantized. The wave is the tetrad eigenfunction of the ECE wave equation or Lemma, the particle is also spacetime, always occupying a finite volume. In the rest frame limit of the Dirac equation this volume is given by Eq. (7) In the standard model the concept of point particle is still used, and this concept is in conflict with relativity because it introduces singularities and the complexity of renormalization.

3. PLANCK CONSTANT, PLANCK-EINSTEIN AND DE BROGLIE EQUATIONS, AND THE SCHRÖDINGER EQUATION.

In the rest frame the ECE lagrangian density is the rest energy divided by the rest volume. Therefore the action in the rest frame is:

$$S_0 = \int \frac{mc}{V_0} \, d^4 x_0 . \qquad - (37)$$

The four volume in the rest frame is:

$$d^4 x_0 = V_0 \, c \, dt_0 . \qquad - (38)$$

Now identify the rest action with the Planck constant:

$$S_0 = \hbar \qquad - (39)$$

and the integral over the time interval with the inverse rest frequency:

$$\int dt_0 = \frac{1}{\omega_0} \qquad - (40)$$

to obtain the Planck Einstein equation in the rest frame:

$$E_0 = \hbar \omega_0 . \qquad - (41)$$

If applied to the photon rest mass this is also known {1-38} as the de Broglie equation for photon rest mass. It is known experimentally that Eq. (41) also holds for the photon when it travels for all practical purposes infinitesimally close to the speed of light with respect to an observer in the rest frame. In this case rest frequency is changed to ω . In special relativity this would be a Lorentz transformation of angular frequency but in general relativity a general coordinate transformation. However, the rest mass of the photon cannot be identically zero in ECE theory, because the action would be identically zero. The rest mass of the photon is very small but not zero. In ECE theory the Planck constant is the rest frame limit of the action:

$$S = c\int T\, d^4x. \quad -(42)$$

In the rest frame:

$$\hbar = mc\int \frac{d^4x_0}{V_0} \quad -(43)$$

and if:

$$\int \frac{d^4x_0}{V_0} = \frac{c}{\omega_0} \quad -(44)$$

we obtain:

$$E_0 = \hbar\omega_0 = mc^2. \quad -(45)$$

More generally, and for any particle, the action that gives the ECE wave equation in any frame of reference is given by:

$$S = \frac{1}{c}\int \mathcal{L}\, d^4x \quad -(46)$$

and is the generalization of the Planck constant to any frame of reference. The Planck constant in the rest frame is:

$$S_0 = \hbar = \frac{1}{c}\int \mathcal{L}_0\, d^4x_0 \quad -(47)$$

where V_0 is the rest volume defined by Eq. (7). Therefore ECE theory shows that the Planck constant in the rest frame must have an internal structure defined by the four volume d^4x_0 and rest volume V_0. The same is true in any other frame, the rest volume being replaced by the volume in that frame of reference. If the four volume is:

$$d^4 x_0 = V_0 \, c \, dt_0 \qquad -(48)$$

it is found that

$$\hbar = mc^2 \int \frac{V_0}{V_0} dt_0 = mc^2 t_0 . \qquad -(49)$$

The time interval t_0 must be a constant for a given mass m. The existence of the Planck constant means that a particle is never quite at rest, it must have a rest frequency defined by Eq. (40) So there must be zero point energy defined by Eq. (45). Classically the particle in the rest frame does not move relative to the observer in the same rest frame, and there is no rest energy in a classical theory. The rest energy mc^2 is the result of special relativity theory as is well known. ECE theory gives both the rest energy and the Planck energy $\hbar\omega_0$, showing that it is a unified field theory . The fact that the Planck constant has an internal structure that depends on volume is of key importance in modifying the Heisenberg commutator equation in accordance with the experimental findings {42} of Croca et al. This modification has been initiated in volume 2 of ref. {1}.

The Fermat principle of least time {48} is the classical principle that governs the propagation of light in optics. The path taken by the light through a medium is such that the time of passage is a minimum. The amplitude of a light wave at point P_1 is related to the amplitude at point P_2 by:

$$\phi(P_2) = e^{i\phi} \phi(P_1) \qquad -(50)$$

where the phase ϕ is defined by:

$$\phi = 2\pi \frac{x}{\lambda} . \qquad -(51)$$

Here x is the coordinate and λ the wavelength $\smile$ {48}. Eq. (50) is the fundamental origin of the Schrödinger equation. Light takes paths such that the phase is minimized. This is the precise statement of the Fermat principle. In the limit of geometrical optics ϕ is infinite, the light appears to travel in straight lines. There is no curvature, and this is a "weak field limit" of ECE theory in which the interval t_0 is minimized.

The propagation of particles is given classically by the Hamilton principle of least action. Particles select paths between two points such that the action associated with that path is a minimum. This classical statement is equivalent to Newtonian dynamics in the weak field limit of ECE field theory. Particles adopt a least path and waves a least time. The reason is the same, the phase ϕ is minimized. So particles and waves become indistinguishable if the phase is made proportional to action and this is the fundamental idea of wave mechanics. Thus ϕ is proportional to S and so the constant of proportionality must have the units of inverse action because ϕ is unitless. In the classical limit ϕ is infinite so the constant of proportionality approaches zero. Schrödinger's equation is recovered ~~form~~ to this argument if:

$$\phi = \frac{S}{\hbar}. \quad -(52)$$

Eq. (50) describes a path from P_1 (x_1, t_1) to P_2 (x_2, t_2) {48}. Thus:

$$\phi(x_2, t_2) = e^{iS/\hbar}\phi(x_1, t_1). \quad -(53)$$

Differentiate {48} Eq. (53) with respect to t_2:

$$\frac{\partial}{\partial t_2}\phi(x_2, t_2) = \frac{\partial}{\partial t_2}\left(e^{iS/\hbar}\phi(x_1, t_1)\right). \quad -(53)$$

Now use the Leibnitz Theorem:

$$\frac{\partial}{\partial t_2}\left(e^{iS/\hbar}\phi(x_1,t_1)\right) = \phi(x_1,t_1)\frac{\partial}{\partial t_2}e^{iS/\hbar} + e^{iS/\hbar}\frac{\partial\phi}{\partial t_2}(x_1,t_1). \quad -(54)$$

Since $\phi(x_1,t_1)$ is not a function of t_2:

$$\frac{\partial\phi}{\partial t_2}(x_1,t_1) = 0 \quad -(55)$$

and

$$\frac{\partial}{\partial t_2}e^{iS/\hbar} = \frac{i}{\hbar}\frac{\partial S}{\partial t_2}\cdot e^{iS/\hbar}. \quad -(56)$$

Thus:

$$\frac{\partial}{\partial t_2}\phi(x_2,t_2) = \frac{i}{\hbar}\frac{\partial S}{\partial t_2}e^{iS/\hbar}\phi(x_1,t_1). \quad -(57)$$

Finally use Eq. (53) to obtain the Schrödinger equation in time dependent form { 48 }:

$$\frac{\partial}{\partial t}\phi = \frac{i}{\hbar}\frac{\partial S}{\partial t}\phi. \quad -(58)$$

This is not strictly a wave equation because a wave equation in mathematics contains second derivatives, but it is the famous equation of non-relativistic quantum mechanics. The more familiar form of the Schrödinger equation is obtained by using { 48 }:

$$E = -\frac{\partial S}{\partial t} \quad -(59)$$

where E is the total energy, the sum of kinetic and potential energy. So Eq. (58) becomes:

$$i\hbar\frac{\partial\phi}{\partial t} = E\phi. \quad -(60)$$

Finally define the operator:

$$H = i\hbar \frac{\partial}{\partial t} \quad -(61)$$

to obtain the familiar:

$$H\phi = E\phi. \quad -(62)$$

It is seen that the Schrödinger equation is a causal differential equation and as such cannot be interpreted as an expression of something that is acausal or unknowable. Using the operator (61) for energy it is seen that the Schrödinger equation is mathematically the same as:

$$[H, t]\phi = i\hbar\phi \quad -(63)$$

where the time t multiplies the function ϕ. Eq. (63) is an example of a Heisenberg commutator equation in the non-relativistic quantum limit. There is no more meaning to Eq. (63) than Eq. (62) because Eq. (63) is a restatement of Eq. (62) and thus contains the same mathematical information. This is the causal deterministic view of Einstein, de Broglie, Schrödinger, Bohm, Vigier and followers. The Copenhagen interpretation of Eq. (63) is that if t is known exactly, E is unknowable, and vice versa. This is the view of Bohr, Heisenberg and followers.

This non-relativistic analysis can be extended to ECE theory {1- 48 } by using the equation for the propagation of the tetrad wave function:

$$q^a_\mu(x^\mu) = e^{iS(x^\mu)/\hbar}\, q^a_\mu(0) \quad -(64)$$

where $S(x^\mu)$ is defined by Eqs. (8) and (9).

Differentiate Eq. (64) to obtain:

$$\partial^{\mu} q^{a}_{\mu}(x^{\mu}) = \frac{i}{\hbar} \partial^{\mu} S \, e^{iS/\hbar} \, q^{a}_{\mu}(0). \quad -(65)$$

The second term disappears in analogy with the derivation of Eq. (58) from Eq. (53).

The following definitions are used:

$$\partial^{\mu} := \left(\frac{1}{c}\frac{\partial}{\partial t_1}, \frac{\partial}{\partial X_1}, \frac{\partial}{\partial Y_1}, \frac{\partial}{\partial Z_1} \right), \quad -(66)$$

$$q^{a}_{\mu}(0) := q^{a}_{\mu}(ct_2, X_2, Y_2, Z_2), \quad -(67)$$

and:

$$\partial^{\mu} q^{a}_{\mu}(0) = 0. \quad -(68)$$

Eq. (65) is the generally covariant Schrödinger equation in which the wave function is the tetrad. Now differentiate Eq. (65) once more:

$$\partial_{\mu}(\partial^{\mu} q^{a}_{\mu}) = \frac{i}{\hbar} \partial_{\mu}((\partial^{\mu} S) q^{a}_{\mu}) \quad -(69)$$

to obtain the following generally covariant wave equation:

$$\Box q^{a}_{\mu} = \frac{i}{\hbar}\left(\Box S + \frac{i}{\hbar} \partial^{\mu} S \, \partial_{\mu} S \right) q^{a}_{\mu} = R q^{a}_{\mu}. \quad -(70)$$

The second equality in Eq. (70) follows from the ECE Lemma. Therefore we obtain the following expression for the scalar curvature in terms of the action:

$$R = \frac{i}{\hbar}\left(\Box S + \frac{i}{\hbar} \partial^{\mu} S \, \partial_{\mu} S \right) \quad -(71)$$

If Eq. (39) is not used, i.e. if it is not assumed a priori that S_0 is $\hbar$, then Eq. (6) becomes:

$$q^a_\mu = \exp\left(i\frac{S}{S_0}\right) q^a_\mu(0) \quad -(72)$$

and the scalar curvature can be expressed as:

$$R = \frac{i}{S_0}\left(\Box S + \frac{i}{S_0}\partial^\mu S\, \partial_\mu S\right). \quad -(73)$$

In the limit of the Dirac equation {1-38}:

$$R \rightarrow -\left(\frac{mc}{\hbar}\right)^2 \quad -(74)$$

and we obtain the wave form of the Dirac equation:

$$\left(\Box + \frac{m^2c^2}{\hbar^2}\right) q^a_\mu = 0. \quad -(75)$$

This limit may be used to identify the Planck constant as:

$$\frac{m^2c^2}{\hbar^2} = -\frac{i}{S_0}\left(\Box S + \frac{i}{S_0}\partial^\mu S\, \partial_\mu S\right) \quad -(76)$$

where

$$S_0 \rightarrow \hbar. \quad -(77)$$

The standard model does not consider the general covariance of the Planck constant, because in the standard model quantum mechanics is not unified with general relativity. The Hamilton and Fermat principles are classical, i.e. non-relativistic. In volume 2 of ref. (1) it was suggested that the key quantity to consider is the density of action, not the action itself. So there may be experimentally observable departures from quantum mechanics when general covariance is properly considered. These may show up in hyperfine spectral structure.