

## Notes for Paper 53: Coulomb's Law in ECE Resonant Form

The origin of Coulomb's Law in ECE field theory is the homogeneous field equation:

$$d \wedge \tilde{F} = \mu_0 J = A^{(0)} (\tilde{R} \wedge \tilde{q} - \omega \wedge \tilde{T}) \quad (1)$$

which in tensorial notation becomes:

$$d_{\mu} F^{a\nu} = \mu_0 J^{a\nu} = A^{(0)} (R^a{}_{b\mu\nu} q^b - \omega^a{}_{\mu b} T^{b\nu}) \quad (2)$$

The notation is fully explained in previous notes and publications, in particular appendix K of volume 1.

The Coulomb Law is obtained by using  $v = 0$

$$d_1 F^{a10} + d_2 F^{a20} + d_3 F^{a30} = \mu_0 J^{a0} \quad (3)$$

where:

$$J^{a0} = \frac{A^{(0)}}{\mu_0} \left( R^a{}_{b10} q^b + R^a{}_{b20} q^b + R^a{}_{b30} q^b - \omega^a{}_{1b} T^{b10} - \omega^a{}_{2b} T^{b20} - \omega^a{}_{3b} T^{b30} \right) \quad (4)$$

In vector notation eq. (3) is:

$$\underline{\nabla} \cdot \underline{E}^a = c \underline{A}^{b'} \cdot \underline{R}^a{}_{b'} - \underline{\omega}^{a'b'} \cdot \underline{E}^b$$

2) Conventionally we have defined:

$$A_{\mu}^{a'} := g_{\mu\nu} A^{\nu a} \quad - (6)$$

$$\omega_{\mu b}^{a'} := g_{\mu\nu} \omega^{\nu a b} \quad - (7)$$

i.e. in previous notes we have defined  $A^{\nu a}$  and  $\omega^{\nu a b}$  as metric free. It follows that  $A_{\mu}^{a'}$  and  $\omega_{\mu b}^{a'}$  quantities have to be defined as in eqs. (6) and (7), so we distinguish them with the tick. This is needed because  $g_{\mu\nu}$  in general is no longer a Minkowski metric.

The resonant version of eq. (5) may now be developed as:

$$\underline{\nabla} \cdot \underline{E}^a + \underline{\omega}^{a'b'} \cdot \underline{E}^b = c \underline{A}^{b'} \cdot \underline{R}^a_b \quad - (8)$$

where

$$\underline{E}^a = - \frac{\partial \underline{A}^a}{\partial t} - c \underline{\nabla} \underline{A}^{aa} - c \omega^{aa b} \underline{A}^b + c \omega^{a b} \underline{A}^{ob} \quad - (9)$$

Eqs (8) and (9) give the linear inhomogeneous differential equation

$$\begin{aligned} & c \underline{\nabla} \cdot \underline{\nabla} \underline{A}^{aa} + \underline{\nabla} \cdot \frac{\partial \underline{A}^a}{\partial t} + c \underline{\nabla} \cdot (\omega^{aa b} \underline{A}^b - \omega^{a b} \underline{A}^{ob}) \\ & + \underline{\omega}^{a'b'} \cdot \left( \frac{\partial \underline{A}^b}{\partial t} + c \underline{\nabla} \underline{A}^{ob} + c \omega^{ob c} \underline{A}^c - c \omega^{b c} \underline{A}^{oc} \right) \\ & = c \underline{A}^{b'} \cdot \underline{R}^a_b \quad - (10) \end{aligned}$$

### 3) Static Electric Field

In the standard model a static electric field is defined by:

$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0 \quad - (11)$$

$$\underline{\nabla} \times \underline{E} = \underline{0} \quad - (12)$$

$$\nabla^2 \Phi = -\frac{\rho}{\epsilon_0} \quad - (13)$$

Eq. (11) is Coulomb's Law in the standard model, and eq. (13) is the Poisson equation. Eq. (12) implies that:

$$\underline{E} = -\underline{\nabla} \Phi \quad - (14)$$

The above is part of the Maxwell Heaviside theory of special relativity. In general relativity the Coulomb Law is unified with gravitation (ECE). More generally in the standard model:

$$\underline{E} = -\underline{\nabla} \Phi - \frac{\partial \mathbf{A}}{\partial t} \quad - (15)$$

so a static electric field means that:

$$\frac{\partial \mathbf{A}}{\partial t} = \underline{0} \quad - (16)$$

Therefore to investigate the effect of a static electric field configuration on gravitation eq. (10) can be approximated.

4) If we assume for the sake of approximation that

$$\underline{\nabla} \cdot \underline{A}^b = 0 \quad - (17)$$

the eq. (10) simplifies further to:

$$\begin{aligned} \underline{\nabla} \cdot \underline{\nabla} A^{0a} - \underline{\nabla} \cdot (\underline{\omega}^a_b A^{0b}) \\ + \underline{\omega}^a_{b'} \cdot \underline{\nabla} A^{0b} + \underline{\omega}^a_{b'} \cdot \underline{\omega}^{0b}_c A^c - \underline{\omega}^a_{b'} \cdot \underline{\omega}^b_c A^c \\ = \underline{A}^{b'} \cdot \underline{R}^a_b = c \mu_0 J^{0a} \end{aligned} \quad - (18)$$

### Limit of Weak Interaction

In the limit of weak interaction between e/n and gravitation eq. (1) simplifies to:

$$d \wedge \tilde{F} = \mu_0 J \rightarrow A^{(0)} (\tilde{R} \wedge \tilde{v})_{\text{grav}} \quad - (19)$$

(see appendix of volume 2). This is because:

$$(\tilde{R} \wedge \tilde{v})_{\text{em}} \sim (\omega \wedge \tilde{T})_{\text{em}} \quad - (20)$$

and:  $\tilde{T}_{\text{grav}} \sim 0. \quad - (21)$

The structure of eq (5) simplifies to:

$$\underline{\nabla} \cdot \underline{E}^a \sim c \underline{A}^{b'} \cdot \underline{R}^a_b \quad - (22)$$

and the spin connection in eq. (9) is

5) approximately dual to the tetrad, because it is the spi connection for  $e/m$ , governed by eq. (20).  
 So the resonant equation simplifies to:

$$\underline{\nabla} \cdot \left( -\frac{\partial \underline{A}^a}{\partial t} - c \underline{\nabla} A^{0a} - c \omega^{0a}{}_b \underline{A}^b + c \underline{\omega}^a{}_b A^{0b} \right) \sim c \underline{A}^{b'} \cdot \underline{R}^a{}_b \quad (23)$$

with:  $\omega^a{}_b \sim -\frac{\kappa}{2} \gamma^c \epsilon^a{}_{bc} \quad (24)$

If we use a static electric field and assume eqn (17), eq. (23) simplifies to:

$$\underline{\nabla} \cdot \underline{\nabla} A^{0a} - \underline{\nabla} \cdot (\underline{\omega}^a{}_b A^{0b}) \sim \underline{A}^{b'} \cdot \underline{R}^a{}_b \quad (25)$$

If we do not assume eq. (17):

$$\underline{\nabla} \cdot \underline{\nabla} A^{0a} + \underline{\nabla} \cdot (\omega^{0a}{}_b \underline{A}^b) - \underline{\nabla} \cdot (\underline{\omega}^a{}_b A^{0b}) \sim \underline{A}^{b'} \cdot \underline{R}^a{}_b \quad (26)$$

Eq. (25) is a Horowitz (aw type eqn. with driving force, eqn. (26) has a damping term. In soft cases resonance occurs, i.e. gravitation is resonantly affected by an electric field.