1)

Notes for Paper 53

The Lorentz Force Equation

Standard Model (Jackson Third Edition)

$$\frac{dp^\mu}{d\tau} = m\frac{dU^\alpha}{d\tau} = \frac{e}{c} F^{\mu\nu} U_\nu \quad -(1)$$

(eq. 11.144 p 557 of Jackson).

Here τ is the proper time as seen in the rest frame:

$$\tau = \left(1 - \frac{v^2}{c^2}\right)^{1/2} t = t/\gamma \quad -(2)$$

and

$$U^\mu = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} (c, \underline{u}). \quad -(3)$$

$$:= \gamma V^\mu$$

So eqn. (1) is:

$$\frac{dp^\mu}{dt} = \frac{e}{c} F^{\mu\nu} V_\nu \quad -(4)$$

where t is the time in the laboratory frame and

$$p^\mu = \gamma m V^\mu \quad -(5)$$

is the relativistic momentum. Here:

$$F^{\mu\nu} = \begin{bmatrix} 0 & -E^1 & -E^2 & -E^3 \\ E^1 & 0 & -cB^3 & cB^2 \\ E^2 & cB^3 & 0 & -cB^1 \\ E^3 & -cB^2 & cB^1 & 0 \end{bmatrix} \quad -(6)$$

and $E^1 = E_x$ etc. So:

$$\frac{dp^\mu}{dt} = \frac{e}{c}\begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -cB_z & cB_y \\ E_y & cB_z & 0 & -cB_x \\ E_z & -cB_y & cB_x & 0 \end{bmatrix}\begin{bmatrix} c \\ -v_x \\ -v_y \\ -v_z \end{bmatrix} \quad -(7)$$

The space part of eq. (7) is:

$$\boxed{\begin{aligned} \frac{d\underline{p}}{dt} &= e\left(\underline{E} + \underline{v} \times \underline{B}\right), && -(8) \\ \underline{p} &= \gamma m \underline{v} && -(9) \end{aligned}}$$

The time part of eq (7) is:

$$\boxed{\frac{dp^0}{dt} = eE = \int \underline{J} \cdot \underline{E}\, dV} \quad -(10)$$

where:

$$p^\mu = \left(\frac{En}{c}, \underline{p}\right). \quad -(11)$$

Eq. (9) is the rate of energy change.

ECE Theory

The electromagnetic field tensor is the

3) Cartan Torsion :

$$F^{a\mu\nu} \text{ (eq. (6))} = cA^{(0)} T^{a\mu\nu} \quad -(12)$$

for each polarization index a. Therefore eq (1) generalizes to:

$$\frac{dp^{a\mu}}{d\tau} = e A^{(0)} T^{a\mu\nu} U_\nu \quad -(13)$$

and eq. (4) to:

$$\boxed{\frac{dp^{a\mu}}{dt} = e A^{(0)} T^{a\mu\nu} V_\nu} \quad -(14)$$

where :

$$T^a = d \wedge q^a + \omega^a{}_b \wedge q^b \quad -(15)$$

$$R^a{}_b = d \wedge \omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b \quad -(16)$$

$$\Box q^a = R q^a. \quad -(17)$$

The torsion tensor is generally covariant, and the most rigorous form of the Lorentz force law in ECE is given by the transformation properties of $T^{\mu\nu}$. These will be given in the next note.