1)

FOURTH RESONANCE EQUATION

(52nd Paper, Notes 8)

This is obtained from:

$$\underline{\nabla} \times \underline{B}^a - \frac{1}{c^2}\frac{\partial \underline{E}^a}{\partial t} = \frac{\mu_0}{c}\underline{\tilde{J}}^a \quad -(1)$$

and is:

$$\boxed{\frac{1}{c^2}\frac{\partial^2 \underline{A}^a}{\partial t^2} + \frac{1}{c}\frac{\partial}{\partial t}\left(\underline{\nabla} A^{0a} + A^{b0}\underline{\omega}^a{}_b - \omega^{a0}{}_b\underline{A}^b\right) + \underline{\nabla} \times \left(\underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b\right) = \frac{\mu_0}{c}\underline{\tilde{J}}^a} \quad -(2)$$

We wish to reduce this to the linear inhomogeneous structure:

$$\boxed{\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = A\cos\omega t} \quad -(3)$$

Approximations

First use: $\omega^a_{\mu b} = -\kappa\,\epsilon^a{}_{bc}\,q^c_\mu \quad -(4)$

so:

$$A^{b0}\underline{\omega}^a{}_b = \omega^{a0}{}_b\underline{A}^b \quad -(5)$$

$$\underline{\omega}^a{}_b \times \underline{A}^b = \underline{0} \quad -(6)$$

Then use:

$$\nabla^2 \underline{A}^a = -\frac{\omega_0^2}{c^2}\underline{A}^a \quad -(7)$$

$$\underline{\nabla} \cdot \underline{A}^a = 0 \quad -(8)$$

$$\frac{\partial A^{0a}}{\partial t} = 0 \quad -(9)$$

with:

$$\nabla \times (\nabla \times A) = -\nabla^2 A + \underline{\nabla}(\underline{\nabla} \cdot A) \quad -(9a)$$

2) Eqn. (2) then simplifies to:

$$\boxed{\frac{1}{c^2}\frac{\partial^2 \underline{A}^a}{\partial t^2} + \frac{\omega_0^2}{c^2}\underline{A}^a = \frac{\mu_0}{c}\underline{\tilde{J}}^a} \quad -(10)$$

This is an undamped, driven oscillator, it has the structure of eqn. (3) with $\beta = 0$. From eqns. (7) and (8):

$$\underline{A}^a = \frac{A^{(0)}}{\sqrt{2}}(\underline{i} - i\underline{j})e^{-i\omega_0 Z/c} \quad -(11)$$

is a possible solution. From the well known analytical solution of eqn (3) (Bernoulli's and Euler, 1739 – 1743):

$$A^{(0)} = A_c^{(0)} + A_p^{(0)} \quad -(12)$$

where:

$$A_c^{(0)} = A_1 e^{i\omega_0 t} + A_2 e^{-i\omega_0 t} \quad -(13)$$

$$A_p^{(0)} = D \quad -(14)$$

assuming

$$\frac{\mu_0}{c}\underline{\tilde{J}}^a = A^a(\underline{i} - i\underline{j})\cos\omega t. \quad -(15)$$

<u>Resonance</u> occurs at:

$$\boxed{\omega_R = \omega_0} \quad -(16)$$

with:

$$\delta = 0, \quad Q \to \infty \quad -(17)$$

and

$$\boxed{D \to \infty} \quad -(18)$$

Thus,

$$\boxed{\underline{\tilde{J}}^a \to \infty} \quad -(19)$$