

331(7): Splittings of the $n \rightarrow n+1$ Series

As shown in note 306(b) the Zeeman lines of each transition are $n^2 + 1$ degenerate in general. However for absorption there is the further restriction of:

$$\Delta l = 1 \quad \text{--- (1)}$$

In addition to: $\Delta m_l = 0, \pm 1 \quad \text{--- (2)}$

Examples of such lines for atomic H are:

- 1) $n=2$ to $n=3$, the H α line at $15,241.4 \text{ cm}^{-1}$ (visible)
- 2) $n=3$ to $n=4$, Infra-red, $5,354.4 \text{ cm}^{-1}$
- 3) $n=4$ to $n=5$, Infra-red, $2,469.1 \text{ cm}^{-1}$
- 4) $n=13$ to $n=14$, Far infra-red, 81.52 cm^{-1}
- 5) $n=50$ to $n=51$, Microwave 1.704 cm^{-1}

It is also possible that $n=2$ goes to $n=4, 5, 6, \dots$ etc. This builds up the well known Lyman, Balmer, Paschen, Pfund series of atomic absorptions.

For $n=2$ to $n=3$ The possible transitions for left CP polarization:

$$\Delta m_l = 1 \quad \text{--- (3)}$$

are the five degenerate transitions:

- a) $2s \rightarrow 2p$ ($n=2, l=0, m=0$ to $n=3, l=1, m=1$)
- b) $2p \rightarrow 3s$ ($n=2, l=1, m=-1$ to $n=3, l=0, m=0$)
- * c) $2p \rightarrow 3d$ ($n=2, l=1, m=-1$ to $n=3, l=2, m=0$)
- * d) $2p \rightarrow 3d$ ($n=2, l=1, m=0$ to $n=3, l=2, m=1$)
- * e) $2p \rightarrow 3d$ ($n=2, l=1, m=1$ to $n=3, l=2, m=2$)

For absorption: $\Delta l = 1 - (4)$

so there are three degenerate lines marked with an asterisk. These are split into three lines by the new relativistic effect, as discussed in note 331(5). Similar results are obtained for:

$$\Delta m = 0 \text{ (linear polarization)} - (5)$$

and right CP polarization:

$$\Delta m = -1 - (6)$$

The new energy levels are defined by:

$$E = -\frac{me^4}{32\pi^2 \epsilon_0^2 \hbar^2 n^2} - \frac{e\hbar}{2m} B_z \left(1 + \frac{2.66567 \times 10^{-5}}{2} \right) - (7)$$

Now consider the $n=4$ to $n=5$ infra-red line at 2469.1 cm^{-1} . We have transitions as in Note 306(b) fm:

$$4s \quad n=4, l=0, m=0$$

$$4p \quad n=4, l=1, m=-1, 0, 1$$

$$4d \quad n=4, l=2, m=-2, -1, 0, 1, 2$$

$$4f \quad n=4, l=3, m=-3, -2, -1, 0, 1, 2$$

to $5s \quad n=5, l=0, m=0$

$$5p \quad n=5, l=1, m=-1, 0, 1$$

$$5d \quad n=5, l=2, m=-2, -1, 0, 1, 2$$

$$5f \quad n=5, l=3, m=-3, -2, -1, 0, 1, 2, 3$$

$$5g \quad n=5, l=4, m=-4, -3, -2, -1, 0, 1, 2, 3, 4$$

3) For

$$\Delta n = 1 - (8).$$

There are seventeen degenerate transitions:

- 1) $4s \rightarrow 5p$ ($n=4, l=0, m=0 \rightarrow n=5, l=1, m=1$)
- 2) $4p \rightarrow 5s$ ($n=4, l=1, m=-1 \rightarrow n=5, l=0, m=0$)
- 3) $4p \rightarrow 5d$ ($n=4, l=1, m=-1 \rightarrow n=5, l=2, m=0$)
- * 4) $4p \rightarrow 5d$ ($\quad \quad \quad 0 \quad \quad \quad \quad \quad 1$)
- * 5) $4p \rightarrow 5d$ ($\quad \quad \quad 1 \quad \quad \quad \quad \quad 2$)
- * 6) $4d \rightarrow 5f$ ($n=4, l=2, m=-2$ to $n=5, l=3, m=-1$)
- * 7) $\quad \quad \quad \quad \quad \quad \quad -1 \quad \quad \quad \quad \quad 0$
- * 8) $\quad \quad \quad \quad \quad \quad \quad 0 \quad \quad \quad \quad \quad 1$
- * 9) $\quad \quad \quad \quad \quad \quad \quad 1 \quad \quad \quad \quad \quad 2$
- * 10) $\quad \quad \quad \quad \quad \quad \quad 2 \quad \quad \quad \quad \quad 3$
- * 11) $4f \rightarrow 5g$ ($n=4, l=3, m=-3$ to $n=5, l=4, m=-2$)
- * 12) $\quad \quad \quad \quad \quad \quad \quad -2 \quad \quad \quad \quad \quad -1$
- * 13) $\quad \quad \quad \quad \quad \quad \quad -1 \quad \quad \quad \quad \quad 0$
- * 14) $\quad \quad \quad \quad \quad \quad \quad 0 \quad \quad \quad \quad \quad 1$
- * 15) $\quad \quad \quad \quad \quad \quad \quad 1 \quad \quad \quad \quad \quad 2$
- * 16) $\quad \quad \quad \quad \quad \quad \quad 2 \quad \quad \quad \quad \quad 3$
- * 17) $\quad \quad \quad \quad \quad \quad \quad 3 \quad \quad \quad \quad \quad 4$

For

$$\Delta l = 1 - (9)$$

There are fourteen degenerate transitions marked with an asterisk.

So eq. (7) splits these into fourteen different lines.

4) Similar results are obtained for :

$$\Delta n = 0, \Delta n = -1 \quad - (10)$$

collect up forty five lines in the relativistic spectrum.

In general there $3(n^2 - 1)$ lines in the relativistic spectrum, occurring in three groups, one symmetric ($\Delta n = 0$) and two asymmetric mirror image groups ($\Delta n = \pm 1$). The table gives the number of lines:

Transition	Wavenumber of Main Line	Number of Lines
$n=2$ to $n=3$	15,241.4	9
$n=3$ to $n=4$	5,334.1	24
$n=4$ to $n=5$	2,469.1	45
$n=13$ to $n=14$	81.52	804
$n=50$ to $n=51$	1.704	7497