

217(4): Inter-relations Between Constants for all x .

Eisner Theory

$$\theta = \frac{1}{x} \cos^{-1} \left[\left[1 - \left(\frac{d}{x\epsilon} \right)^2 \left(\frac{1}{b^2} - \left(1 - \frac{r_0}{r} \right) \left(\frac{1}{a^2} + \frac{1}{r^2} \right) \right) \right]^{1/2} \right] - (1)$$

$$a = \frac{L}{mc}, \quad b = \frac{Lc}{E} - (2)$$

$$r_0 = 2mb/c^2 - (3)$$

Orbital Section

$$\theta = \frac{1}{x} \cos^{-1} \left(\frac{1}{\epsilon} \left(\frac{d}{r} - 1 \right) \right) - (4)$$

The constants d and ϵ must be those for the precessing ellipse. These must be calculated from the Lagrangian:

$$L = T - U - (5)$$

where $T = \frac{1}{2} m v^2 = \frac{1}{2} m (\dot{r}^2 + \dot{\theta}^2 r^2) - (6)$

$$U = -\frac{kx^2}{r} - \frac{k(1-x^2)d}{2r^2} - (7)$$

The Lagrangian theory can be summarized as:

$$\frac{d^2}{d\theta^2} \left(\frac{1}{r} \right) + \frac{1}{r} = -\frac{mr^2}{L} F(r) - (8)$$

where: $\frac{1}{r} = \frac{1}{d} (1 + \epsilon \cos(x\theta)) - (9)$

2) In eq. (8):

$$L = m r^2 \dot{\theta} \quad - (10)$$

= constant of motion.

Eqs. (8) and (9) give:

$$F(r) = - \frac{k}{r^2} \left(x^2 + (1-x^2) \frac{d}{r} \right), \quad - (11)$$

$$u(r) = - \partial F(r) / \partial r. \quad - (12)$$

Therefore:

$$\mathcal{L} = \frac{1}{2} m \dot{r}^2 + \frac{L^2}{2mr^2} + \frac{x^2 k}{r} + \frac{(1-x^2) d k}{2r^2} \quad - (13)$$

$$= \frac{1}{2} m \dot{r}^2 + \frac{L_1^2}{2mr^2} + \frac{k_1}{r}$$

where:

$$k_1 = x^2 k \quad - (14)$$

$$L_1^2 = L^2 + m k d (1-x^2) \quad - (15)$$

The Hamiltonian corresponding to eq. (13) is:

$$H = \frac{1}{2} m \dot{r}^2 + \frac{L^2}{2mr^2} - \frac{x^2 k}{r} - \frac{(1-x^2) d k}{2r^2} \quad - (16)$$

and this is calculated for eq. (9). Therefore
 the Hamiltonian (16) gives the orbit (9). Therefore
 Hamiltonian is the total energy E:

$$H = E - (17)$$

and E is a constant of motion. The Newtonian theory is:

$$E = \frac{1}{2} m \dot{r}^2 + \frac{L^2}{2mr^2} - \frac{k}{r} \quad - (17)$$

The structure of eq. (16) is:

$$E = \frac{1}{2} m \dot{r}^2 + \frac{L_1^2}{2mr^2} - \frac{k_1}{r} \quad - (18)$$

where $k_1 = \alpha^2 k, \quad - (19)$

$$L_1^2 = L^2 - m k d (1 - \alpha^2). \quad - (20)$$

It follows from eq. (17) that:

$$r = \frac{d}{1 + \epsilon \cos \theta} \quad - (21)$$

where $d = \frac{L^2}{mk}, \quad \epsilon = \left(1 + \frac{2EL^2}{mk^2} \right)^{1/2} \quad - (22)$

i.e.

$$\cos \theta = \frac{\frac{L^2}{mk r} - 1}{\left(1 + \frac{2EL^2}{mk^2} \right)^{1/2}} \quad - (23)$$

and

$$\theta(r) = \int \frac{L}{r^2} \left(2m \left(E + \frac{k}{r} - \frac{L^2}{2mr^2} \right) \right)^{-1/2} dr \quad - (24)$$

4) Eq. (24) can be from:

$$\frac{d\theta}{dr} = \frac{dr}{dt} \frac{dt}{d\theta}, \quad - (25)$$

$$\frac{d\theta}{dt} = \frac{L}{mr^2} \quad - (26)$$

$$\frac{dr}{dt} = \left(\frac{2}{r} (E - u) - \frac{L^2}{m^2 r^3} \right)^{1/2} \quad - (27)$$

$$u = -\frac{k}{r} \quad - (28)$$

Eq. (18) has the same mathematical structure as
 Eq. (17) with:
 $L \rightarrow L_1, \quad k \rightarrow k_1, \quad - (29)$
 so eqs. (24) to (28) will also have the same
 structure. So:

$$\theta_1(r) = \int \frac{L_1}{r^2} \left(2m \left(E + \frac{k_1}{r} - \frac{L_1^2}{2mr^3} \right) \right)^{-1/2} dr \quad - (30)$$

and

$$\theta_1 = \int \frac{x L_1}{r^2} \left(2m \left(E + \frac{k_1}{r} - \frac{L_1^2}{2mr^3} \right) \right)^{-1/2} dr \quad - (31)$$

Now rewrite eq. (31) in exactly the

5) Eq. (31) has the same structure as eq. (24), so:

$$\cos \theta_1 = \frac{1}{\epsilon_1} \left(\frac{d_1}{r} - 1 \right) \quad - (32)$$

i.e.
$$r = \frac{d_1}{1 + \epsilon_1 \cos \theta_1} \quad - (32)$$

However, it is known that θ_1 result comes from eqs. (8) and (9), so:

$$\theta_1 = x\theta \quad - (33)$$

and in eq. (9):

$$d \rightarrow d_1 \quad - (34)$$

$$\epsilon \rightarrow \epsilon_1 \quad - (35)$$

so

$$\boxed{r = \frac{d_1}{1 + \epsilon_1 \cos(x\theta)}} \quad - (36)$$

where:

$$d_1 = \frac{L_1^2}{x^2 m^2 M G} = \frac{L_1^2}{m k_1}, \quad - (37)$$

$$\epsilon_1 = \left(1 + \frac{2EL_1^2}{m k_1^2} \right)^{1/2}, \quad - (38)$$

$$L_1^2 = L^2 - m k d (1 - x^2) \quad - (39)$$

$$= x^2 L^2$$

6) Final result:

$$d_1 = d, \quad \epsilon_1 = \epsilon \quad - (40)$$

$$\text{so} \quad r = \frac{d}{1 + \epsilon \cos(x\theta)} \quad - (41)$$

is given by eq (7), QED. This is
a self-consistent result, so the mathematics
are correct.

COMPARISON

1) Re Covert Conical Section

$$\theta = \frac{1}{2\pi} \cos^{-1} \left(\frac{1}{\epsilon} \left(\frac{d}{r} - 1 \right) \right) \quad - (42)$$

$$d = \frac{L^2}{m^2 M G} ; \quad \epsilon = \left(1 + \frac{2EL^2}{m^3 M^2 G^2} \right)^{1/2} \quad - (43)$$

m = mass of planet

M = mass of sun

G = Newton constant

E = constant total energy

L = constant total angular momentum

7) 2) Einsteinian General Relativity (EGR)

$$\theta = \frac{1}{x} \cos^{-1} \left[\left[1 - \left(\frac{d}{xf} \right)^2 \left(\frac{1}{b^2} - \left(1 - \frac{r_0}{r} \right) \left(\frac{1}{a^2} + \frac{1}{r^2} \right) \right) \right]^{1/2} \right]$$

$$a = \frac{L}{mc}, \quad b = \frac{Lc}{E}, \quad r_0 = \frac{2MG}{c^2}$$

— (44)

Valid for all x .

IT IS OBVIOUS THAT EGR IS
WILDLY INCORRECT. WE NOW
 KNOW THAT EQ. (42) HAS A
VAST RANGE OF POSSIBLE RESULTS