

206(10): General Metric Definition

In general: $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$ — (1)

Consider the case of cylindrical polar coordinates in the plane

$$dz^2 = 0, \quad \text{--- (2)}$$

i.e. $ds^2 = c^2 dt^2 - dr^2 - r^2 d\theta^2$, — (3)

and $g_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -r^2 \end{bmatrix}$ — (4)

then $\left. \begin{aligned} g_{00} &= 1, & g_{11} &= -1, & g_{22} &= -r^2 \\ dx^0 &= c dt, & dx^1 &= dr, & dx^2 &= d\theta \end{aligned} \right\}$ — (5)

By defining:

$$f = \frac{dr}{d\theta} \quad \text{--- (6)}$$

it is found that the constrained ds^2 is:

$$ds^2 = c^2 dt^2 - \left(1 + \frac{r^2}{f^2}\right) dr^2 \quad \text{--- (7)}$$

and $g_{\mu\nu} = \begin{bmatrix} 1 & 0 \\ 0 & -\left(1 + \frac{r^2}{f^2}\right) \end{bmatrix}$ — (8)

so $\left. \begin{aligned} g_{00} &= 1, & g_{11} &= -\left(1 + \frac{r^2}{f^2}\right) \end{aligned} \right\}$ — (9)

$$dx^0 = c dt, \quad dx^1 = dr$$