

# 197(1) : Force Law for Any Orbit.

From UFT 196 the force law for any orbit is:

$$-\frac{m F(r) r^2}{L^2} = \left( \frac{1}{1 + \omega r t_f} \right) \left( \frac{d^2}{d\theta^2} \left( \frac{1}{r} \right) + \frac{1}{r} \right) + \frac{1}{2} \omega r t_f \left( \frac{d\omega}{dr} \right) \left( \frac{1}{(1 + \omega r t_f)^2} \right) \left( \frac{1}{r^2} + \left( \frac{d}{d\theta} \left( \frac{1}{r} \right) \right)^2 \right) \quad - (1)$$

in the notation of that paper.

In order to make a first investigation of this equation assume that:

$$\omega = -\frac{1}{r} \quad - (2)$$

as in some previous papers of the UFT series. Then

$$\frac{d\omega}{dr} = \frac{1}{r^2} \quad - (3)$$

and:

$$-\frac{m F(r) r^2}{L^2} = \left( 1 - \frac{c t_f}{r} \right)^{-1} \left( \frac{d^2}{d\theta^2} \left( \frac{1}{r} \right) + \frac{1}{r} \right) + \frac{1}{2} c t_f \left( 1 - \frac{c t_f}{r} \right)^{-2} \left( \frac{1}{r^2} + \left( \frac{d}{d\theta} \left( \frac{1}{r} \right) \right)^2 \right) \quad - (4)$$

If it is assumed further that:

$$c t_f / r \ll 1 \quad - (5)$$

and that the second term in eq. (4) is much smaller than the first term, then:

$$2) -m \frac{F(r)}{L^2} r^2 \left(1 - \frac{ct_g}{r}\right) \sim \frac{d^2}{d\theta^2} \left(\frac{1}{r}\right) + \frac{1}{r} \quad - (6)$$

For a precessing ellipse:

$$\frac{1}{r} = \frac{1}{d} (1 + \epsilon \cos(x\theta)) \quad - (7)$$

$$\text{so } \frac{d}{d\theta} \left(\frac{1}{r}\right) = -\frac{x\epsilon}{d} \sin(x\theta) \quad - (8)$$

$$\frac{d^2}{d\theta^2} \left(\frac{1}{r}\right) = -\frac{x^2 \epsilon}{d} \cos(x\theta) \quad - (9)$$

Resubst:

$$-m \frac{F(r)}{L^2} r^2 \left(1 - \frac{ct_g}{r}\right) = -\frac{x^2 \epsilon}{d} \cos(x\theta) + \frac{1}{d} + \frac{\epsilon \cos(x\theta)}{d} \quad - (10)$$

$$\text{so: } F(r) = \left(1 - \frac{ct_g}{r}\right)^{-1} \left[ -\frac{m M_b x^2}{r^2} + \frac{(x^2 - 1)L^2}{m r^3} \right] \quad - (11)$$

$$d = \frac{L^2}{m^2 M_b} \quad - (12)$$

using

It can be seen that the force law is eq. (11) is the same as that found in previous notes, but

it is modified by  $\left(1 - \frac{ct_g}{r}\right)^{-1}$ .

A more complete treatment requires the complete  
eq. (4). If:

$$ct_g / r \ll 1 \quad - (13)$$

then:

$$F(r) \sim \left(1 + \frac{ct_g}{r}\right) \left[ -\frac{m M_1 G x^2}{r^2} + \frac{(x^2 - 1) L^2}{m r^3} \right] - (14)$$

so it becomes a sum of inverse square, cube and first  
power terms.

As far as is known at present this is the  
only correct relativistic description of a precessing  
elliptical orbit. It depends on a characteristic  
time  $t_g$  which does not appear in the ordinary  
theory of general relativity.

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