Notes 124(1): Spiral Galaxy

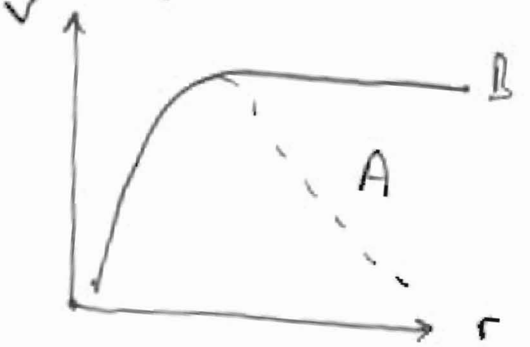
The basic equation is:

$$\underline{\nabla} \cdot \underline{g} = 4\pi G (R - \omega T) \quad -(1)$$

where:

$$\boxed{\underline{g} = ck\, \underline{J}} \quad -(2)$$

where $\underline{J}$ is an angular momentum. The velocity curve of the galaxy is:



The Newtonian region A is described by:

$$\underline{\nabla} \cdot \underline{g} = 4\pi G R = 4\pi G \rho \quad -(3)$$

i.e.

$$\boxed{\underline{\nabla} \cdot \underline{J} = 4\pi ck G R} \quad -(4)$$

In order to describe the region B, we use

$$R = \omega T \quad -(5)$$

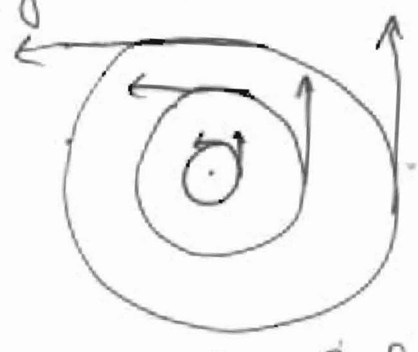
so

$$\boxed{\underline{\nabla} \cdot \underline{g} = \underline{\nabla} \cdot \underline{J} = 0} \quad -(6)$$

Therefore:

$$\underline{J} = -J_y \underline{i} + J_x \underline{j} \quad -(7)$$

and look as follows:



This is beginning to look like a spiral galaxy
a whirlpool.

If the angular momentum is constant:

$$|\underline{J}| = (J_x^2 + J_y^2)^{1/2} \quad -(8)$$

$$= \text{constant},$$

i.e

$$J = mrv = mr^2\omega \quad -(9)$$

$$= \text{constant}$$

and

$$v = \omega r \quad -(10)$$

where

$$\omega = \frac{d\theta}{dt} \quad -(11)$$

If

$$\omega r = \text{constant} \quad -(12)$$

then

$$\boxed{v = \text{constant}} \quad -(13)$$

as in region B.

We have:

$$J = mrv \quad -(14)$$

and if v is constant, J is proportional to r.
The angular velocity is:

$$\frac{d\theta}{dt} = \frac{mv}{r} = \frac{\text{constant}}{r} \quad -(15)$$

So:

$$\theta = \frac{\text{constant}}{r}\int_0^{\tau} dt \quad -(16)$$

$$\boxed{\theta = \frac{\text{constant} \cdot \tau}{r}} \quad -(17)$$

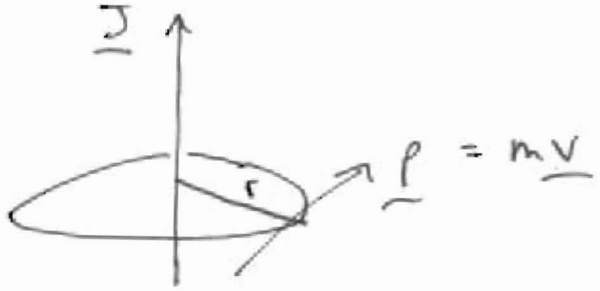
which is a spiral as in paper 76.

124(2): Angular Momentum as a Constant of Motion

Consider a particle of mass m moving in a central force field described by the potential U. If the latter depends only on the distance of the particle from the force centre and not on orientation, the angular momentum of the system is conserved:

$$\underline{J} = \underline{r} \times \underline{p} = \text{constant} \qquad -(1)$$

(J.B. Marion and S.T. Thornton, "Classical Dynamics", HCB 1988, 3rd ed., page 246).



The Lagrangian analysis of this problem defines the lagrangian in plane polar coordinates r and θ as follows:

$$\mathcal{L} = \frac{1}{2} m \left(\dot{r}^2 + r^2 \dot{\theta}^2 \right) - U(r) \qquad -(2)$$

The angular momentum conjugate to the coordinate θ is conserved:

$$\dot{p}_\theta = \frac{\partial \mathcal{L}}{\partial \theta} = 0 = \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\theta}} \qquad -(3)$$

i.e.

$$J = p_\theta = \frac{\partial \mathcal{L}}{\partial \dot{\theta}} = m r^2 \dot{\theta} = \text{constant} \qquad -(4)$$

The angular velocity is defined by:

$$\omega = \dot{\theta} = \frac{d\theta}{dt} \qquad -(5)$$

so

$$\boxed{J = m r^2 \omega = m r v = \text{constant}} \qquad -(6)$$

The angular momentum J is the first integral of the motion. The infinitesimal area dA is:

$$dA = \frac{1}{2} r^2 d\theta \quad -(7)$$

and the areal velocity is:

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \frac{1}{2} r^2 \omega \quad -(8)$$

$$\boxed{\frac{dA}{dt} = \frac{J}{2m} = \text{constant}} \quad -(9)$$

This is Kepler's second law of planetary motion. This is true for all central-force motion, not just for an inverse square law.

If the plane of, for example, a galaxy is in X - Y, the angular momentum is:

$$\underline{J} = m r^2 \omega \, \underline{k} \quad -(10)$$

where:

$$\underset{\sim}{r} = r_X \underline{i} + r_Y \underline{j}, \quad -(11)$$

$$r = |\underline{r}| = \left(r_X^2 + r_Y^2\right)^{1/2} \quad -(12)$$

so

$$\underline{\nabla} \cdot \underline{J} = \frac{\partial J}{\partial Z} = 0 \quad -(13)$$

and

$$\underline{p} = p_X \underline{i} + p_Y \underline{k}. \quad -(14)$$

The ECE equation of motion is:

$$\underline{\nabla} \cdot \underline{g} = c^2 (R - \omega T) \quad -(15)$$

3)

where:

$$\underline{g} = c^2 \left(T^{010} \underline{i} + T^{020} \underline{j} + T^{030} \underline{k} \right). \quad -(16)$$

Now integrate over volume:

$$T^{i0} = \int T^{0i0} \, dV = V T^{0i0} \quad -(17)$$

$i = 1, 2, 3.$

and define the angular momentum tensor:

$$J^{i0} = \frac{c}{k} T^{i0} \quad -(18)$$

Therefore:

$$\boxed{\underline{g} = \frac{ck}{V} \underline{J}} \quad -(19)$$

Units Check

$g = m s^{-2}$; $c = m s^{-1}$, $k = m\, kgm^{-1}$, ✓✓
$J = kgm\, m^2 s^{-1}$, $V = m^3$

Therefore:

$$\underline{\nabla} \cdot \underline{g} = \frac{ck}{V} \underline{\nabla} \cdot \underline{J} = 4\pi G \rho \quad -(20)$$

where: $$G = \frac{c^2 k}{8\pi} \quad -(21)$$

Therefore

$$\boxed{\underline{\nabla} \cdot \underline{J} = \rho = \frac{1}{2} V c \rho} \quad -(22)$$

4) and:

$$\boxed{\underline{\nabla} \cdot \underline{J} = \frac{V c}{k} \left(R - \omega T \right)} \quad -(23)$$

If it assumed that the mass density is

$$\rho = \frac{m}{V} \quad -(24)$$

then:

$$\boxed{\underline{\nabla} \cdot \underline{J} = \frac{1}{2} m c} \quad -(25)$$

Denote:

$$V \left(R - \omega T \right) := \left(R - \omega T \right)' \quad -(26)$$

then:

$$\underline{\nabla} \cdot \underline{J} = \frac{c}{k} \left(R - \omega T \right)' \quad -(27)$$

$$= \frac{1}{2} m c$$

and

$$\boxed{m = \frac{2}{k} \left(R - \omega T \right)'} \quad -(28)$$

Therefore mass is proportional to $(R - \omega T)'$. The fundamental dynamics of the Newton and Coulomb laws are given by eq. (25), where mass m acts as a source to the field $\underline{J}$.

5) Central Motion

In central motion:

$$\underline{\nabla} \cdot \underline{J} = 0 \quad -(29)$$

and so:

$$\boxed{R = \omega T} \quad -(30)$$

is a general law of central motion. Therefore Kepler's second law is eq. (30). Eq. (30) means that:

$$m = 0 \quad -(31)$$

meaning that there is no mass outside the plane of the orbit.

Kepler's First and Third Laws

These depend specifically on the inverse square law of Newton. If Newtonian dynamics are defined by:

$$\omega \to 0 \quad -(32)$$

(spin connection goes to zero), then the first and third laws are given by:

$$\underline{\nabla} \cdot \underline{J} = \frac{c}{k} R, \quad -(33)$$

$$R^{1} = R^{10} + R^{20} + R^{30} \quad -(34)$$

124(3): Relation between Torsion and Spin Connection for General Central Orbits.

The general relation between force laws and orbit is given from a Lagrangian analysis by eq. (7.20) of Marion and Thornton. This can be expressed as:

$$F(r) = -mv^2 \left(\frac{d^2}{d\theta^2} \left(\frac{1}{r} \right) + \frac{1}{r} \right) \quad -(1)$$

where m is the mass of the attracting particle. The potential energy is: -(2)

$$U(r) = -\int F\,dr = m \int v^2 \left(\frac{d^2}{d\theta^2} \left(\frac{1}{r} \right) + \frac{1}{r} \right) dr$$

and the gravitational potential is:

$$\Phi = \frac{1}{m} U(r) = \int v^2 \left(\frac{d^2}{d\theta^2} \left(\frac{1}{r} \right) + \frac{1}{r} \right) dr \quad -(3)$$

Marion and Thornton write eq. (1) as:

$$\frac{d^2}{d\theta^2} \left(\frac{1}{r} \right) + \frac{1}{r} = -\frac{mr^2}{J^2} F(r) \quad -(4)$$

where

$$J = mr^2 \omega = mr^2 \dot{\theta} = \text{constant.} \quad -(5)$$

Therefore:

$$F(r) = -r^2\omega^2 m\left(\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right)+\frac{1}{r}\right). \quad -(6)$$

Define:

$$\boxed{v = r\omega} \quad -(7)$$

to be the orbital linear velocity. The angular velocity is:

$$\boxed{\omega = \frac{d\theta}{dt}} \quad -(8)$$

In ECE theory: −(9)

$$\boxed{\underline{g} = -\underline{\nabla}\,\underline{\Phi} + \underline{\omega}\,\underline{\Phi} = c^2\,\underset{\sim}{T}}.$$

Eq. (9) gives a relation between the torsion $\underset{\sim}{T}$ and the spin connection $\underline{\omega}$ for a given Φ, defined by eq. (3). From Kepler's second law:

$$\frac{dA}{dt} = \frac{1}{2}r^2\omega = \text{constant} \quad -(10)$$

Therefore:

$$\boxed{v = \frac{2}{r}\frac{dA}{dt}} \quad -(11)$$

Therefore

$$v = \frac{k}{r} \quad -(12)$$

where:

$$k = 2\frac{dA}{dt} = \text{constant} \quad -(13)$$

for all central orbits.

3) Therefore in eq. (3):

$$\Phi = \int \frac{k^2}{r^2}\left(\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r}\right) dr \quad -(14)$$

where:

$$k = 2\frac{dA}{dt}. \quad -(15)$$

Since k is a constant:

$$\boxed{\Phi = k^2 \int \left(\frac{1}{r^2}\left(\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r}\right)\right) dr} \quad -(16)$$

Newtonian Orbits

These are given by:

$$\frac{1}{r} = \frac{1}{\alpha}\left(1 + \epsilon\cos\theta\right) \quad -(17)$$

(M&T eq. (7.41)), and by:

$$\boxed{\Phi = -\frac{Gm}{r}} \quad -(18)$$

(M&T eq. (4.6)). The Newtonian orbits are particular cases of eq. (16) (M&T page 250)

Log Spiral Orbits

These are:

$$r = k\exp(\alpha\theta) \quad -(19)$$

and

$$\Phi = k^2(1+\alpha^2)\int \frac{1}{r^3} dr$$

$$\boxed{\Phi = -\frac{1}{2}k^2(1+\alpha^2)\frac{1}{r^2}} \quad -(20)$$

24(4): Kepler's Equation

In astronomy we need the function $\theta(t)$, so the direction of an orbiting object may be found at any time. Planetary motion for example is described by:

$$\frac{\alpha}{r} = 1 + \epsilon \cos\theta \quad — (1)$$

i.e. the equation of a conic section with one focus at the origin (Marion & Thornton eq. (7.41)). Here ϵ is the eccentricity and 2α is the latus rectum. Define the period of the orbit as τ. This is the time taken for the radius vector to sweep out the entire area πab of an elliptical orbit. By Kepler's second law, the area $(\pi ab/\tau)t$ is swept out (M & T page 261). Thus:

$$\int dA = \frac{\pi ab}{\tau} t \quad — (2)$$

where

$$dA = \frac{1}{2} r^2 d\theta . \quad — (3)$$

If $\theta = 0$ at $t = 0$:

$$\frac{\pi ab}{\tau} t = \frac{1}{2} \int_0^\theta r^2 d\theta \quad — (4)$$

where, from eq. (1):

$$r = \frac{\alpha}{1 + \epsilon\cos\theta} \quad — (5)$$

so

$$\frac{\pi ab}{\tau} t = \frac{\alpha^2}{2} \int_0^\theta \frac{d\theta}{(1+\epsilon\cos\theta)^2} \quad — (6)$$

Now use:

$$ab = \alpha^2 (1-\epsilon^2)^{-3/2} \quad -(7)$$

So:

$$\boxed{\frac{2\pi t}{\tau} = 2\tan^{-1}\left(\left(\frac{1-\epsilon}{1+\epsilon}\right)^{1/2}\tan\frac{\theta}{2}\right) - \frac{\epsilon(1-\epsilon^2)^{1/2}\sin\theta}{1+\epsilon\cos\theta}} \quad -(8)$$

(M & T eq. (7.53)).

This equation must be inverted to give θ as a function of t. It may be possible to use Maxima to do this. An approximate result is:

$$\theta(t) = \frac{2\pi t}{\tau} + 2\epsilon\sin\left(\frac{2\pi t}{\tau}\right) + \frac{5}{4}\epsilon^2\sin\left(\frac{4\pi t}{\tau}\right) + \frac{1}{12}\epsilon^3\left(13\sin\left(\frac{6\pi t}{\tau}\right) - 3\sin\left(\frac{2\pi t}{\tau}\right)\right) + \ldots \quad -(9)$$

This problem was also solved by <u>Kepler's Equation</u>:

$$\boxed{\frac{2\pi t}{\tau} = \phi - \epsilon\sin\phi} \quad -(10)$$

where:

$$\tan\frac{\theta}{2} = \left(\frac{1+\epsilon}{1-\epsilon}\right)^{1/2}\tan\frac{\phi}{2} \quad -(11)$$

So ϕ is found as a function of t from eq. (10), by inversion, and ϕ is related to θ by eq. (11).

Velocity as a Function of the Radius Vector

This is found from Kepler's equation using:

$$v^2 = \dot{x}^2 + \dot{y}^2 \quad -(12)$$

$$= a^2\dot{\phi}^2(1-\epsilon^2\cos^2\phi) \quad -(13)$$

(M & T eq. (7.68)). So:

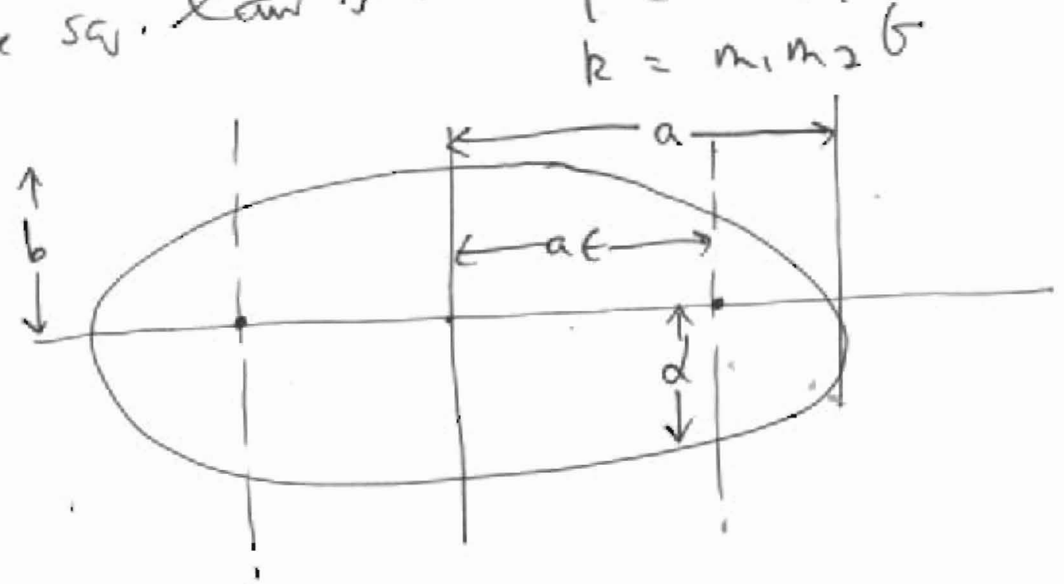
$$v^2 = \frac{k}{\mu}\left(\frac{2}{r} - \frac{1}{a}\right) \quad -(14)$$

Kepler's third law is:

$$\tau^2 = \frac{4\pi^2\mu}{k}a^3 \quad -(15)$$

The inverse sq. law is: $F = -k/r^2$

$k = m_1 m_2 G$



Animation Method for Any Central Force Law

The angular velocity ω defined by:

$$\omega = \frac{d\theta}{dt} = \frac{J}{mr^2} = \frac{\text{constant}}{r^2} \quad -(1)$$

so:

$$\boxed{\theta = \text{constant} \int \frac{1}{r^2}\, dt} \quad -(2)$$

In their chapter seven, Marion and Thornton give:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{mr^2}{J^2} F(r) \quad -(3)$$

which is valid for any central force law $F(r)$. This gives plenty of scope for animations.

From eq. (3):

$$\boxed{\frac{d\theta}{dt} = \frac{J}{mr^2} = -JF(r)\left(\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r}\right)^{-1}} \quad -(4)$$

where

$$\boxed{J = mr^2\omega = \text{constant of motion}} \quad -(5)$$

So the orbit for any central force law is:

$$\boxed{\theta = -J \int F(r)\left(\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r}\right)^{-1} dt} \quad -(6)$$

The Inverse Square Force Law

This is:

$$F(r) = -\frac{k}{r^2} = -\frac{m_1 m_2 G}{r^2} \quad -(7)$$

2) Eq. (7) gives:

$$\frac{1}{r} = \frac{1}{\alpha}\left(1+\epsilon\cos\theta\right) \quad -(8)$$

where α and ϵ are the half latus rectum and eccentricity of an ellipse. So the dependence of $1/r$ on θ is given by eq. (8) for the inverse square law. If the orbit is a circle:

$$\epsilon = 0 \quad -(9)$$

so:

$$\theta = -J\int rF(r)\,dt \quad -(10)$$

$$= Jk\int \frac{1}{r}\,dt$$

$$\boxed{\theta = \frac{Jkt}{r}} \quad -(11)$$

because r is constant in time for a circular orbit.

More generally, r is a function of t in eq. (6). This can be seen from the fact that θ is a function of t in eq. (8). The dependence of r on t for the inverse square law is given by Kepler's equation:

$$v^2 = \frac{k}{m}\left(\frac{2}{r}-\frac{1}{a}\right) \quad -(12)$$

where

$$\underline{v} = \frac{dx}{dt}\,\underline{i} + \frac{dy}{dt}\,\underline{j} \quad -(13)$$

$$v^2 = \dot{x}^2 + \dot{y}^2 \quad -(14)$$

So :

$$\frac{dr}{dt} = \left(\frac{k}{m} \left(\frac{2}{r} - \frac{1}{a} \right) \right)^{1/2}$$

$$\frac{dt}{dr} = \left(\frac{k}{m} \left(\frac{2}{r} - \frac{1}{a} \right) \right)^{-1/2} \quad -(15)$$

$$\boxed{t = \int \left(\frac{k}{m} \left(\frac{2}{r} - \frac{1}{a} \right) \right)^{-1/2} dr} \quad -(16)$$

1) The dependence of t on r may be animated from eq. (16). From eqns. (6) and (15):

$$\boxed{\theta = -J \int F(r) \left(\frac{d^2}{d\theta^2} \left(\frac{1}{r} \right) + \frac{1}{r} \right)^{-1} \left(\frac{k}{m} \left(\frac{2}{r} - \frac{1}{a} \right) \right)^{-1/2} dr} \quad -(17)$$

2) and the dependence of θ on r may be animated from this equation.

3) The dependence of r on t may be found by inverting the results of eq. (16) and parametrizing numerically.

4) The simplest method is to use eq. (2) and eq. (16). The latter is integrated by Maxima to give t as a function of r, and the result inverted to give r as a function of t.

4) Log Spiral Orbit

This is worked out completely by M & T, pp. 250 and 251. If a particle moves on a log spiral:

$$r = k \exp(\alpha\theta) \quad -(1)$$

where k and α are constants. Eq. (2) gives:

$$F(r) = -\frac{J^2}{mr^3}(1+\alpha^2) \quad -(2)$$

and

$$\theta = \frac{1}{2\alpha}\log_e\left(\frac{2\alpha J t}{mk^2} + c\right) \quad -(3)$$

with

$$r = \left(\frac{2\alpha J}{m}t + k^2 c\right)^{1/2} \quad -(4)$$

Animate eqns. (1), (3) and (4).

These were first discussed by Roger Cotes (1682 – 1716) and the orbits are Cotes' spirals.

—— 2 ——

4 (6): Effect of Constant Spacetime Torsion on the Newtonian Attraction of Stars in a Galaxy

In general, the total energy in a central orbit of any kind is constant:

$$E = T + U = \text{constant} \quad -(1)$$

$$E = \frac{1}{2} m \dot{r}^2 + \frac{1}{2} \frac{J}{mr^2} + U(r) \quad -(2)$$

Therefore:

$$\frac{dr}{dt} = \left(\frac{2}{m} (E - U) - \frac{J^2}{m^2 r^2} \right)^{1/2} \quad -(3)$$

where E and J are constants. The inverse square potential is:

$$U = m \Phi = - \frac{G m^2}{r} \quad -(4)$$

Therefore, as in M & T chapter 7:

$$\theta = \int \frac{(J / r^2) \, dr}{2m \left(E - U(r) - \frac{J^2}{2mr^2} \right)^{1/2}} \quad -(5)$$

One may animate θ as a function of r.

The effective potential is:

$$V(r) = U(r) + \frac{J^2}{2mr^2} \quad -(6)$$

$$V(r) = - \frac{Gm^2}{r} + \frac{J^2}{2mr^2}$$

and consists of the attraction and centrifugal repulsion, i.e. respectively negative and positive.

The Effect of Constant Spacetime Torsion

From previous work it is known that torsion, when integrated over volume, is proportional to angular momentum. Therefore in a galaxy, there is an additional repulsive potential due to constant spacetime torsion:

$$\boxed{V(\text{torsion}) = \frac{J_T^2}{2mr^2}} \quad -(7)$$

The extra repulsive force is:

$$F(\text{torsion}) = -\frac{\partial V(\text{torsion})}{\partial r}$$

$$= +\frac{J_T^2}{mr^3} \quad -(8)$$

From problem (7.22) of m & T, if:

$$J_0^2 := J_T^2(1+\alpha_0) \quad -(9)$$

then the orbit produced by eq. (8) is a logarithmic spiral:

$$\boxed{r = k\exp(\alpha_0\theta)} \quad -(10)$$

Therefore this simple model produces the main features of a spiral galaxy. The Newtonian curve of θ versus r is given by eq. (5), and the spiral orbits by eq. (10):

$$\theta = \frac{1}{\alpha_0} \log_e \frac{r}{R} \quad - (11)$$

Eq. (5) gives:

$$\cos\theta = \frac{1}{\epsilon}\left(\frac{\alpha}{r} - 1\right)$$

$$\theta = \cos^{-1}\left(\frac{1}{\epsilon}\left(\frac{\alpha}{r} - 1\right)\right) \quad - (12)$$

Animations can be made of θ as a function of r from eqns. (11) and (12).

The arms of a whirlpool galaxy are logarithmic spirals. We have:

$$\frac{dr}{d\theta} = br \quad - (13)$$

and

$$v = \frac{dr}{d\theta}\frac{d\theta}{dt} = \omega\frac{dr}{d\theta} = b\omega r$$

$$= \text{constant} \quad - (14)$$

---

1) Note 124(8): Logarithmic Spiral Orbit

Consider the lagrangian

$$\mathcal{L} = \frac{1}{2} m \left(\dot{r}^2 - r^2 \dot{\theta}^2 \right) \quad (1)$$

and the Euler Lagrange equation

$$\frac{\partial \mathcal{L}}{\partial r} = \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{r}} \quad (2)$$

Thus:

$$\frac{\partial \mathcal{L}}{\partial \dot{r}} = m \dot{r}, \quad \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{r}} = m \ddot{r} \quad (3)$$

$$\frac{\partial \mathcal{L}}{\partial r} = -m r \dot{\theta}^2 \quad (4)$$

Thus:

$$\boxed{\ddot{r} = -r \dot{\theta}^2} \quad (5)$$

Making the transform of variable:

$$r = \frac{1}{u} \quad (6)$$

and using the method in Marion and Thornton page 249:

$$\ddot{r} = -\frac{J^2}{m^2} u^2 \frac{d^2 u}{d\theta^2} \quad (7)$$

$$r \dot{\theta}^2 = \frac{J^2}{m^2} u^3 \quad (8)$$

The centrifugal force is:

2)

Using eq. (8)

$$F = \frac{\partial U}{\partial r} = m r \dot{\theta} = \frac{J^2}{m r^3} \quad — (9)$$

which is the centrifugal force outwards.

Therefore

$$m\ddot{r} - \frac{\partial U}{\partial r} = 0 \quad — (10)$$

and the orbital equation is

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) = \frac{1}{r} \quad — (11)$$

This is satisfied by the log spiral

$$r = e^{\theta} \quad (12)$$

We have:

$$r(t) = \left(\frac{2J}{m}t + C\right)^{1/2} \quad — (13)$$

and

$$\theta(t) = \frac{1}{2}\log_e\left(\frac{2J t}{m} + C\right) \quad — (14)$$

giving the evolution of a whirlpool galaxy.

Reprint from Proc. 4th International New Energy Conference, Denver, CO, May 2?, 1997, p. 5-51. Slightly corrected in June 1997 and in August 1997.

mass represents ... "switches per second" of action quanta across the boundary of the mass, as a flux exchange between the mass and its environment, where one switch equals h/(4π). The same early paper, though very crude, derives Newton's three laws of motion in relativistic form and the square law of gravitation.

55. See Bearden, 1973, ibid. for an early indication of the mechanism that generates the rate of flow of time for an interacting object. For a somewhat expanded treatment, see T.E. Bearden, "Vacuum Engines and Priore's Methodology: The True Science of Energy-Medicine. Parts I and II." Explore!, 6(1), 1995, p. 66-76; 6(2), 1995, p. 50-62, passim.

56. T.E. Bearden, "A Redefinition of the Energy Anasis, Leading to a Fundamentally New Class of Nuclear Interactions," Proceedings of the 27th Intersociety Energy Conversion Engineering Conference (IECEC-92), Town & Country Hotel Convention Center, San Diego, California, 1992, p. 4.303-4.310.

57. Oddly, the conventional practice of simply discarding vector zero resultant summations of nonzero vector systems actually discards vacuum engines and their utilization. Thus, the enforced symmetry (such as the Lorentz regauging condition) in EM modeling discards one of the most useful things in all of electrodynamics!

58. Clean Energy Technologies Inc. (CETI) of Sarasota, Florida has begun marketing the Patterson Power Cell Research Kit, code-named RIFEX, which allows researchers to test the cold fusion and the formation of new nuclear species. The kit goes for $3,750, which also includes a one-year research license, on-site training, and technical support from CETI, as well as other benefits. For details, phone numbers and web sites, refer to David Kestenbaum, "Cold Fusion: Science or Religion?" R&D Magazine, 39(5), Apr. 1997, p. 51-56.

59. The sharp-eyed reader will recognize that we have just given the precise mechanism actually involved in homeopathy's process of potentizing. The present extended EM approach can be applied to homeopathy to yield a rigorous explanatory model for it.

60. H.E. Puthoff, "Space Propulsion: Can Empty Space Itself Provide a Solution?" Ad Astra, 9(1), Feb. 1997, p. 42-46.

61. H.E. Puthoff, "Source of Vacuum Electromagnetic Zero-point Energy," Physical Review A, 40(9), Nov. 1, 1989, p. 4857-4862.

62. T.D. Lee, 1981, ibid. Chapter 25: Outlook: Possibility of Vacuum Engineering," p. 824-828. Also, on p. 383, Lee points out that the microscopic structure of the scalar vacuum field (i.e., of vacuum charge) is not utilized. By the use of EM templates and vacuum engines, we have indicated a feasible method of macroscopically and microscopically performing and developing Lee's vacuum engineering as a new technology.

63. With sufficient research, the vacuum engine type approach could be developed to yield an effective method of totally disposing of radioactive wastes, as previously

3) The velocity of a star is:

$$v^2 = \dot{r}^2 + r^2 \dot{\theta}^2 \quad (15)$$

and may be calculated from eqs. (13) and (14).

Animations

Eqs. (13) to (15) may be animated. If the velocity v becomes constant, then

$$\dot{r}^2 + r^2 \dot{\theta}^2 = \text{constant} \quad (16)$$

and it is seen that eqs. (13), (14) and (16) give an equation in t. At some critical time the galaxy has evolved to a constant v, and the arms of the log spiral become straightened out as observed.

Reprint from Proc. 4th International New Energy Conference, Denver, CO, May 23-26, 1997, p. 5-31. Slightly corrected in June 1997 and in August 1997.

T.E. Bearden, AIDS: Biological Warfare, Tesla Book Co., Chula Vista, California, 1988, p. 428-443. See particularly Morris Kline, Mathematics: The Loss of Certainty, Oxford University Press, New York, 1980; and G. Spencer Brown, Laws of Form, Julian Press, New York, 1972.

44. E.g., see Bunge, 1967, ibid., p. 180-181.

45. In various articles in The Electrician, a technical magazine roughly equivalent to today's Scientific American.

46. J.H. Poynting, "On the transfer of energy in the electromagnetic field," Philosophical Transactions of the Royal Society of London, Vol. 175, Part II, 1884, p. 343-361.

47. See editorial, "The Transfer of Energy," The Electrician, Vol. 27, July 10, 1891, p. 270-272.

48. Also, it has not been clearly realized that only collected energy can be dissipated or changed in form.

49. "Changing the form" of energy actually is "changing the form of the collecting" of energy.

50. E.g., see Robert Bruce Lindsay and Henry Margenau, Foundations of Physics, Dover Publications, New York, 1963, p. 283 which emphasizes that a "field of force" at any point is actually defined only for the case when a unit mass is present at that point. Why such an astounding, well-known, fundamental physics error as "forces in the vacuum" continues to be propagated in all our scientific journals and textbooks is an inexplicable mystery.

51. Rigorously, we do not dissipate energy in any load. Instead, we dissipate the on-going energy-collecting process of the electrons [illegible] transiting through the load.

52. If the flow is made nonlinear in a local area, the gradient can be affected (by a sort of Fresnel bunching), and the energy collecting magnetostatic of a particle can be increased. See cited work by Bohren, 1983, ibid. and by Paul and Fischer, 1983, ibid.

53. No tension in such a "taut string" exists without equal and opposite restoring reaction forces in the body of a holder for the taut string. By assuming independent taut strings and discarding the string holder, Maxwell erroneously discarded Newton's third law reaction from electrodynamics. Fully one-half the electrodynamics, the energy, the forces, etc. was erroneously omitted by Maxwell, and continues to be omitted in every electrodynamics textbook.

54. For a scholarly treatise on the concept of mass, see Max Jammer, Concepts of Mass, in Classical and Modern Physics, Harvard University Press, Cambridge, Massachusetts, 1961. For an early force-free definition of mass, suitable for defining mass as a localized asymmetry in spacetime, see T.E. Bearden, Quiton/Perception Physics: A Theory of Existence, Perception, and Physical Phenomena, National Technical Information System, Report AD-763210, 1973, p. 11. One Kilogram of

1) 124(9): Velocity Curve of a Log Spiral Orbit.

The velocity of the star in a logarithmic spiral orbit is:

$$v^2 = \dot{r}^2 + r^2 \dot{\theta}^2 \quad — (1)$$

where:

$$\dot{\theta} = \frac{J}{m r_0^2} e^{-2\theta\alpha} \quad — (2)$$

(Marion and Thornton, eq. (7.23), page 250),

and:

$$\dot{r} = \frac{\alpha_0 r_0 J}{m r^2} e^{\alpha\theta} \quad — (3)$$

(Marion and Thornton, eq. (7.28), page 251).

The log spiral is:

$$\boxed{r = r_0 e^{\alpha\theta}} \quad — (4)$$

so

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) = \frac{\alpha^2}{r} \quad — (5)$$

Therefore:

$$\boxed{v^2 = \left(\frac{\alpha_0 r_0 J}{m}\right)^2 \frac{e^{2\alpha\theta}}{r^4} + \left(\frac{J}{m r_0^2}\right)^2 r^2 e^{-4\alpha\theta}}$$

— (6)

2)

Define:

$$A = \frac{\alpha_0 r_0 J}{m}, \quad B = \frac{J}{m r_0^2} \qquad -(7)$$

Then:

$$v^2 = \left(\frac{A e^{\alpha\theta}}{r^2}\right)^2 + \left(B r e^{-2\alpha\theta}\right)^2 \qquad -(8)$$

Limits

1) As

$$r \to \infty \qquad -(9)$$

$$v \to B r e^{-\alpha\theta} \qquad -(10)$$

2) As

$$r \to 0 \qquad -(11)$$

$$v \to \infty \qquad -(12)$$

In the limit (9):

$$v \to \frac{J r}{m r_0^2} e^{-\alpha\theta} \qquad -(13)$$

Using eq. (4):

$$v \to \frac{J}{m r_0} = \text{constant}$$

for any r. The graph of v against r is a plateau as observed experimentally.

1) 124(10): The Potential Energy Generated by Spacetime Torsion.

Consider a star moving along the radial direction $\underline{e}_r$ with velocity:

$$\underline{v}_r = \dot{r}\,\underline{e}_r \quad -(1)$$

By Newton's first law it would move along this direction permanently unless acted upon by an external force, $\underline{F}$. This force produces work done on the star:

$$W_{12} = \int_1^2 \underline{F} \cdot d\underline{r}, \quad -(2)$$

work which transforms the star from condition 1 to 2. The kinetic energy generated by $\underline{v}$ of eq. (1) is:

$$T = \frac{1}{2} m \dot{r}^2 \quad -(3)$$

The torsion of spacetime does work on the star while keeping T constant. The potential energy transferred to the star from spacetime is defined by:

$$\int_1^2 \underline{F} \cdot d\underline{r} = U_1 - U_2 \quad -(4)$$

so:

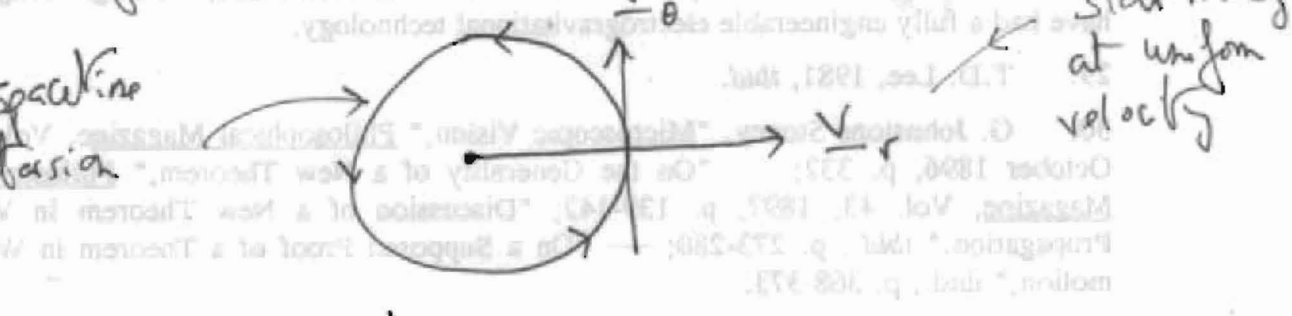
$$\underline{F} = -\underline{\nabla} U \quad -(5)$$

This potential energy moves the star in a direction transverse to $\underline{e}_r$, producing the

2) velocity:

$$\underline{v}_\theta = r\dot{\theta}\,\underline{e}_\theta \quad -(6)$$

This process may be thought of as a particle moving outward from the centre of a rotating platform:



The total velocity is:

$$\underline{v} = \dot{r}\,\underline{e}_r + r\dot{\theta}\,\underline{e}_\theta \quad -(7).$$

and the total energy is:

$$E = \frac{1}{2} m\, \underline{v}\cdot\underline{v} \quad -(8)$$

$$E = \frac{1}{2} m \left(\dot{r}^2 + r^2\dot{\theta}^2\right).$$

Thus

$$E = T + U \quad -(9)$$

where:

$$T = \frac{1}{2} m \dot{r}^2 \quad -(10)$$

$$U = \frac{1}{2} m r^2 \dot{\theta}^2 \quad -(11)$$

The lagrangian is:

$$\mathcal{L} = T - U = \frac{1}{2} m \left(\dot{r}^2 - r^2\dot{\theta}^2\right) \quad -(12)$$

3) The Euler Lagrange equations are:

$$\frac{\partial \mathcal{L}}{\partial r} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{r}} \quad -(13)$$

and

$$\frac{\partial \mathcal{L}}{\partial \theta} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\theta}} \quad -(14)$$

The motion takes place in a plane, so:

$$J = \frac{\partial \mathcal{L}}{\partial \dot{\theta}} = \text{constant} \quad -(15)$$

$$= m r^2 \dot{\theta}$$

the conservation of angular momentum is:

$$\frac{dJ}{dt} = 0 \quad -(16)$$

and the conservation of energy is:

$$\frac{dE}{dt} = 0 \quad -(17)$$

The potential energy is therefore

$$U = \frac{1}{2} m r^2 \dot{\theta}^2 = \frac{1}{2}\frac{J^2}{m r^2} \quad -(18)$$

and

$$F = \frac{J^2}{2 m r^3} \quad -(19)$$

1) 124(11): Force Law and Orbit due to Constant Torsion

The Euler Lagrange equation is

$$\frac{\partial \mathcal{L}}{\partial r} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{r}} \quad (1)$$

wrt

$$\mathcal{L} = \frac{1}{2} m \left(\dot{r}^2 - r^2 \dot{\theta}^2 \right) \quad (2)$$

So:

$$F = m\ddot{r} = -\frac{\partial U}{\partial r} \quad (3)$$

wrt

$$W_{12} = \int_1^2 \underline{F} \cdot d\underline{r} \quad (4)$$

and

$$\underline{F} = -\nabla U \quad (5)$$

Therefore:

$$\boxed{\int_1^2 \underline{F} \cdot d\underline{r} = U_1 - U_2} \quad (6)$$

The work done is the change of potential energy:

$$W_{12} = U_2 - U_1 \quad (7)$$

The work is done by the star if $U_1 > U_2$. The force is:

$$\boxed{\int_1^2 \underline{F} \cdot d\underline{r} = -\int_1^2 dU = U_1 - U_2} \quad (8)$$

2) If F is positive valued the force is repulsive by convention. Therefore a repulsive force means that U_1 is greater than U_2 by convention. If F is negative valued the force is attractive by convention, so U_2 is greater than U_1 by convention. The initial state of potential is chosen such that:

$$U_1 = 0 \quad -(9)$$

and the final state is:

$$U_2 = \frac{1}{2} m r^2 \dot{\theta}^2 \quad -(10)$$

Therefore:

$$U_2 > U_1 \quad -(11)$$

and:

$$\boxed{W_{12} > 0.} \quad -(12)$$

This means that work has been done on the star by the force of spacetime. By convention, the force is negative valued, meaning that the star is attracted by the whirling spacetime.

Therefore:

$$F = m r = -\frac{dU}{dr} = -m r \dot{\theta}^2 \quad -(13)$$

Now make the change:

$$u = \frac{1}{r} \quad -(14)$$

so

$$\frac{du}{d\theta} = -\frac{1}{r^2}\frac{dr}{d\theta} = -\frac{1}{r^2}\frac{dr}{dt}\frac{dt}{d\theta} = -\frac{1}{r^2}\frac{\dot{r}}{\dot{\theta}} \quad -(15)$$

3) So $\frac{du}{d\theta} = -\frac{m}{J}\dot{r}$ — (16)

and $\frac{d^2u}{d\theta^2} = -\frac{m^2 r^2}{J^2}\ddot{r}$ — (17)

So $F(r) = -\frac{J^2}{m r^2}\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right)$ — (18)

If the orbit is a logarithmic spiral

$r = r_0 \exp(\alpha\theta)$ — (19)

and $\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) = \frac{\alpha^2}{r}$ — (20)

So $F(r) = -\frac{\alpha^2 J^2}{m r^3}$ — (21)

The potential is $U(r) = \frac{\alpha^2 J^2}{m r^2}$ — (22)

So $\alpha \; 1/\sqrt{2}$ — (23)

The velocity is $v^2 = \dot{r}^2 + r^2\dot{\theta}^2$

$v^2 = \left(\frac{A e^{\alpha\theta}}{r^2}\right)^2 + \left(B r e^{-\alpha\theta}\right)^2$ — (24)

$v \underset{r\to\infty}{\longrightarrow} B r e^{-\alpha\theta} = \frac{J}{m r_0}$

1) 124(12): Force Law for a Logarithmic Spiral
(Marion and Thornton page 250)

In this development consider a star moving with velocity v in a plane. Let work be done on the star by a potential:

$$U(r) = -\frac{J^2(1+\alpha^2)}{2m}\frac{1}{r^2} \quad -(1)$$

so:

$$F(r) = -\frac{J^2}{m r^3}(1+\alpha^2) \quad -(2)$$

the orbit is the logarithmic spiral

$$r = r_0 \exp(\alpha\theta) \quad -(3)$$

Eq. (1) is the potential energy due to constraint, spacetime torsion.

We have

$$\theta(t) = \frac{1}{2\alpha}\log_e\left(\frac{2\alpha J t}{m r_0^2} + c\right) \quad -(4)$$

$$r(t) = \left(\frac{2\alpha J t + r_0^2 c}{m}\right)^{1/2} \quad -(5)$$

Animate eqns. (4) and (5).

Also:

$$\dot{\theta} = \frac{J}{m r^2} \quad -(6)$$

$$\dot{r} = \frac{\alpha J}{m r} \quad -(7)$$

2)

$$v^2 = \dot{r}^2 + r^2 \dot{\theta}^2 \quad — (8)$$

$$= (1 + d^2) \frac{J^2}{m^2 r^2}$$

so:

$$J = m v r (1 + d^2)^{-1/2} \quad — (9)$$

Here J, d, r, and C are constants. Eq. (9) shows that the orbit evolves to that of a circle of radius $(1 + d^2)^{-1/2} r$, a circular orbit. v and r are constant.

Let:

$$R = (1 + d^2)^{-1/2} r \quad — (10)$$

and so:

$$J = m v R = \text{constant} \quad — (11)$$

The basic equation is M8 eq. (7.21):

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{m r^2}{J^2} F(r) \quad — (12)$$

124(13): Resonance in the Inverse Square Law of Gravitational Attraction

Start with the equation:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{mr^2}{J^2}F(r) \quad -(1)$$

Kepler's laws and Newtonian dynamics are given by:

$$F(r) = -\frac{k}{r^2} \quad -(2)$$

which is the inverse square law of gravitational attraction.

Therefore:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = \left(\frac{mk}{J^2}\right) \quad -(3)$$

Now introduce an oscillatory structure into the familiar inverse square law:

$$F(r) = -\frac{k}{r^2}\cos(\omega\theta) \quad -(4)$$

Then:

$$\boxed{\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = \left(\frac{mk}{J^2}\right)\cos(\omega\theta)} \quad -(5)$$

This is a Bernoulli Euler resonance equation.

At resonance:

2)

$$\frac{1}{r} = \frac{nk \cos(\alpha\theta)}{s^2 (\alpha^2 - 1)^{1/2}} \quad (6)$$

so if: $\alpha^2 = 1, \alpha = \pm 1$ — (7)

then $\frac{1}{r} \to \infty, \; r \to 0$ — (8)

and the force $F(r)$ becomes negative infinite and the mass m is attracted to the mass M by an infinite force, and the system implodes.

Application to Counter Gravitation

When:

$$\cos(\alpha\theta) < 0 \quad (9)$$

then

$$F(r) > 0 \quad (10)$$

and the force becomes repulsive. At ~~the~~ the resonance point (6) the object m is repelled from M. This is an example of resonant counter-gravitation, where the system explodes.