

# 1) 121 (6) : Derivation of Covariant Theorems of Physics

In ECE theory the covariant theorems of physics are based on the tetrad postulate of geometry:

$$D_{\mu} g^{\alpha}_{\nu} = 0 \quad - (1)$$

which is true in any frame of reference. Now use the definition

$$g^{\mu}_{\alpha} g^{\alpha}_{\nu} = \delta^{\mu}_{\nu} \quad - (2)$$

where:

$$\left. \begin{aligned} \delta^{\mu}_{\nu} &= 1 \text{ if } \mu = \nu \\ \delta^{\mu}_{\nu} &= 0 \text{ if } \mu \neq \nu \end{aligned} \right\} - (3)$$

To find that:

$$g^{\mu\nu} = g^{\mu\sigma} g^{\sigma}_{\nu} = g^{\mu\nu} \quad - (4)$$

So:

$$g^{\mu\nu} = g^{\mu\nu} \quad - (5)$$

$$g_{\mu\nu} = g_{\mu\nu} \quad - (6)$$

These are useful relations which show that the tetrad in the base manifold is the metric, and the inverse tetrad  $g^{\mu\nu}$  is the inverse metric.

Therefore:

$$g^{\mu\nu} g_{\mu\nu} = 4 \quad - (7)$$

$$g^{\mu\nu} g_{\mu\nu} = 4 \quad - (8)$$

Now we use the metric compatibility condition:

$$2) \quad D_\rho g_{\mu\nu} = D_\rho g^{\mu\nu} = 0 \quad - (9)$$

$$\alpha \quad D^\rho g_{\mu\nu} = D^\rho g^{\mu\nu} = 0 \quad - (10)$$

to find that

$$D^\mu g_{\mu\nu} = 0 \quad - (11)$$

where:

$$\rho = \mu \quad - (12)$$

Summation over repeated indices occurs in eq

(11). However, by convention:

$$g^\mu_\alpha g^\alpha_\mu = 1 \quad - (13)$$

by definition. Eq. (13) must be true in order for eq. (1) to be true. (see note 121(5))

The fundamental ECE hypothesis is:

$$A^\alpha_\mu = A^{(0)} g^\alpha_\mu \quad - (14)$$

so:

$$A_{\mu\nu} = A^{(0)} g_{\mu\nu} \quad - (15)$$

and

$$D^\mu A_{\mu\nu} = 0 \quad - (16)$$

3) Eq. (16) is one of the fundamental conservation theorems of ECE theory. It can be seen that it follows from metric compatibility, a geometrical property. The conservation theorems of physics follow from geometry. In Cartesian geometry, the conservation theorem is:

$$D_{\mu} A^{\mu} = 0 \quad - (17)$$

which may be developed as:

$$\square A^{\mu} = R A^{\mu} \quad - (18)$$

and

$$(\square + kT) A^{\mu} = 0 \quad - (19)$$

The wave equations of physics are conservation theorems of physics.

The conservation of canonical energy - momentum density follows from:

$$T^{\mu}_{\nu} = T^{(\nu)}_{\mu} \quad - (20)$$

so:

$$\square^{\mu} T_{\mu\nu} = 0 \quad - (21)$$

which is the covariant Noether theorem.

+) Therefore:

$$\boxed{\square T^a = R T^a} \quad - (22)$$

i.e. canonical energy momentum density is quantized and also stochastic.

The conservation of charge-current density follows from:

$$J^a_{,\mu} = J^{(0)} v^a_{,\mu} \quad - (23)$$

so

$$\boxed{D^\mu J_{\mu\nu} = 0} \quad - (24)$$

or

$$\boxed{\square J^a_{,\mu} = R J^a_{,\mu}} \quad - (25)$$

i.e. charge-current density is quantized and stochastic.

The dual identity of gravity is:

$$D_\mu T^{\kappa\nu} = R^{\kappa\nu}_{,\mu} \quad - (26)$$

and it is possible to define a curvature:

$$R^{\kappa\nu} = R^{\kappa\nu}_{,\mu} \quad - (27)$$

by summation over internal indices (i.e. by

5) index contraction. By hypothesis similar to that of Euler:

$$R^{\kappa\omega} = R T^{\kappa\omega} \quad - (28)$$

$$= D_{\mu} T^{\kappa\omega}$$

Here:

$$T^{\kappa\omega} = R J^{\kappa\omega} \quad - (29)$$

where:

$$J^{\kappa\omega} = -\frac{1}{2} (T^{\kappa\mu} x^{\omega} - T^{\omega\mu} x^{\kappa}) \quad - (30)$$

is the canonical angular energy-momentum density tensor.

So:

$$D_{\mu} J^{\kappa\omega} = T^{\kappa\omega} \quad - (30)$$

or:

$$D^{\kappa} J_{\mu\kappa\omega} = T_{\mu\omega} \quad - (31)$$

and the conservation of canonical angular energy-momentum density is:

$$D^{\mu} (D^{\kappa} J_{\mu\kappa\omega}) = D^{\mu} T_{\mu\omega} = 0 \quad - (32)$$