Note 115(11): The Equivalence of Lorentz Transformation and Hodge duality.

Consider the Faraday law of induction in frame K:

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0} \quad -(1)$$

and carry out the Lorentz transform of fields into frame K′ to give:

$$\underline{\nabla} \times \left(\underline{E} + \underline{v} \times \underline{B} \right) + \frac{\partial}{\partial t} \left(\underline{B} - \frac{1}{c^2} \underline{v} \times \underline{E} \right) = \underline{0} \quad -(2)$$

Using eq. (1):

$$\underline{\nabla} \times \left(\underline{v} \times \underline{B} \right) - \frac{1}{c^2} \frac{\partial}{\partial t} \left(\underline{v} \times \underline{E} \right) = \underline{0} . \quad -(3)$$

Now use the vector identities:

$$\underline{\nabla} \times (\underline{a} \times \underline{b}) = \underline{a} \, \underline{\nabla} \cdot \underline{b} - \underline{b} \, \underline{\nabla} \cdot \underline{a} + (\underline{b} \cdot \underline{\nabla}) \underline{a} - (\underline{a} \cdot \underline{\nabla}) \underline{b} \quad -(4)$$

and

$$\underline{a} \times (\underline{\nabla} \times \underline{a}) = \frac{1}{2} \underline{\nabla} a^2 - (\underline{a} \cdot \underline{\nabla}) \underline{a} \quad -(5)$$

("Vector Analysis Problem Solver", pp. 337 and 339).

Therefore:

$$\underline{\nabla} \times (\underline{v} \times \underline{B}) = \underline{v} \, \underline{\nabla} \cdot \underline{B} - \underline{B} \, \underline{\nabla} \cdot \underline{v} + (\underline{B} \cdot \underline{\nabla}) \underline{v} - (\underline{v} \cdot \underline{\nabla}) \underline{B} \quad -(6)$$

2) For a constant $\underline{v}$:

$$\underline{\nabla} \times (\underline{v} \times \underline{B}) = -(\underline{v} \cdot \underline{\nabla})\underline{B} \quad -(7)$$

From eq. (5):

$$\underline{v} \times (\underline{\nabla} \times \underline{B}) = \frac{1}{2} \underline{\nabla}(\underline{v} \cdot \underline{B}) - (\underline{v} \cdot \underline{\nabla})\underline{B}$$

$$= -(\underline{v} \cdot \underline{\nabla})\underline{B} \quad -(8)$$

for constant $\underline{v}$.

Therefore:

$$\underline{\nabla} \times (\underline{v} \times \underline{B}) = \underline{v} \times (\underline{\nabla} \times \underline{B}) \quad -(9)$$

For constant $\underline{v}$:

$$\frac{\partial}{\partial t}(\underline{v} \times \underline{E}) = \underline{v} \times \frac{\partial \underline{E}}{\partial t} \quad -(10)$$

so eq. (3) is:

$$\underline{v} \times \left(\underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} \right) = \underline{0} \quad -(11)$$

which is the Ampère Maxwell law in free space:

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} = \underline{0} \quad -(12)$$

Q.E.D. Eq. (12) is the <u>Hodge dual</u> of eq. (1) as is well known.

3) Result

The Lorentz transform of eq. (1) is:

$$\underline{\nabla} \times \underline{E}' + \frac{\partial \underline{B}'}{\partial t} = \underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} + \underline{v} \times \left(\underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} \right) \quad -(13)$$

$$= \underline{0}$$

with:

$$\underline{E}' = \underline{E} + \underline{v} \times \underline{B}, \quad -(14)$$

$$\underline{B}' = \underline{B} - \frac{1}{c^2} \underline{v} \times \underline{E}. \quad -(15)$$

Here K′ is a frame moving at $\underline{v}$ with respect to frame K. In the Faraday disk generator:

$$\frac{\partial \underline{B}}{\partial t} = \underline{0}, \quad \underline{\nabla} \times \underline{B} = 0 \quad -(16)$$

whether or not the magnet spins around its own Z axis. So under no circumstance can the standard model produce induction of $\underline{E}$ in the disk, either in frame K or K′.

This is the Faraday paradox, which is not resolved by Lorentz transformation. It is resolved only in ECE theory.