1) III(7): General Metric for Spherically Symmetric Spacetime

The most general metric has been given by Crothers. Here we use empirical satellite data and a metric:

$$ds^2 = -m c^2 dt^2 + n dr^2 + r^2 d\Omega^2 \quad -(1)$$

as given by Carroll in chapter 7. The Minkowski metric is

$$ds^2 = -c^2 dt^2 + dr^2 + r^2 d\Omega^2; \quad -(2)$$

so in this case:

$$m = 1. \quad -(3)$$

Write eq. (3) as:

$$\boxed{rm = \int dr} \quad -(4)$$

where the constant of integration is assumed to be zero, i.e.

$$rm = r. \quad -(5)$$

Now assume a constant of integration μ:

$$rm = r + \mu, \quad -(6)$$

i.e.

$$\boxed{m = 1 + \frac{\mu}{r}}. \quad -(7)$$

Assume that:

$$n = 1/m = \left(1 + \frac{\mu}{r}\right)^{-1} \quad -(8)$$

then:

$$\boxed{ds^2 = -\left(1 + \frac{\mu}{r}\right)c^2 dt^2 + \left(1 + \frac{\mu}{r}\right)^{-1} dr^2 + r^2 d\Omega^2} \quad -(9)$$

and when

$$\mu = 2MG / c^2 \quad -(10)$$

this is the so-called Schwarzschild metric.

Eq. (9) is a particular solution of eq. (1) and is purely geometrical.