108(7): Second Method of Calculating the Orbit of a Binary Pulsar.

The second method proceeds from:

$$\frac{1}{2} m \left(\frac{dr}{d\tau} \right)^2 = E_n - V \quad — (1)$$

where:

$$E_n = \frac{1}{2} m c^2 \left(1 - \frac{r_s}{r} \right)^2 \left(\frac{dt}{d\lambda} \right)^2, \quad — (2)$$

$$V = mc^2 \left(\frac{1}{2} - \frac{r_s}{r} \right) + \frac{mL^2}{2r^2} - mL^2 \frac{r_s}{r^3} \quad — (3)$$

Here E_n and L are constants of motion and

$$L = r^2 \frac{d\phi}{d\tau}. \quad — (4)$$

Use:

$$\frac{dr}{d\phi} = \frac{dr}{d\tau} \frac{d\tau}{d\phi} = \frac{r^2}{L} \frac{dr}{d\tau} \quad — (4).$$

So:

$$\boxed{\frac{dr}{d\phi} = \frac{r^2}{L} \left(\frac{2}{m} \left(E_n - V \right) \right)^{1/2}} \quad — (5)$$

and

$$\boxed{\phi = \int \frac{L}{r^2} \left(\frac{2}{m} \left(E_n - V \right) \right)^{-1/2} dr} \quad — (6)$$

Eq (6) can be integrated numerically to give the light deflection due to gravitation and the advance of the perihelion. Use:

$$|r_s| = \frac{T}{R} \quad — (7)$$