1)

Paper 105, Notes 3 : Null Geodesic Condition

The basic method used for light deflection by gravitation uses the null geodesic condition:

$$ds^2 = -g_{00} c^2 dt^2 + g_{11} dX^2 + g_{22} dY^2 + g_{33} dZ^2 \quad -(1)$$
$$= 0$$

i.e. $$g_{00} c^2 dt^2 = g_{11} dX^2 + g_{22} dY^2 + g_{33} dZ^2. \quad -(2)$$

Integrating:
$$g_{00} c^2 t^2 = g_{11} X^2 + g_{22} Y^2 + g_{33} Z^2. \quad -(3)$$

Without loss of generality consider the Z axis only:
$$g_{00} c^2 dt^2 = g_{33} Z^2 \quad -(4)$$

The ECE equation describing the deflection of light by gravitation is found from the Bianchi identity:
$$d \wedge T := R \wedge q - \omega \wedge T \quad -(5)$$
and its Hodge dual:
$$d \wedge \tilde{T} := \tilde{R} \wedge q - \omega \wedge \tilde{T} \quad -(6)$$

Here $\wedge$ denotes the wedge product of Cartan's differential geometry. The general definition of the wedge product is given by Carroll in his chapter one as the antisymmetrized tensor

2) product :

$$(A \wedge B)_{\mu_1 \cdots \mu_{p+q}} = \frac{(p+q)!}{p!\,q!} A_{[\mu_1 \cdots \mu_p} B_{\mu_{p+1} \cdots \mu_{p+q}]} \quad -(7)$$

and as given in Carroll eq. (1.81) its symmetry is :

$$A \wedge B = (-1)^{pq} B \wedge A \quad -(8)$$

It follows that :

$$\tilde{R} \wedge q = q \wedge \tilde{R} \quad -(9)$$

because : $p = 2, q = 1 \quad -(10)$

in eq. (8)

Therefore the light-like null geodesic condition is equivalent to :

$$\boxed{\tilde{R} \wedge q = -\omega \wedge \tilde{T}} \quad -(11)$$

Restricting attention to the Z axis eq (11)'s relevant tensor elements are :

$$\boxed{R^0{}_3{}^{30} = -\omega^0{}_{30} T^{030}} \quad -(12)$$

This means that :

$$\underline{\nabla} \cdot \underline{g} = 2c^2 R^0{}_3{}^{30} \quad -(13)$$

3) and the effective g is doubled for light interacting with any mass M, as observed in NASA Cassini.

Comparing eqs. (4) and (12):

$$R^{0}{}_{3}{}^{30} = \frac{1}{g_{33} Z^2}, \quad -(14)$$

$$-\omega^{0}{}_{30} T^{030} = \frac{1}{g_{00} c^2 t^2}. \quad -(15)$$

Adopting the simplest possible spin connection

$$\omega^{0}{}_{30} = -\frac{1}{Z} \quad -(16)$$

as in previous work on the Coulomb law (paper 63)

$$T^{030} = \frac{Z}{g_{00} c^2 t^2} = Z R^{0}{}_{3}{}^{30} \quad -(17)$$

so

$$\boxed{R^{0}{}_{3}{}^{30} = \frac{1}{g_{33} Z^2}, \quad T^{030} = \frac{1}{Z g_{33}}, \quad \omega^{0}{}_{30} = -\frac{1}{Z}.}$$